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Bayesian new hierarchical Laplace prior distributions of Tobit Quantile Regression

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Abstract: In current paper, we introduced a new method linked with a new hierarchical Laplace prior distributions via scale mixture of Uniform distribution mixing with standard exponential distribution . This mixture to Laplace prior distributions provide us attractive algorithm ,it have a good features to being efficiency to achieving variables selection and coefficient estimations in Tobit quantile regression model . simulation examples and real data set are employed to evaluation our method with Bayesian and non-Bayesian methods in variable selection filed . Both simulation approach and real data shown the our proposed method have best performance compared with other methods .

Key Words: Tobit quantile regression, hierarchical Posteriors distribution, a new hierarchical prior , MCMC algorithm,

1-Introduction

The classical Tobit regression model (TRM) that is effected by a set of assumption. Such as, it is very sensitive to outlier value , This means the (TRM) doesn't robust against outlier value. (Wooldridge, J. (2002)). Also,

it is sensitive to heteroskedastic error term (Long, J. S., & Ervin, L. H. (2000)). Also, TQR is affected when the error distributions are violates normal error condition. From last speech , T R M does not robust with econometric problems and assumptions random error breakthrough (Sune Karlsson, A. (2014)). To overcome all these problems Tobit quantile regression model TQRM have been used. TQRM considers a good statistical tool for estimating the relationship between response variable and a set of independent variables with infinity quantile levels. Powell (1986)), first introduced T QR M, it is known in all application sciences. such as Medical Sciences, Astronomical sciences and econometrics, etc. TQRM estimated its parameters via many estimation methods that is introduced by many researchers , for example Hahn (1995)) , (Buchinsky and Hahn (1998)), (Bilias et al. (2000)), (Chernozhukov and Hansen (2008)), etc.

recently, the variables selection (VS) approaches have been proposed, this technique provide us great solutions for excluding weak explanatory variables from our model for a best explanation. Because VS has a high quality for building regression models. Recently, some researchers combine the variable selection method with regression models. Such us , least absolute shrinkage and selection operator (lasso) proposed by Tibshirani, (1996) , Smoothy cliffed Atsolut Deviation (SCAD) proposed by(Fan and Li, (2001)) and elastic net approach proposed by (Park and Casella, (2008)) the above methods are combined with classical regression model . Many researchers are extended these method with TQRM via Bayesian approach such as, (Alhamzawi, (2013)) is proposed Bayesian adaptive lasso in T Q R M .Also, (Alhamzawi and Yu, (2014)), proposed a Bayesian g-prior distribution technique with T Q R M. Also, (Alhamzawi, (2014)) proposed Bayesian elastic net penalty in T Q RM. ALheseini fadel((2017)), proposed Bayesian composite TQRM). Also ALheseini fadel((2017)) proposed Bayesian new lasso in TQRM. And also (ALheseini fadel et al. (2020)), (Remah Oday and Fadel Al-Hussaini (2021)) are introduced a new scale mixture of uniforms distribution mixing with standard exponential distribution on with variances in quantile regression model via Bayesian method . All methods that mentioned above focus on scale mixture distribution for Laplace distribution. Because this procedure proved us constructing efficient MCMC algorithm for variable selection and coefficient estimation in TQRM. In current paper , we extended a new formulation of Laplace distribution (scale mixture of uniforms distribution mixing with standard exponential distribution) in tobit quantile regression via Bayesian approach. This paper is orderly via five sections . In first section we focused some concepts in TR . Second section concept TQRM have been presented . Third section concentrate on Hierarchical Prior distribution. Four section concentrate Hierarchical full posteriors distribution. The simulation example and real data have been shown in fifth sections. The Conclusions and recommendations have been shown in six sections

2-Concept Tobit Regression Model

Since seminal work of James Tobin (1958) left censored regression model(Tobit model) is became very known in many applied Sciences. The Tobit model is a good tool for censored data at zero point. This model defined according mathematical function as follows:

$$y_i = \begin{cases} y_i^* = \alpha + \beta x_i^T + \epsilon_i & if\ y_i^* > 0 \\ 0 & if\ y_i^* \leq 0 \end{cases} \quad \dots\dots\dots \quad [1]$$

where, y_i^* is the latent variable of TR, x_i^T is 1×p a vector of the independent variables, α is intercept term β are a vector unknown coefficients of TR model, ϵ_i is a random error term distributed with normal distribution by mean 0 and variance (σ^2).Therefore, the latent variable y_i^* is distributed normal with the mean $(\alpha + \beta x_i^T)$ and the variance (σ^2).

From equation (1), the T R model is deal with latent variabley_i^*. The latent variable y_i^* is observed $if\ y_i^* > 0$. and the latent variable doesn't observed $if\ y_i^* \leq 0$. The latent variable y_i^* is distributed normally with the mean $(\alpha + \beta x_i)$ and the variance (σ^2), where $y_i^* \sim N(\alpha + \beta x_i, \sigma^2)$. The probability density function belongs to latent variable y_i^* at $y_i = y_i^*$ $if\ y_i^* > 0$ is:

$$f(y_i) = \frac{1}{\sqrt{2\pi\sigma^2}}\ e^{\frac{-(y_i - \hat{y}_i)^2}{2\sigma^2}} \quad \text{, where } \hat{y}_i = \alpha + \beta x_i), \quad \dots[2]$$

We can rewrite the equation (2) as follow, $f(y_i) = \frac{1}{\sigma}\phi\left(\frac{y_i - (\alpha + x_i^T\beta)}{\sigma}\right)$,
The shape of probability density function (pdf) of normal distribution

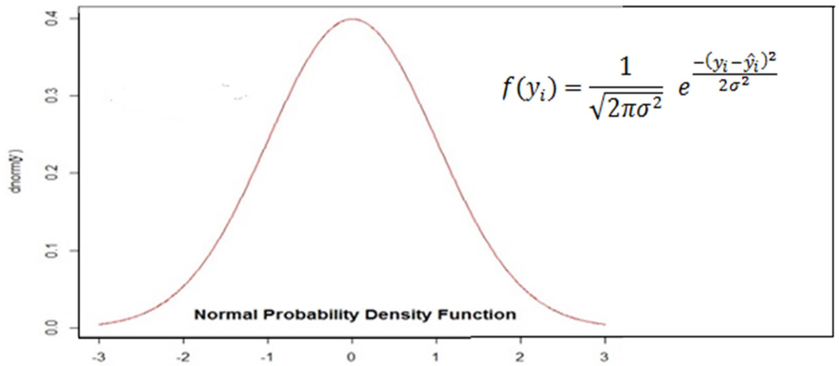

Figure 1. shown the shape of the normal probability density function

The part unobserved deal with cumulative distribution function for the normal distribution.

4

$$pro(y_i = 0) \quad if \quad pro(y_i^* \leq 0) \longrightarrow \Phi\left(\frac{y_i - \hat{y}_i}{\sigma}\right) = \Phi\left(\frac{0 - \hat{y}_i}{\sigma}\right)$$
$$= \Phi\left(\frac{-\hat{y}_i}{\sigma}\right) = 1 - \Phi\left(\frac{\hat{y}_i}{\sigma}\right) \quad \ldots\ldots \quad [3]$$

$\Phi(.)$ is cumulative distribution function (cdf). As flowing figure

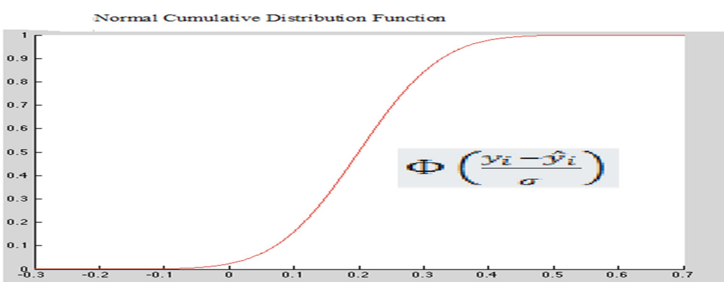

Figure 2. shown the shape of the normal cumulative distribution function

The T R model in equation (1) is contain observe function and unobserved function , see equations (2) and (3) respectively. The T R model is mixture between normal probability density function and normal cumulative distribution function, as follows:

$$p(y_i) = \left[\frac{1}{\sigma}\phi\left(\frac{Y-(\alpha+x_i^T\beta)}{\sigma}\right)\right]\left[1-\Phi\left(\frac{(\alpha+x_i^T\beta)}{\sigma}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots \quad [4]$$

We can estimate the coefficients of Tobit regression model by many estimation method.

3-Concept Tobit Quantile Regression Model

The Tobit regression model is very sensitive to violation some normal assumption to overcoming this problem tobit quantile regression model (TQRM)have been employed . The TQRM can cover all regression area ,because TQRM can estimate infinity from Tobit regression lines, when $0 < \tau < 1$, τ is Tobit quantile level . At each Tobit quantile levels, there is TQRM. The TQRM defined as follows

$$y_i = \begin{cases} y_i^* = \alpha_\tau + \beta_\tau x_i^T + \epsilon_i & if\ y_i^* > 0 \\ 0 & if\ y_i^* \le 0 \end{cases} \quad \dots\dots\dots \quad [5]$$

We can rewrite equation (4) another formula as follows:

$$y = \max(0, y_i^*) \quad \dots\dots \quad [6]$$

$$y_i^* = \alpha_\tau + x_i^T \beta_\tau + \epsilon_i,$$

where , α_τ is intercept term , β_τ are vector unknown parameters of TQRM , and τ is Tobit quantile level belonging to the open interval (0,1). We can estimated of coefficients of TQRM via minimizing the following loss function.

$$\min_{\alpha_\theta, \beta_\theta} = \sum_{i=1}^{n} \rho_\tau \left(y_i - max\{0, y_i^*\}\right) \dots\dots\dots \quad [7]$$

The loss function in equation (6) is not differentiable (0) point, see (Koenker, (2005)). (Koenker and D'Orey, (1987)) Show the minimization of the equation [6] by a linear programming approach. Yu and Stander, (2007)) and proposed the Bayesian approach for estimating in TQRM. Yu and Moyeed (2001and others researchers saw the loss function in equation (6) very closed from asymmetric Laplace distribution (ALD), from this result,

y_i belong to ALD with (pdf), as follows :

$$(y|X, \alpha, \beta, \tau) = \tau^n (1-\tau)^n exp\left\{-\sum_{i=1}^{n} \rho\tau(y_i - max\{0, \alpha_\tau + x_i^T \beta_\tau + \epsilon_i\}\right\} \dots\dots \quad [8]$$

The maximizing of the likelihood function in equation (7) is equivalent to minimizing loss function in equation [6]. Many researchers doesn't focus on equation (7) directly, Because it provide as hard MCMC algorithm. Therefore, Most of the researchers in field of variables selection focus on (Kozumi and Kobayashi, (2011)) proposition. Summary of this proposition the likelihood function in equation [7] is possible to being as in the follows.

$$y_{i=max\{0, y_i^*\}}, \quad i=1,\dots,n,$$

$$y_i^* | \alpha_\tau, \beta_\tau, m_i \sim N(\alpha_\tau + x_i^T \beta_\tau + (1-2\tau)m_i, 2m_i) \quad \dots\dots \quad [9]$$

The probability density function ($f(y_i^* | \alpha_\tau, \beta_\tau, m_i)$) is

$$f(y_i^* | \alpha_\tau, x_i^T, \beta_\tau, m_i) = \frac{1}{\sqrt{4\pi m_i}} e^{-\frac{\left(y_i^* - \alpha_\theta - x_i^T \beta_\theta - (1-2\theta)m_i\right)^2}{4m_i}} \quad -\infty \le y_i^* \le \infty \quad \dots,, \quad [10]$$

The likelihood function of ($f(y_i^* | \alpha_\tau, \beta_\tau, m_i)$) is

$$f(y_i^*|\alpha_\tau, x_i^T, \beta_\tau, m_i) = \left[\frac{1}{\sqrt{4\pi m_i}}\right]^n e^{-\sum_1^n \frac{\left(y_i^* - \alpha_\tau - x_i^T\beta_\tau - (1-2\tau)m_i\right)^2}{4m_i}} \quad .... \quad [11]$$

The equation (10) is very important for achieving Bayesian coefficient estimation of TQRM.

4-Hierarchical Prior distribution

Tibshirani, (1996) is give note for the researchers who work with Bayesian variable selection filed, the prior distribution is Laplace distribution. The Laplace probability density function can be written as:

$$f(x) = \frac{\lambda}{2}\exp(-\lambda|x|) \quad \dots\dots\dots. \quad [12]$$

The Laplace prior onβ_τ takes the form $f(\beta_\tau) = \frac{\lambda}{2}\exp(-\lambda|\beta_\tau|)$. But estimation with Laplace prior distribution directly is be very hard to obtain a good MCMC algorithm. Therefore, many researchers are used transformation of Laplace prior distribution see (Andrews and Mallows (1974)) ,they can reformulation Laplace prior distribution from a two parts. The first part belong to prior distribution of β_τ parameters which it is distributed standard normal distribution. The second part s_j is distributed exponential prior distribution. Mallick and Yi, (2014)) are reformulated a Laplace prior distribution of β_τ from two parts first belong the uniform distribution and second part belong to Gamma distribution when $\lambda = 2$.

Remah Oday and Fadel Al-Hussaini 2021 used another transformation of the Laplace prior density as scale mixture of Uniform distribution mixing with standard exponential distribution. in current paper ,we will used mixture between Uniform distribution mixing with standard exponential distribution with TQRM.

Hierarchical Full Posteriors Distribution

We will know, the Bayesian framework is focus on The likelihood function and hierarchical prior distribution to getting hierarchical Posteriors distribution . The conditional posterior distributions are defined mathematical formula:

$$f(\beta_\tau|y) = \frac{f(y,\beta_\tau)}{f(y)} = \frac{f(y|\beta_\tau)*H(\beta_\tau)}{f(y)} =\propto f(y|\beta_\tau)\, H(\beta_\tau). \quad \dots\dots \quad [13]$$

where:$\propto$ is proportional

$H(\beta_\tau)$ is hierarchical prior distributions and $f(y|\beta_\tau)$ is the likelihood function ,$f(\beta_\tau|y)$ is the hierarchical posterior distributions . The figure 3, show simple clarification about Bayes theorem.

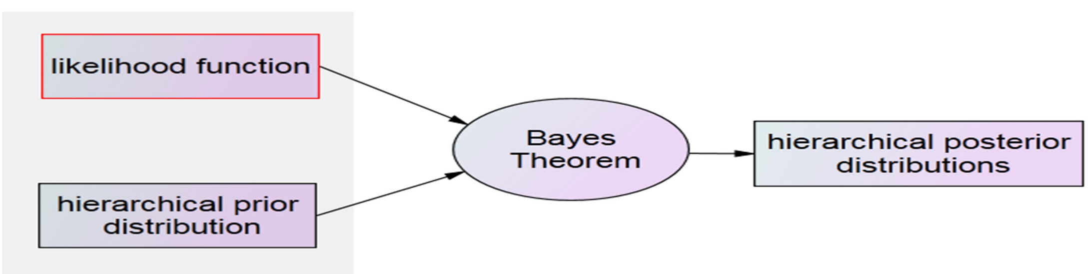

Figure -3- Show Bayes Theorem

Bayesian approach is played good role for parameters estimation in regression models even when the sample size is small. Also Bayesian approach makes updating the parameters via prior distributions in a dynamic (Draper and Smith (1981)). The hierarchical posterior distribution is important part for estimation the parameters by Bayesian approach. In this paper , we used the likelihood function shown in equation (10) and a new hierarchical Laplace prior distributions that it is scale mixture of Uniform distribution mixing with standard exponential distribution. From this parts, we will obtained hierarchical posterior distributions. The full posterior distributions of variable (y) is distributed normal distribution with mean $(x_i^T\beta_\tau-(1-2\tau)m_i)$ and variance $(2m_i)$. The parameter m_i is distributed General Inverse Gaussian with rate parameter $(\frac{(y_i^*-x_i^t\beta_\tau)^2}{2\tau})$, and shape parameter $\left(\frac{(1-\tau)^2}{\tau^2}+2\right)$. The parameter δ_j is distributed truncated standard exponential . The parameter β_τ is distributed of multivariate normal posterior with mean $\left(\sum_{i=1}^{n}\frac{x_i(x_i\beta_\tau+\tau m_i)}{(1-\tau)m_i}\right)$ and variance $(\sum_{i=1}^{n}\frac{x_ix_i^t}{\tau m_i})$. The parameter $\boldsymbol{\lambda_j}$ is distributed truncated gamma distribution. The above Gibbs sampler is efficient and simple algorithm for estimation and variables estimation to TQRM.

Simulation approach

The our proposed method(New Lasso T .Q) is consider a good method in field of variables selection, it give us efficient and simple MCMC algorithm, to prove this claim the simulation examples and real data set have been used. New Lasso T .Q is compared with three last methods in the same field. First method (crq) is introduced by Powell, (1986)) with R package proposed by (Koenkers, (2011). The second method is that proposed by Alhamzawi, (2014) name it BAnet . Third method is introduced by fadel alhuseini (2017) that name it (new B L Tobit Q Reg) . In current study, we will used two criteria, the Root Mean Square Error symbolizes it (RMSE) and median of mean absolute deviations symbolizes it (MMAD). The RMSE is calculated by the following mathematical function $RMSE(\beta,\hat{\beta})=\sqrt{E\left[(\hat{\beta}-\beta^T)(\hat{\beta}-\beta^T)^t\right]}$. The MMAD is calculated by the following mathematical function MMAD $=median\left(mean(\left|x^t\hat{\beta}-x^t\beta^T\right|)\right)$, where β^T is true parameters and $\hat{\beta}$ is estimated parameters. The true model used in generation the data via the following model:

$$y_i=\begin{cases} y_i^* & ,if\ \ y_i^*>0 \\ 0 & ,if\ \ y_i^*\le 0\end{cases} \quad . \qquad ,$$

$$y_i^*=x^t\beta_\tau+\epsilon_i \qquad ,i=1,2,3,\ldots\ldots.100$$

where y_i is the response variable ,and y_i^* is the latent variable , x^t explanatory variables generated from a multivariate normal distribution with mean zero and $cov(x_i,x_j)=0.5^{|i-j|}$. β_τ is unknown vector of parameters .$\epsilon_i, i=1,\ldots\ldots\ldots,100$ is random error term. In current study ,it are generated from three different error term distributions:a $\epsilon_i\sim N(3,2)$, normal distribution with mean 3 and variance 2, $\epsilon_i\sim\chi^2_{(4)}$, a chi-square distribution with four degrees of freedom,a, $\varepsilon_i\sim Laplace(1,1)$,
Laplace distribution with location parameter 1 and scale parameter 1. In this simulation examples three quantile levels have been used (first law quantile level $\tau=0.25$, second intermediate quantile level $\tau=0.55$ and third high quantile level $\tau=0.95$)
In current study two simulation examples have been used :

1- Simulation 1 (sparse vase): $\beta=(1,0,1.5,0,0,0,1,0)^t$
where $y_i^*=x_{1i}+1.5x_{3i}+x_{7i}+\epsilon_i$

2- Simulation 2 (dense case): $\beta=\underbrace{(0.85,\ldots.,0.85)^t}_{8}$

where $y_i^*=0.85x_{1i}+0.85x_{2i}+0.85x_{3i}+0.85x_{4i}+0.85x_{5i}+0.85x_{6i}+0.85x_{7i}+0.85x_{8i}+\epsilon_i$

We run the algorithm MCMC algorithm 13000 iterations first 3000 iterations exclude-in .

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Table .1. The root mean square error (RMSE) and median of mean absolute deviations (MMADs) for the our simulation examples

	Methods	$\epsilon_i \sim N(3,2)$,	$\epsilon_i \sim \chi^2_{(4)}$	$\varepsilon_i \sim Laplace(1,1)$
Sim.1	$\text{crq}_{\tau_1} = 0.25$	1.762 (0.871)	1.591 (0.954)	0.956 (0.768)
	$\text{crq}_{\tau_2} = 0.55$	1.653 (0.892)	1.653 (0.835)	0.892 (0.682)
	$crq_{\tau_3} = 0.95$	1.622 (0.946)	1.577 (0.793)	0.892 (0.788)
	$BAnet_{\tau_1} = 0.25$	0.972 (0.682)	0.845 (0.693)	0.792 (0.574)
	$BAnet_{\tau_2} = 0.55$	0.787 (0.575)	0.755 (0.654)	0.788 (0.564)
	$BAnet_{\tau_3} = 0.95$	0.877 (0.687)	0.877 (0.745)	0.893 (0.677)
	$\text{New B L Tobit Q Reg}_{\tau_1} = 0.25$	0.865 (0.646)	0.803 (0.563)	0.782 (0.564)
	$\text{New B L Tobit Q Reg}_{\tau_2} = 0.55$	0.723 (0.641)	0.797 (0.571)	0.788 (0.609)
	$\text{New B L Tobit Q Reg}_{\tau_3} = 0.95$	0.845 (0.623)	0.858 (0.653)	0.725 (0.594)
	$\text{New Lasso T . Q}_{\tau_1} = 0.25$	0.535 (0.363)	0.564 (0.369)	0.589 (0.377)
	$\text{New Lasso T . Q}_{\tau_2} = 0.55$	0.512 (0.343)	0.472 (0.308)	0.573 (0.387)
	$\text{New Lasso T . Q}_{\tau_3} = 0.95$	0.428 (0.320)	0.493 (0.376)	0.532 (0.374)
Sim.2	$\text{crq}_{\tau_1} = 0.25$	1.241 (0.845)	1. 459 (0.947)	1.364 (0.944)
	$\text{crq}_{\tau_2} = 0.55$	1. 065 (0.861)	1.252 (0.927)	1.257 (0.895)
	$crq_{\tau_3} = 0.95$	1.156 (0.827)	1.179 (0.822)	1.468 (0. 858)
	$BAnet_{\tau_1} = 0.25$	0.966 (0.789)	0.963 (0.755)	0.867 (0.648)
	$BAnet_{\tau_2} = 0.55$	0.918 (0.795)	0.926 (0.701)	0.869 (0.718)
	$BAnet_{\tau_3} = 0.95$	0.890 (0.711)	0.909 (0.689)	0.822 (0.726)
	$\text{New B L Tobit Q Reg}_{\tau_1} = 0.25$	0.855 (0.692)	0.799 (0.619)	0.803 (0.697)
	$\text{New B L Tobit Q Reg}_{\tau_2} = 0.55$	0.724 (0.528)	0.734 (0.547)	0.747 (0.594)
	$\text{New B L Tobit Q Reg}_{\tau_3} = 0.95$	0.705 (0.512)	0.711 (0.511)	0.693 (0.537)
	$\text{New Lasso T . Q}_{\tau_1} = 0.25$	0.594 (0.430)	0.583 (0.396)	0.585 (0.385)
	$\text{New Lasso T . Q}_{\tau_2} = 0.55$	0.466 (0.395)	0.437 (0.385)	0.486 (0.296)
	$\text{New Lasso T . Q}_{\tau_3} = 0.95$	0.424 (0.294)	0.439 (0.269)	0.395 (0.295)

Note: In the parentheses are RMSE

From the results listed in above table ,we can see. Generally in two simulation examples, we see the RMSE is generated by our proposed method (New Lasso T . Q) much smaller than the RMSE is generated by other methods (crq, *BAnet and* New B L Tobit Q Reg . Also, MMAD is generated by our proposed method (New Lasso T . Q) much smaller than the MMAD is generated by other methods (crq, *BAnet and* New B L Tobit Q Reg. Therefore , the our proposed method have a good performance for variables selection and parameters estimation compared with other methods, for all quantile levels and error distributions under consideration.

Instead of focus at the RMSE and the MMAD, the coefficients estimation of our model under studied in direct way have been used via the following figures for second simulation example for all quantile levels and error distributions under consideration.

From the figure -1- ,the black line is belong the true parameters , but the green line is belong to coefficient estimated by our proposed method . The rest line belong to other methods. From the below figure , we see clearly the green line is very closed from black line .This mean , the coefficient estimated by our proposed method (New Lasso T . Q) is very closed from true parameters . Therefore, the our proposed method have a good performance compared with other method . We see from the below figure the green line that belong to our proposed is very closed from of black line that belong true parameters

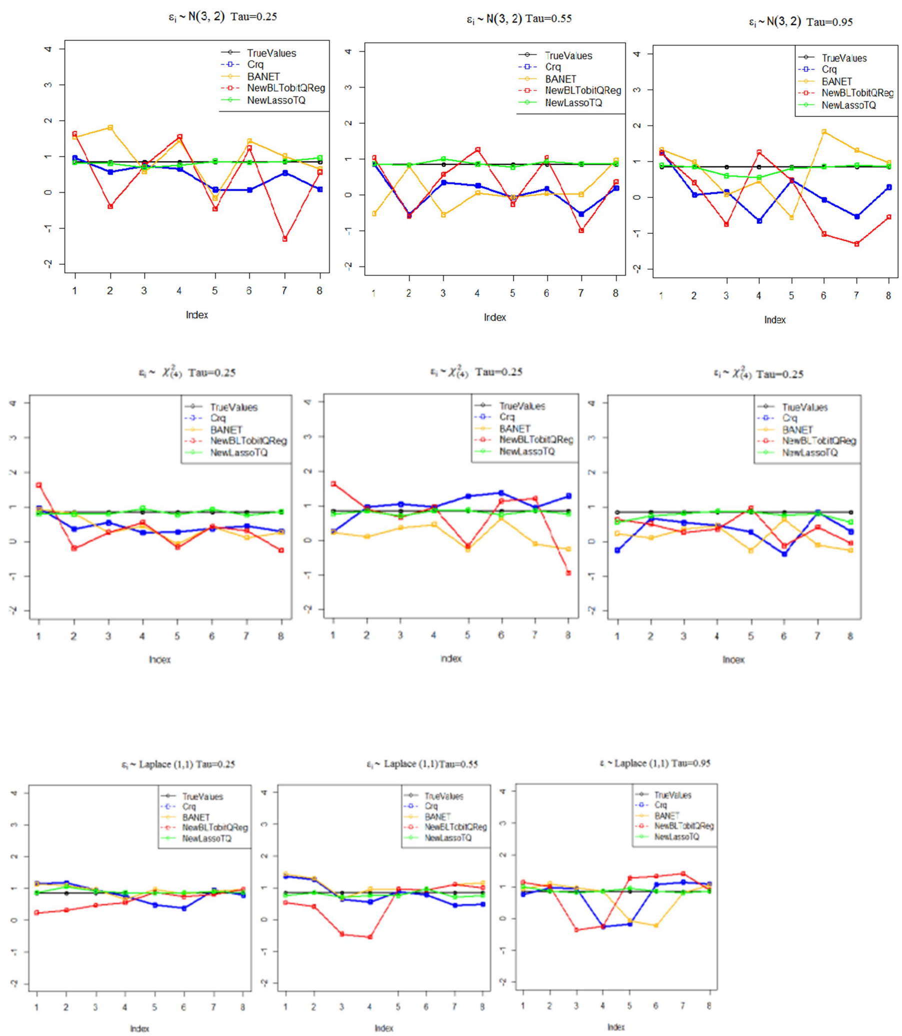

Figure -1- show the parameters estimated via our proposed method and other methods for second simulation example for all quantile levels and error distributions under consideration.

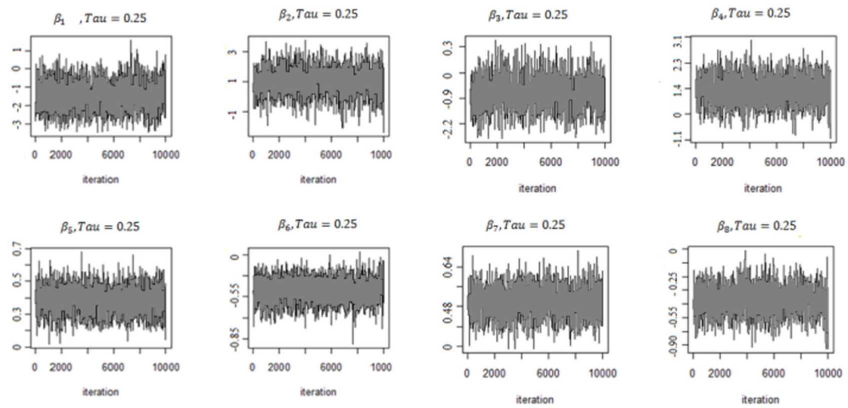

Figure 2. Trace plots with (0.25) quantile level

Figure 2. displayed the trace plots for second simulation example with Tau=0.25 , which are very stationary via all iterations. Therefore, the Gibbs sampling algorithm is efficient to implement and, it is simple .

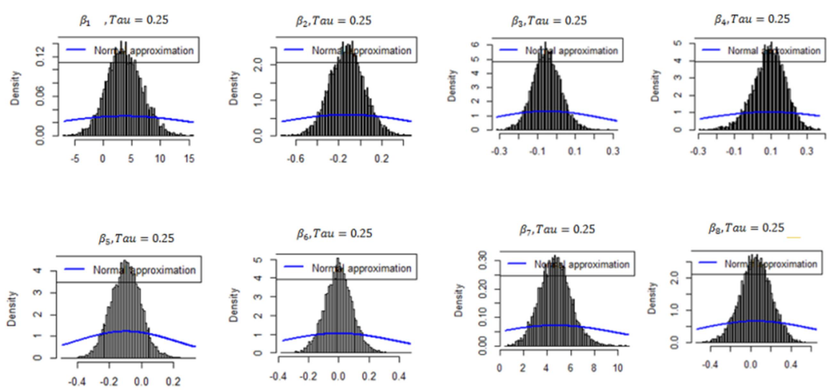

Figure 3. Histograms of parameter estimates with (0.25) quantile level

Figure 3. displayed the histograms for second simulation example with Tau=0.25 , the histograms of parameters estimated are very closed from normal distribution .

Real data set

In this section, we will study the factors that effected on abortion. This phenomenon is censored at zero. The sample size of current study is 200 observations, 90 observations are from the left- censoring at zero point by percentage 45% from original real dataset. The rest observations (110) are non-censoring at zero point. The real data set contain one response variable (number of abortions),and set of independent variable are displayed in below table

Table .2 show the name and symbol independent variables

i	Name variable	Symbol variable
1	Mother blood group	x_1
2	Mother blood sugar	x_2
3	Mother weight	x_3
4	sequences of birth	x_4
5	Mother age at birth	x_5
6	Mother blood hemoglobin	x_6
7	Mother blood platelet	x_7
8	Mother white blood cells	x_8

The data are collected from Al-Muthanna birth Hospital. We will employed this real data to assess these methods under study.Via employing the Mean Square Error (MSE) and the standard deviation (S.D)as shown in the table below.

Table .3 The mean square error (MSE) and standard deviation (S.D) for real data

	Methods	MSE	$S.D$
Real Data	$\mathrm{crq}_{\tau_1} = 0.25$	2.563 (1.683)	2.493 (1.542)
	$\mathrm{crq}_{\tau_2} = 0.55$	2.485 (1.782)	1.723 (1.534)
	$crq_{\tau_3} = 0.95$	2.144 (1.376)	1.477 (1.562)
	$BAnet_{\tau_1} = 0.25$	1.734 (0.892)	1.643 (0.836)
	$BAnet_{\tau_2} = 0.55$	1.573 (0.784)	1.466 (0.742)
	$BAnet_{\tau_3} = 0.95$	1.462 (0.764)	1.453 (0.792)
	$\text{New B L Tobit Q Reg}_{\tau_1} = 0.25$	0.956 (0.706)	0.929 (0.763)
	$\text{New B L Tobit Q Reg}_{\tau_2} = 0.55$	0.945 (0.523)	0.763 (0.464)
	$\text{New B L Tobit Q Reg}_{\tau_3} = 0.95$	0.827 (0.467)	0.737 (0.434)
	$\text{New Lasso T.Q}_{\tau_1} = 0.25$	0.646 (0.442)	0.691 (0.461)
	$\text{New Lasso T.Q}_{\tau_2} = 0.55$	0.610 (0.411)	0.541 (0.471)
	$\text{New Lasso T.Q}_{\tau_3} = 0.95$	0.561 (0.367)	0.518 (0.417)

We see the MSE is generated by our proposed method (New Lasso T . Q) much smaller than the MSE is generated by other methods (crq, $BAnet$ and New B L Tobit Q Reg . Also, S.D is generated by our proposed method (New Lasso T . Q) much smaller than the S.D is generated by other methods (crq, $BAnet$ and New B L Tobit Q Reg. Therefore , the our proposed method have a good performance for variables selection and parameters estimation compared with other methods, for all quantile levels until with real dataset.

Conclusions and recommendations

Conclusions

The main conclusion of this paper, we introduced new method of the Bayesian regularization in Tobit quantile regression analysis. Via we employed new formulation to Laplace distribution and mixing it with Tobit quantile regression model. We see the our proposed method (New Lasso T . Q) have good performance compared with other methods in same filed, especially with high quantile level. Also , we find from the a real data set our proposed method (New Lasso T . Q) superior compared with other method.

Recommendations

we recommend the used of the proposed Gibbs sampler model under a new scale mixture with kind of regression linear and non-linear models, such as, composite quantile regression , Binary quantile regression , etic. Also, we recommend to use our proposed method in other fields, such as medicinal field , biological filed and economic filed, etc .

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