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Robust sparse sliced inverse regression with application

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بِسۡمِ ٱللَّهِ ٱلرَّحۡمَٰنِ ٱلرَّحِيمِ

{وَالرَّاسِخُونَ فِي الْعِلْمِ يَقُولُونَ آمَنَّا بِهِ كُلٌّ مِّنْ عِندِ رَبِّنَا وَمَا يَذَّكَّرُ إِلَّا أُولُو الْأَلْبَابِ}

صدق الله العظيم

آل عمران : الآية (7)

List of abbreviations

Adaptive Lasso	Adaptive Least absolute shrinkage and selection Operator
Ad EN	Adaptive Elastic net
AIC	Akaike information criterion
Ave 0's	Average number of zero coefficients
BIC	Bayesian Information Criterion
CD	Curse of Dimensionality
DR	Dimension Reduction
EN	Elastic Net
HD	High Dimensional
i.i.d	Independent Identically Distributed
Lasso	Least Absolute Shrinkage and Selection Operator
MAVE	Minimum Average Variance Estimator
MCP	Minimax concave penalty
MSE	Mean Squared Error
OP's	Oracle Properties
RSIR	Regularized Sliced Inverse Regression
SAVE	Sliced Average Variance Estimator
SCAD	Smoothly Clipped Absolute Deviation
SD	Standard Deviation
SDR	Sufficient Dimension Reduction
SIR	Sliced Inverse Regression
SIR- Lasso	Sparse Sliced Inverse Regression with Lasso penalty
SPSIR	Sparse Sliced Inverse Regression
SSIR	Shrinkage Sliced Inverse Regression
SSIR-EN	Sparse Sliced Inverse Regression with Elastic Net penalty

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Dedication

*In the name of Allah most gracious most merciful *

* To my supervisor prof.DR. Ali Alkenani.

*To my respected teachers.

*To my husband (Haider Abed Albaqer) I want to tell you that you are my support after god you make me laugh, wipe my tears and keep me strong no matter what .

*To my husband family thanks for everything (my uncle &aunt).

*To my parents and my beloved children ((Baqer & juri)).

*To my colleagues & friends .

*This Work is dedicated to them.

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Abstract

In regression applications, the sliced inverse regression(SIR) is a method for reducing the dimensions without losing any information about the regression. Although, the SIR has been proven as an efficient method to deal with the high dimensionality problems, but it suffers that it gives directions contains all the original predictors. Many researchers suggested approaches to dealing with this problem by combining variable selection methods with SIR. One of these methods combined the SIR method with Elastic Net penalty(SIR-EN). The SIR –EN is an efficient method without assuming a parametric model. It produces accurate and sparse solutions when the predictors are highly correlated under sufficient dimension reduction settings. However, the SSIR-EN is not robust to outliers because of the method use the loss function which is sensitive to outliers in data.

As a result, we suggested RSSIR-EN as a robust version of SSIR-EN for outliers in both the dependent variable and the independent variables.

Chapter one

(Introduction, Reduce Dimensionality methods and literature review)

1-Introduction

Due to the explosion of large information in the past decades, high-dimensional regression analysis problems appear in several applications. The sufficient dimension reduction(SDR) theory has received great attention in high – dimensional regression. The basic idea of SDR is to exchange X with d-dimensional orthogonal dropping PsX onto S, where d<p and p is a number of predictors, without losing information about the conditional distribution of Y|X and without assuming any parameter pattern. Assume Y is a response variable, $X = \left(X_1, X_2, \ldots., X_p\right)^T$ is a predictors vector. SDR aims to find the central subspace $S_{Y|X}$and $S_{Y|X}$ is the intersection of all subspaces S that achieve $Y \perp\!\!\!\perp X|P_s X$, when $\perp\!\!\!\perp$ it indicates independence. Therefore, $P_\beta X$ extracts that information from X to Y, where β is the basis of $S_{Y|X}$(Cook, 1998).There are several suggested methods to find $S_{Y|X}$. One of the most important of these methods which has proven to be effective in dealing with high dimensions is SIR method (Li, 1991). SIR is one of the useful tools, which help to solve the problem of the "high dimensions". It is applied in different fields, including economics and bioinformatics. The results of SIR are linear sums of all the original variables, which may cause difficulty in interpreting the results of SIR. For this reason, there a need to reduce the number of non-zero coefficients in the SIR directions. Many researchers suggested approaches to dealing with this problem by combining variable selection methods with SIR. Alkenani (2021) proposed RSIR-Lasso method that does not have the ability to select groups of highly correlated predictors. Alkenani and Hassel (2020) proposed SIR-EN method which deals with correlated predictors but this method sensitive to outliers and are not robust because the method uses the least squares loss

function which is sensitive to outliers in data . It is necessary to deal with this problem and solve this problem. The squared loss criterion is used between the covariates and response. Also, the classical estimates of the sample mean and the sample variance of X is used within the least squares formula. These are all sensitive to outliers and are not robust. In this research, we proposed robust method of SIR method with Elastic Net(EN) by using Tukey biweight criterion instead the squared loss criterion. If the derivative of the loss function is descending, the loss function is robust and insensitive to outliers in X and Y (Rousseeuw and Yohai, 1984). Tukey biweight function has this property.

The problem of the study

The curse of dimensions is a problem for most statistical methods. Also, regression analysis in some applications becomes very difficult because of high dimensional data. The problem occurs when the dimensions increase so quickly that the data becomes sparse. Alkenani (2021) proposed RSIR-Lasso method that does not have the ability to select groups of highly correlated predictors. Alkenani and Hassel (2020) proposed the SIR-EN method, which deals with the correlated predictors, but this method is sensitive to outliers and is not robust. It is necessary to deal with this problem and solve this problem by dimension reduction.

The aim of the study

This study aims to develop the SIR-EN(Alkenani and Hassel, 2020) method that is sensitive to outliers in data and not robust. Therefore, we proposed Robust sparse sliced inverse regression with Elastic Net (RSSIR-EN) as a robust version of SIR-EN, which is a robust method for outliers in X and Y under the SDR settings .

1-Variable Selection

One of the important methods in selecting predictors that used in building a multiple regression model, for the case of high- dimensional data that contains important and unimportant variables, in which we define sub-sets of the original set of predictors by entering them into the building model. That gives a clear interpretation of the data, through providing information for the important variables to obtain statistical models, that contain the least number of important variables, which are of high accuracy for prediction pattern. Also saves money and time (Jabbar and Alkenani, 2020).

There are many methods of V.S that proposed by researchers, which are classified as follows:

Variable selection	**Classical methods**	**Step wise selection procedure**
		Backward elimination procedure
		Forward seleection
		Akaike Information Criteria
		Bayesian Information Criterion
	Regularized methods	**Lasso**
		Smoothly Clipped Absolute Deviation
		Adaptive Lasso
		Elastic Net
		Adaptive Elastic Net
		Minimax Concave Penalty

The mentioned methods were prepared by the researcher and are part of large number of variable selection methods.

2-Variable Extraction

Variable Extraction is the process of transforming (projection) variables to less number of variables. Variable Extraction(V.E) shares the objectives of V.S. Also, the major difference is that the results should be determined in terms of all variables. It also refers to the process of finding the transformation that is projecting data from the original space to the feature space. This approach tries to enable the picturing of data by minimizing vector X from the P-dimension predictor without loss of information.

There are many methods of V.E that suggested by a number of researchers that can be classified according to the following:-

Vraiable Extraction	**Sufficient Dimension Reduction**	**Central Sapspace**	**SIR** **SAVE** **PHD** **IPHD**
		Central Mean Sapspace	**MAVE** **OPG**
	Dimension Reduction	**Factor Analysis**	
		Principal Component Analysis	
		Single Index Models	
		Canonical Correlation Analysis	

3- Methods for variable selection

V.S is an important approach that enables the researchers to save money, make the interpreting of the results easy and a low-cost model (Guyon and Elisseeff, 2003).

3-1-Classical methods

These methods are important, but they have disadvantages including instability, work discrete and takes a long time. Classical methods such as step-wise selection (Efroymson, 1960), Backward elimination procedure, Forward selection, Akaike information criterion (AIC) (Akaike, 1973) and Bayesian information criterion (BIC) (Schwarz,1978).

3-1-1-Stepwise selection procedure:

This method can be considered an evaluation of the forward selection method. It was suggested by (Efroymson, 1960) to develop its efficiency. The distinction between the two methods is that all the independent variables at the end of every step are checked by relying on the choice of (Fpartial); We re-assessed again because there are strong relationships between the independent variables are introduced in the previous steps. Thus, stepwise was considered the best approach to choosing a good regression equation.

3-1-2-Forward selection:

This method is based on starting without independent variables and the independent variables are chosen to be included in the equation one by one based on the comparison ($F_{partial}$) for each of the variables with a value ($F_{tabular}$). The maximum value is chosen ($F_{tabular}$) which is referred to as (F_{IN}), for each step and after making sure that the value is higher than (F_{IN}), the variable in question is entered into equation and the steps continue to show the independent variables one by one to the point of reaching the top ($F_{partial}$) less than (F_{IN}) according to the following formula:

$$F^* = \frac{SSR(x1\,)}{\frac{SSE(x1)}{n-2}}, \qquad (1\text{-}1)$$

where SSR: the sum of squares for the regression(the deviations shown),

SSE : the sum of squares for the errors(the unclarified deviations),

n: sample size.

3-1-3-Backward elimination procedure:

It is one of the easy methods to choose variables and can be applied easily. It begins with full model which takes into account all the independent variables in the equation and then the variables are deleting one after the other based on the value of ($F_{tabular}$), which is referred to as (F_{IN}).

The steps for this process are as follows:

1- Begin with all independent variable in the regression model and calculate the value of ($F_{i\,partial}$) for every variable depending on the formula below:

$$F_{i\,partial} = SSR\left[\frac{x_i}{all\ other\ explanatory\ variables}\right] / \frac{SSE(x_1,\ldots,x_k)}{n-k-1},\ \ i = 1,\ldots,p \quad (1\text{-}2)$$

Then, choosing a variable that has the minimum value of ($F_{partial}$) and comparing with (F_{IN}), when ($F_{partial}$) < (F_{IN}) the relevant variable deleting from equation and moving to the degree freedom (1) for numerator and (n-k-1) for denominator.

2- It includes all the independent variable except those omitted in the first step. ($F_{partial}$) included the remaining variables from step 1. Choose the smallest value from($F_{partial}$) and compare with (F_{IN}) to the degree of freedom of numerator (1) and denominator (n-k-2). If it proves that ($F_{partial}$) < (F_{IN}), deletes the variable in question and goes to step (3). This process continues until the lowest value ($F_{partial}$) > (F_{IN}) after which the process stops.

3-1-4-Akaike Information Criteria(AIC):

The AIC is introduced by (Akaike, 1973). It is a criterion that used to compare the quality of models and rank models from best to worst. AIC is a technique to selection good model. Also, the good model is selected through the model that contains lowest of AIC. It is done in the following from:

$$AIC(K) = -2Ln(L) + 2K, \qquad (1\text{-}3)$$

where K is number of model parameters,

L is the value of MLE.

3-1-5-Bayesian Information Criterion(BIC):

The BIC is a criterion of choosing the model from specific set of models that introduced by (Schwarz, 1978). The Bayesian Information Criterion is similar to the Akaike Information Criteria in choosing the models, but there is a difference that BIC including sample size which gives Bayesian Information Criterion an advantage over Akaike Information Criteria. The good model according to Bayesian Information Criteria is the model with the lowest BIC value.

$$BIC(K) = -2Ln(L) + K\ Ln(n), \qquad (1\text{-}4)$$

3-2-Regularization Methods:

The first method of regularization methods suggested by (Donoho and Johnstone, 1994). This approach can be defined as the procedure used to solve the model complexity problem. The model with high complexity tends to have high variance and low bias. Whereas a low complexity model tends to have low variance and high bias, thus regularization ways are often used to control model complexity by adding a penalty term to the loss function of OLS. In regularization methods, variable selection (V.S) is carried out through a parameter estimation process (Wang and Yin, 2008).Therefore, the advantages of this approach are that it is stable and works continuously in the least time. Regularization methods provide a tool with which we can improve the model's ability to interpret and predict accuracy. Examples of regularizations method are Lasso(Least absolute shrinkage and selection operator) proposed by (Tibshirani, 1996), SCAD(Smoothly Clipped Absolute Deviation) proposed by (Fan and Li, 2001), EN(Elastic Net) proposed by (Zou and Hastie, 2005), MCP(Minimax Concave Penalty) proposed by (Zhang, 2010) and others.

3-2-1-Lasso:

Lasso is one of the regularization methods that was suggested by (Tibshirani, 1996), Which is used to estimate parameters and select variables simultaneously in the model. Also, the Lasso estimator is adding l_1 norm to the least squares loss function. Moreover, In Lasso the residual sum squares has been minimized subject to $\sum_{j=1}^{p}|\beta_j|$ being less than a constant. Based on this condition, the Lasso shrinks some of it's coefficients and eliminates others by zeroing it's coefficients. Lasso formula:

$$\hat{\beta}(lasso) = argmin_{\beta} \sum_{i=1}^{n}(y_i - x_i^T\beta)^2 + \lambda \sum_{j=1}^{p}|\beta_j| \ , \qquad (1\text{-}5)$$

Where $\lambda \geq 0$ is the tuning parameter $j = 1, \ldots, p$.

The value of λ has an important role in how much weight you put to the penalty for the coefficients. This penalty reduces some coefficients value to zero.

3-2-2-Adaptive lasso:

A new version of Lasso that suggested by (Zou, 2006), he referred to it the Adaptive Lasso. Due to the estimation of Lasso are biased to large coefficients. Adaptive Lasso control the biasedness of the Lasso estimation where adaptive weights are used to penalize the various coefficients of the penalty l_1. Zou (2006) explained that this method has the oracle property. The adaptive Lasso can be resolved by a similar efficient equation that solves Lasso through adding weight. In the following equation:

$$\hat{\beta}(Adaptive\ Lasso) = argmin_{\beta} \sum_{i=1}^{n}(y_i - x_i^T \beta)^2 + \lambda \sum_{j=1}^{p} \widetilde{\omega}_j \left|\beta_j\right| , \quad (1\text{-}6)$$

where the tuning parameter is λ, j=1,…,p. Also, $\widetilde{\omega}_j$ is adaptive weight and $\widetilde{\omega}_j = \frac{1}{\left|\widetilde{\beta_j}\right|^{\delta}}$, $\tilde{\beta}$ is a non-penalized regression estimation and δ is contraction parameter.

3-2-2-Elastic Net(EN):

Elastic Net is a regression method that use for variable selection and estimation the parameters at same time. This method suggested by (Zou and Hastie, 2005). The method combines the Lasso and Ridge methods. Ridge method is used to deal with data in groups and when p > n and Lasso method is used for selection the important variables and discarding the unimportant ones by zeroing them. There are some cases in which the Lasso doesn't work and they are as follow:

1- In the case p > n, the Lasso choose nearly n variables.

2-If the set of variables is highly correlated, then lasso selects only one variable from the set and discarding the rest of variables.

In general, the EN is work in the case of variables in groups and the case of highly correlated data. The equation for EN as follows:

$$\widehat{\beta}\,(EN) = argmin_{\beta}\ \sum_{i=1}^{n}\left(y_i - x_i^T\beta\right)^2 + \lambda_1 \sum_{k=1}^{p}\beta_k^2 + \lambda_2 \sum_{k=1}^{p}|\beta_k|, \qquad (1\text{-}7)$$

where β_k^2 and $|\beta_k|$ are related with Lasso and Ridge parameter, respectively. Also, λ_1, λ_2 are the tuning parameters.

4-Literature Review

Li(1991) proposed SIR method as an eigenvalues problem. Cook(2004) proposed SIR as a least squares problem. Under setting SDR with the regularized method, researcher have suggested many methods that are no less important, but these methods suffer from drawbacks. For example, Ni et.al(2005) reformulate SIR equation by using a standard Lasso algorithm. SIR-Lasso suggested by (Lin et.al, 2019) this method suffers from Lasso does not have the property of selecting predictors with non-zero coefficients with probability equal to one. SIR-AL suggested by (Alkenani and Salman, 2021) this approaches suffers from ineffectiveness with highly correlated data. RSIR-Lasso suggested by (Alkenani, 2021) this method suffer from the inability to select groups of highly correlated predictors. SIR-EN suggested by (Akenani and Hassel, 2020) this method suffers from sensitive to outliers and is not robust, that combines the sparse SIR and EN penalty. However, the previous approaches are not robust and sensitive to the presence of outliers in variables. Therefore, several robust studies introduced such as, Gather et.al, (2002) studied SIR's sensitivity to outliers, also suggested a robust version for SIR. Yohai and Sertter(2005) proposed another a robust version of SIR. Prendergast(2005) studied the influence function of SIR, when the derivative of the loss function is redescending, it is robust and insensitive to outliers in Y and X (Rousseeuw and Yohai, 1984). This property is existed in Tukey's biweight loss function (Tukey, 1960). Alkenani (2021) suggested robust shrinkage for SIR through combining Lasso with Tukey biweight creterion for SIR. The drawback of this method is that it does not deal with data in groups and also data with high correlations. For this reason, we propose a robust method for variable selection under SDR settings deals with grouped predictors. The proposed method (RSSIR – EN) is a robust version of SSIR-EN (Alkenani and Hassel,2020) . It is an effective approach when the predictors are highly correlated under SDR settings. Furthermore, this approach works under different error distributions settings.

Chapter two

(Brief Review of SDR, IR, SIR, SSSIR, SSIR-EN and RSSIR-EN the proposed method)

2-1-Sufficient Dimension Reduction(SDR)

Cook (1998) suggested (SDR) method that used to solve the problem of high-dimensional data to obtain low- dimensionality without losing the regression information. The basic concept of SDR in regression is to replace high-dimensional predictors(original) with low dimensional predictors without losing regression information. There are many suggested ways to find SDR. The main objective of the SDR function is to find subset of predictor (S), whether related to estimate central subspace ($S_{Y|X}$) or to estimate central mean subspace ($S_{E(Y|X)}$) that important in determining the conditional distribution related to Y|X or E(Y|X). Some method of SDR such as SAVE that proposed by (Cook and Weisberg, 1991) and SIR that proposed by (Li, 1991). SIR is one method for estimating dimensionality reduction subspace through inverse conditional distribution relate to X|Y.

2-2- Inverse Regression(IR)

The normal regression is known to deal with Y given X, in another meaning E(Y|X). However, if the number of predictors is large, the problem of the curse of dimensionality appears, when the predictors increase , we get large amounts of data, which leads to sparsity of data and the difficulty of analyzing it as well as leads to breakdown of many statistical methods. The properties of the regression separating the data. To solve this problem, we use inverse regression to deal with X given Y instead of Y given X. In another meaning it deal with E(X|Y). We used invers regression to obtain a one-dimensional regression model. We use the following mode;

$$Y = f\left(\beta_1 X, \beta_2 X, \dots, \beta_p X\right) + \varepsilon, \qquad (2\text{-}1)$$

Given that the trajectory of the inverse regression curve E (X|Y) as y changes. The center of this curve is at E (E (X|Y)) = E (X). In general, it is a curve of p-dimension in $\mathbb{R}^P$. We will see that it lies on a subspace of K-dimension, K < P, if the following condition is satisfied: For b any in $\mathbb{R}^P$, the conditional expectation $E(bX \mid \beta_1 X, \beta_2 X, \dots, \beta_K X)$ linearity in$\beta_1 X, \beta_2 X, \dots, \beta_K$; for some constants $C_0, C_1, \dots, C_K$, $E(bX \mid \beta_1 X, \beta_2 X, \dots, \beta_K X) = C_0 + C_1\beta_1 X + C_2\beta_2 X + \cdots + C_K\beta_K X$ Li (1991) states that this condition is satisfied when the x distribution is elliptical (for example, a normal distribution).

2-3- Sliced Inverse Regression(SIR)

For finding the central subspace $S_{Y|X}$, SIR method is suggested by (Li, 1991). This method requires $Z = \Sigma^{\frac{-1}{2}}\big(X - E(X)\big)$, under the linear condition $E(Z/PgZ) = PgZ$, where $\sum_x = Cov(X)$ is a population covariance matrix of X and g is a basis to $S_{Y|Z}$. $S_{Y|Z}$is the central subspace of regression Y on Z. This condition connects with the inverse regression of Z on Y. The kernel matrix of SIR is M and $M = Cov[E(Z|Y)]$, $span(M) \subseteq S_{Y|Z}$. We took a random sample of size n of (X, Y), which has a joint distribution. Let $\bar{X}$ is the sample mean of X, the sample version of Z is $\hat{Z} = \hat{\Sigma}^{-\frac{1}{2}}(\mathrm{X} - \bar{\mathrm{X}})$ and $\hat{\Sigma}$ is the estimated covariance matrix of X. Assume h be the number of slices also n_y is a number of observations in yth slice. Let $\widehat{M} = \sum_{y=1}^{h} \hat{f}_y \hat{Z}_y \hat{Z}_y^T$ is an estimator of M, where $\hat{f}_y = n_y|n$ and $\hat{Z}_y$ is the average of Z in slice y. Let $\hat{\delta}_1 > \hat{\delta}_2 > \cdots > \hat{\delta}_p \geq 0$ are the eigenvalues corresponding to the eigenvectors $\hat{v}_1, \hat{v}_2, \dots, \hat{v}_p$ of $\widehat{M}$. If d of $S_{Y|Z}$ is known and $span(\hat{\beta}) = span(\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_d)$ is a consistent estimator of $S_{Y|X}$, where$\hat{\beta}_i = \hat{\Sigma}^{\frac{-1}{2}} \hat{v}_i$. The SIR method provides the estimator $span(\hat{\beta})$ of $S_{Y|X}$. Generally, $\hat{\beta} \in \mathbb{R}^{p \times d}$ has nonzero elements, when the number of predictions is huge or when the number of predictions is highly correlated, we only take the important predictions that we need to make 'sufficient predictors' combining the regularizations methods with SIR method is the solution to compress number of the coefficients of $\hat{\beta}$ to 0's. The SIR was formulated by (Cook, 2004) as a regression type minimization problem(least squares problem) as follows :

$$F(A,C) = \sum_{y=1}^{h} \left\| \hat{f}_y^{\frac{1}{2}} \hat{Z}_y - AC_y \right\|^2, \tag{2-2}$$

Over $A \in \mathbb{R}^{p\times d}$ and$C_y \in \mathbb{R}^d$ with $C = (C_1, \dots, C_h)$. Let $\hat{A}$ and $\hat{C}$ are the values of A and C that minimize F. Then $span(\hat{A})$ equals the space spanned by the d largest eigenvectors of M. By focusing on the coefficients of X variables, (Ni et.al,2005)reformulate$F(A,C)$as::

$$G(B,C) = \sum_{y=1}^{h} \left(\hat{f}_y^{\frac{1}{2}} \hat{\Sigma}^{\frac{-1}{2}} \hat{Z}_y - BCy \right)^T \hat{\Sigma} \left(\hat{f}_y^{\frac{1}{2}} \hat{\Sigma}^{\frac{-1}{2}} \hat{Z}_y - BC_y \right), \tag{2-3}$$

Where $B \in \mathbb{R}^{p\times d}$. The value of B, which minimizes (2-3) is $\hat{\beta}$ and $span(\hat{\beta}) = span(\hat{\Sigma}^{\frac{-1}{2}} \hat{A})$ is the estimator of $S_{Y|X}$.

2-4- Shrinkage Sliced Inverse Regression(SSIR)

Ni et al. (2005) suggested shrinkage sliced inverse regression(SSIR) for finding $S_{Y|X}$ as $span(diag(\tilde{\alpha})\hat{\beta})$, where the shrinkage indices $\tilde{\alpha} = (\tilde{\alpha}_1, \dots, \tilde{\alpha}_p)^T \in \mathbb{R}^p$ are determined by minimizing

$$\sum_{y=1}^{h} \left\| \hat{f}_y^{\frac{1}{2}} \hat{Z}_y - \hat{\Sigma}^{\frac{1}{2}} diag(\hat{B}\hat{C}_y)\alpha \right\| + \lambda \sum_{i=1}^{p} |\alpha_i|, \tag{2-4}$$

Where $\hat{B}$ and $\hat{C} = (\hat{C}_1, \dots, \hat{C}_h)$ minimize(2). The minimization of (2-4) can be done by using a standard Lasso algorithm, let $\tilde{Y} = vec(\hat{f}_1^{\frac{1}{2}} \hat{Z}_1, \dots, \hat{f}_h^{\frac{1}{2}} \hat{Z}_h) \in \mathbb{R}^{ph}$, and

$\tilde{X} = (diag(\hat{B}\hat{C}_1)\hat{\Sigma}^{\frac{1}{2}}, \dots, diag(\hat{B}\hat{C}_h)\hat{\Sigma}^{\frac{1}{2}})^T \in \mathbb{R}^{ph\times p}$. Where $vec(.)$ is a matrix operator that it puts the columns of the matrix in the single vector. Also, the vector α is the estimator of the lasso in the regression $\tilde{Y}$ and $\tilde{X}$.

2-5- Sparse Sliced Inverse Regression with Elastic Net (SSIR-EN)

Under setting SDR with regularized method. Researchers have suggested many methods that are no less important, but these methods suffer from drawbacks. For example, SIR-Lasso suggested by (Lin et.al, 2019) this method suffer from Lasso does not have the property of selecting predictors with non-zero coefficients with probability equal to one, this property is called the oracle property (Fan and Li, 2001). SIR-AL suggested by (Alkenani and Salman, 2021) this method suffer from ineffectiveness with highly correlated data. RSIR-Lasso suggested by (Alkenani, 2021) this method suffer from the inability to select groups of highly correlated predictors. SIR-EN suggested by (Akenani and Hassel, 2020) this method suffer from sensitive to outliers and is not robust. The SIR-EN method in the following:

$$\sum_{y=1}^{h}\left\|\hat{f}_y^{\frac{1}{2}}\hat{Z}_y - \hat{\Sigma}^{\frac{1}{2}} diag(\hat{B}\hat{C}_y)\alpha\right\|^2 + \lambda_1 \sum_{j=1}^{p} \alpha_j^2 + \lambda_2 \sum_{j=1}^{p} |\alpha_j|, \qquad (2\text{-}5)$$

where the first part is the loss function in the SIR and the second part is EN, which consists of the ridge penalization function and Lasso penalty function.

2-6- Robust Sparse Slice Invers Regression with ElasticNet (RSSIR-EN)

SIR use the classical estimates of the sample mean and the sample covariance. Also, it uses the squared loss between the response variable and the covariates. The classical estimates for the mean and covariance and loss squared criterion are very sensitive to outliers and they are not robust .

Gather et.al, (2002) studied SIR's sensitivity to outliers, also suggested a robust version for SIR. Yohai and Sertter(2005) proposed another a robust version of SIR. Prendergast(2005) studied the influence function of SIR. When the derivative of the loss function is redescending, it is robust and insensitive to outliers in Y and X (Rousseeuw and Yohai, 1984). This property is existed in Tukey's biweight loss function (Tukey, 1960). We exchange the loss squared function with Tukey’s biweight function in(2-5), that achieve the robustness against outliers in X and Y. Alkenani (2021) suggested robust shrinkage for SIR through combining Lasso with Tukey biweight criterion for SIR. The drawback of this method is that it does not deal with data in groups and also data with high correlations. For this reason, we propose a robust method for variable selection under SDR settings deals with grouped predictors. The proposed method (RSSIR – EN) is a robust version of SSIR-EN (Alkenani and Hassel,2020).

In this study, we replace the classical estimates of sample mean with a robust estimator such as the median and replace the classical estimates of sample covariance matrix with robust covariance matrix estimator as ball covariance. The estimates of suggested RSSIR-EN can be obtained by minimizing the following .

$$\sum_{y=1}^{h} \rho\left(\frac{\hat{f}_y^{\frac{1}{2}} \widehat{ROZ}_y - \widehat{RO\Sigma}^{\frac{1}{2}} diag(\hat{B}\hat{C}_y)\alpha}{\hat{\sigma}}\right) + \lambda_1 \sum_{j=1}^{p} \alpha_j^2 + \lambda_2 \sum_{j=1}^{p} |\alpha_j|, \qquad (2\text{-}6)$$

The minimizing of (2-6) contains two parts. The first part is robust SIR by using Tukey’s biweight function and the second part is Elastic Net penalty function, where, ρ is Tukey’s biweight function .

$\hat{\sigma}$ is a robust estimate of σ and MAD is used as an estimate for σ, where MAD is the median absolute deviation .

$\widehat{ROZ}_y$is a robust versions of $\hat{Z}_y$.

$\widehat{RO\Sigma}^{\frac{1}{2}}$ is a robust version of $\hat{\Sigma}^{\frac{1}{2}}$.
$\lambda_1, \lambda_2 \geq 0$ is the tuning parameters of EN .
The function of Tukey's biweight is as follows:

$$\rho_c(u) = \begin{Bmatrix} \left(\frac{c^2}{6}\right)\left\{1 - \left[1 - \left(\frac{u}{6}\right)^2\right]^3\right\} & if\ |u| \leq c \\ \frac{c^2}{6} & if\ |u| \leq c \end{Bmatrix} \tag{2-7}$$

where c controls the robustness level and c= 4.685.

2-7-Selection the tuning parameter λ

There are some information criterion for example, generalized cross validation(GCV) which is proposed by (Ni et.al, 2005), Akaike's information criterion(AIC) which is proposed by (Akaike, 1973), Bayesian information criterion(BIC) which is proposed by (Schwarz, 1978) , Residual information criterion(RIC) which is proposed by (Shi and Tsai, 2002) and Robust residual information criterion(RRIC) which is proposed by (Alkenani, 2020). These criterion information are proposed to selection λ according to the following formulas ;

$$GCV = \frac{RSS}{n\{1-p(\lambda)/n\}^2} \tag{2-8}$$

$$AIC = n\log(RSS/n) + 2p(\lambda) \tag{2-9}$$

$$BIC = n\log(RSS/n) + \log(n)p(\lambda), \tag{2-10}$$

$$RIC = \{n - p(\lambda)\}\log(RSS/\{n - p(\lambda)\} + p(\lambda)\{\log(n) - 1\} + \frac{4}{\{n-p(\lambda)-2\}}, \tag{2-11}$$

where $RSS = \sum_{y=1}^{h}\left\|\hat{f}_y^{1/2}\hat{Z}_y - \hat{\Sigma}^{\frac{1}{2}}diag(\hat{B}\hat{C}_y)\alpha\right\|^2$ is the residual sum of squares of lasso fit and $p(\lambda)$ denotes to the number of non-zero coefficients .
The simulation results of (Alkenani, 2020) show that using RRIC for selection λ gives better performance and consistent results for SIR-EN. In this paper, we employed RRIC which is proposed by (Alkenani, 2020) in our simulations, which is as follows:

$$RRIC = \{n - p(\lambda)\}\log(RRSS/\{n - p(\lambda)\}) + p(\lambda)\ \{\log(n) - 1\} + \frac{4}{\{n-p(\lambda)-2}, \tag{2-12}$$

$$RRSS = \sum_{y=1}^{h} \rho \left(\frac{\hat{f}_y^{1/2} \widehat{ROZ}_y - \widehat{RO\Sigma}^{\frac{1}{2}} diag(\hat{B}\hat{C}_y)\alpha}{\hat{\sigma}} \right) \quad (2\text{-}13)$$

$$\widehat{ROZ}_y = \widehat{BCov}_n^{\frac{-1}{2}} (X - median(X)), \text{ and } \widehat{RO\Sigma}^{\frac{1}{2}} = \widehat{BCov}_n^{\frac{-1}{2}} \quad (2\text{-}14)$$

Chapter three

(Simulation, Real data application)

3-1-Simulation

In this part, the main purpose of this section is compare the performance of the proposed method (RSSIR-EN) with RSSIR-Lasso and SSIR-EN methods, in terms the efficiency and variables selection. In all examples, we employed a robust RIC that proposed by (Alkenani, 2021) for the tuning parameter. The R code for SSIR-Lasso is made by (Ni et.al, 2005). The R code for SIR-AL is made by (Alkenani and Salman, 2021). The R code for RSIR-L is made by (Alkenani,2021) . The R code for SSIR-EN is made by (Alkenani and Hassel, 2020). . The R code for RSSIR-EN is made by (Alkenani and Alkim, 2023). In term of variable selection, the average number of zeros coefficients(Ave0's) is reported. In term of prediction accuracy, the mean squared error (MSE) is reported. Four distributions are assumed for ε and X.

Dist.1. The standard normal distribution N(0,1).

Dist.2. $t_3/\sqrt{3}$, t-distribution with 3 degree of freedom.

Dist.3. $(1-\alpha)N(0,1)+\alpha\, N(0,10^2)$

Dist.4. $(1-\alpha)N(0,1)+\alpha\, U(-50,50)$, $(1-\alpha)$ from standard normal and α from normal with mean 0 and variance 100 for (Dist.3) and uniform(-50,50) for (Dist.4). (Wang and Yao, 2013)

There are two examples as follows:

Example 1. Let d=1,iteration=500, p=40 and n=50,100 and200. Consider the model,

$$Y=1+2(\theta^T X+3)\times \log(3|\theta^T X|)+\varepsilon$$

$$\theta = \Big(\underbrace{0, \dots, 0}_{10}, \underbrace{2, \dots, 2}_{10}, \underbrace{0, \dots, 0}_{10}, \underbrace{2, \dots, 2}_{10}\Big)^T,$$

With pairwise correlation (Xi ,X j) = 0.90 for all i and j. (Alkenani and Rahman,2021).

<u>Example 2.</u> Let d =1,iteration=500, p =40 and n=50,100 and 200. Consider the model,

$$Y = 1 + 2(\theta^T X + 3) \times \log(3|\theta^T X| + 1) + \varepsilon \,.$$

$$\theta = \Big(\underbrace{3, \dots, 3}_{15}, \underbrace{0, \dots, 0}_{25}\Big)^T,$$

$x_i = z_1 + \varepsilon_i,\ i = 1, \dots, 5,$

$x_i = z_2 + \varepsilon_i,\ i = 6, \dots, 10,$

$x_i = z_3 + \varepsilon_i,\ i = 11, \dots, 15,$

$x_i,\ i = 16, \dots, 40,$

For $i = 1, \dots, 15$, five predictors within each group and there are three groups in this model. There are 25 zero predictors where (Xi ,X j) = 0.90 for all i and j. (Alkenani and Rahman,2021).

Table1: The results of example 1, based on Ave0's, and MSE when n = 50 and α =0.05, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	4.487977e-04	1.52
	RSSIR-Lasso	4.809671e-05	3.62
	RSSIR –EN	2.037116e-05	3.99
2	SSIR-EN	0.04487381	1.39
	RSSIR-Lasso	4.824503e-05	3.01
	RSSIR –EN	2.045203e-05	5.04
3	SSIR-EN	0.04483268	1.53
	RSSIR-Lasso	4.7773e-05	3.02
	RSSIR –EN	2.022285e-05	6.75
4	SSIR-EN	0.04484397	1.56
	RSSIR-Lasso	4.768485e-05	3.22
	RSSIR -EN	1.999591e-05	6.32

Table2: The results of example 1, based on Ave0's, and MSE when n = 50 and α =0.10, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	6.4855383e-05	2.57
	RSSIR-Lasso	5.166768e-05	5.03
	RSSIR –EN	1.832232e-05	6.52
2	SSIR-EN	0.04854842	2.46
	RSSIR-Lasso	5.243622e-05	5.02
	RSSIR –EN	1.831266e-05	6.33
3	SSIR-EN	0.04846247	2.44
	RSSIR-Lasso	5.019152e-05	5.02
	RSSIR –EN	1.785424e-05	6.57
4	SSIR-EN	0.04847204	2.19
	RSSIR-Lasso	5.05871e-05	5.10
	RSSIR –EN	1.797277e-05	6.14

Table3: The results of example 1, based on Ave0's, and MSE when n = 50 and α =0.15, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	7.517086e-05	2.58
	RSSIR-Lasso	6.830077e-05	5.04
	RSSIR –EN	7.305918e-06	6.67
2	SSIR-EN	0.07517058	2.04
	RSSIR-Lasso	6.82998e-05	5.02
	RSSIR –EN	7.254978e-06	6.16
3	SSIR-EN	0.07502427	2.48
	RSSIR-Lasso	6.775194e-05	6.02
	RSSIR –EN	7.201068e-06	6.07
4	SSIR-EN	0.07506487	2.94
	RSSIR-Lasso	6.76135e-05	6.02
	RSSIR –EN	7.199168e-06	6.47

Table4: The results of example 1, based on Ave0's, and MSE when n = 50 and α =0.20, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	9.393552e-05	3.50
	RSSIR-Lasso	8.979675e-05	5.76
	RSSIR –EN	5.203075e-06	5.814
2	SSIR-EN	0.09393545	3.48
	RSSIR-Lasso	8.973807e-05	6.26
	RSSIR –EN	5.111469e-06	7.36
3	SSIR-EN	0.09264675	3.37
	RSSIR-Lasso	8.71893e-05	6.02
	RSSIR –EN	5.028723e-06	8.97
4	SSIR-EN	0.09384761	3.49
	RSSIR-Lasso	9.049795e-05	4.04
	RSSIR –EN	4.941705e-06	8.63

Table5:The results of example 1, based on Ave0's, and MSE when n = 50 and α =0.25, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	5.254388e-05	4.40
	RSSIR-Lasso	4.595704e-05	6.92
	RSSIR –EN	4.261535e-05	7.31
2	SSIR-EN	0.1254414	4.53
	RSSIR-Lasso	0.0001302293	6.01
	RSSIR –EN	4.492947e-06	7.45
3	SSIR-EN	0.1191665	4.47
	RSSIR-Lasso	0.0001157054	6.02
	RSSIR –EN	4.708869e-06	7.06
4	SSIR-EN	0.1254033	5.45
	RSSIR-Lasso	0.0001304494	7.04
	RSSIR –EN	4.488822e-06	7.44

Table6:The results of example 1, based on Ave0's, and MSE when n = 50 and α =0.30, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.215631e-06	5.21
	RSSIR-Lasso	1.268868e-06	6.36
	RSSIR –EN	1.115171e-06	6.42
2	SSIR-EN	0.160634	5.48
	RSSIR-Lasso	0.0001462529	6.02
	RSSIR –EN	2.948553e-06	7.73
3	SSIR-EN	0.122353	5.47
	RSSIR-Lasso	0.0001189397	6.03
	RSSIR –EN	4.567956e-06	8.95
4	SSIR-EN	0.1292572	5.53
	RSSIR-Lasso	0.0001287179	7.02
	RSSIR –EN	4.358948e-06	8.52

Table7: The results of example 1, based on Ave0's, and MSE when n = 50 and α =0.35, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.437201e-04	5.35
	RSSIR-Lasso	1.328295e-05	6.05
	RSSIR –EN	1.969545e-06	7.27
2	SSIR-EN	0.1437244	5.30
	RSSIR-Lasso	0.000132213	6.03
	RSSIR –EN	3.934988e-06	8.36
3	SSIR-EN	0.1373105	5.51
	RSSIR-Lasso	0.0001336923	7.04
	RSSIR –EN	3.993004e-06	9.01
4	SSIR-EN	0.1436664	6.33
	RSSIR-Lasso	0.0001332097	8.01
	RSSIR –EN	3.880173e-06	9.64

Table8:The results of example 1, based on Ave0's, and MSE when n = 100 and α =0.05, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	2.242592e-05	7.50
	RSSIR-Lasso	2.172927e-05	8.036
	RSSIR –EN	5.463114e-06	8.89
2	SSIR-EN	0.02683092	7.54
	RSSIR-Lasso	2.735996e-05	8.02
	RSSIR –EN	1.121969e-05	9.38
3	SSIR-EN	0.03984808	7.48
	RSSIR-Lasso	4.137197e-05	8.04
	RSSIR –EN	3.213351e-06	10.90
4	SSIR-EN	0.0228326	7.42
	RSSIR-Lasso	2.505456e-05	9.03
	RSSIR –EN	1.722706e-05	10.70

Table9:The results of example 1, based on Ave0's, and MSE when n = 100 and α =0.10, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	5.471879e-05	7.59
	RSSIR-Lasso	5.028328e-05	9.04
	RSSIR –EN	6.315377e-06	10.08
2	SSIR-EN	0.05472697	8.62
	RSSIR-Lasso	5.034745e-05	9.03
	RSSIR –EN	6.38995e-06	10.04
3	SSIR-EN	0.05464707	8.59
	RSSIR-Lasso	5.017851e-05	10.03
	RSSIR –EN	6.064363e-06	10.07
4	SSIR-EN	0.05465397	8.55
	RSSIR-Lasso	5.003533e-05	10.03
	RSSIR –EN	6.061648e-06	11.41

Table10:The results of example 1, based on Ave0's, and MSE when n = 100 and α =0.15, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	8.668329e-05	9.51
	RSSIR-Lasso	8.319031e-05	10.05
	RSSIR –EN	2.55889e-06	10.90
2	SSIR-EN	0.08368376	9.60
	RSSIR-Lasso	8.602455e-05	10.04
	RSSIR –EN	2.634705e-06	11.92
3	SSIR-EN	0.08323136	9.51
	RSSIR-Lasso	8.147524e-05	10.04
	RSSIR –EN	2.540456e-06	12.80
4	SSIR-EN	0.08354529	10.70
	RSSIR-Lasso	8.214336e-05	11.04
	RSSIR –EN	2.551316e-06	12.45

Table11:The results of example 1, based on Ave0's, and MSE when n = 100 and α =0.20, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.127955e-04	10.43
	RSSIR-Lasso	1.147732e-05	11.04
	RSSIR –EN	1.852285e-06	12.60
2	SSIR-EN	0.1127933	10.56
	RSSIR-Lasso	0.000114415	11.02
	RSSIR –EN	1.865391e-06	12.14
3	SSIR-EN	0.1102235	10.53
	RSSIR-Lasso	0.0001089213	11.05
	RSSIR –EN	1.682965e-06	12.71
4	SSIR-EN	0.1123473	10.33
	RSSIR-Lasso	0.0001124245	11.02
	RSSIR –EN	1.691537e-06	12.46

Table12:The results of example 1, based on Ave0's, and MSE when n = 100 and α =0.25, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.184495e-05	10.60
	RSSIR-Lasso	1.020961e-05	11.04
	RSSIR –EN	1.269786e-06	12.05
2	SSIR-EN	0.1184512	11.57
	RSSIR-Lasso	0.0001218806	12.05
	RSSIR –EN	1.277237e-06	13.38
3	SSIR-EN	0.11415	11.41
	RSSIR-Lasso	0.0001130521	12.03
	RSSIR –EN	1.24263e-06	13.12
4	SSIR-EN	0.1173952	11.37
	RSSIR-Lasso	0.000117889	13.02
	RSSIR –EN	1.225129e-06	13.93

Table13:The results of example 1, based on Ave0's, and MSE when n = 100 and α =0.30, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.429102e-05	10.53
	RSSIR-Lasso	1.404676e-05	12.04
	RSSIR –EN	1.005395e-06	13.73
2	SSIR-EN	0.1429153	11.52
	RSSIR-Lasso	0.0001408843	12.04
	RSSIR –EN	1.019099e-06	13.18
3	SSIR-EN	0.1391714	11.55
	RSSIR-Lasso	0.0001337227	13.03
	RSSIR –EN	1.020009e-06	13.88
4	SSIR-EN	0.1428816	11.39
	RSSIR-Lasso	0.0001404269	13.01
	RSSIR –EN	1.003903e-06	14.13

Table14:The results of example 1, based on Ave0's, and MSE when n = 100 and α =0.35, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.49855e-04	11.44
	RSSIR-Lasso	1.449111e-04	13.04
	RSSIR –EN	1.062438e-05	13.61
2	SSIR-EN	0.1498605	12.27
	RSSIR-Lasso	0.000142984	13.03
	RSSIR –EN	1.088247e-06	14.14
3	SSIR-EN	0.1496073	12.66
	RSSIR-Lasso	0.0001452582	13.03
	RSSIR –EN	1.014076e-06	14.78
4	SSIR-EN	0.1497792	12.50
	RSSIR-Lasso	0.0001462159	14.02
	RSSIR –EN	1.01036e-06	14.06

Table15:The results of example 1, based on Ave0's, and MSE when n = 200 and α =0.05, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	3.171039e-05	12.39
	RSSIR-Lasso	3.162419e-05	14.03
	RSSIR -EN	5.088275e-06	14.73
2	SSIR-EN	0.03171494	13.42
	RSSIR-Lasso	3.189382e-05	14.02
	RSSIR -EN	5.191862e-06	16.44
3	SSIR-EN	0.03166915	13.38
	RSSIR-Lasso	3.126864e-05	14.02
	RSSIR -EN	4.929933e-06	16.96
4	SSIR-EN	0.0228974	13.53
	RSSIR-Lasso	2.472147e-05	15.04
	RSSIR -EN	8.055795e-06	16.58

Table16:The results of example 1, based on Ave0's, and MSE when n = 200 and α =0.10, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	4.578004e-04	13.47
	RSSIR-Lasso	4.75026e-05	15.03
	RSSIR -EN	1.821982e-05	16.36
2	SSIR-EN	0.04578566	14.56
	RSSIR-Lasso	4.775888e-05	15.03
	RSSIR -EN	1.891925e-06	16.96
3	SSIR-EN	0.04570286	14.49
	RSSIR-Lasso	4.772843e-05	16.04
	RSSIR -EN	1.795677e-06	16.12
4	SSIR-EN	0.04571188	14.60
	RSSIR-Lasso	4.78515e-05	16.06
	RSSIR -EN	1.805547e-06	17.08

Table17:The results of example 1, based on Ave0’s, and MSE when n = 200 and α =0.15, for dist3 and dist4.

Dist	Method	MSE	Ave. 0’s
1	SSIR-EN	6.662301e-05	14.4
	RSSIR-Lasso	4.697981e-05	16.01
	RSSIR –EN	1.188927e-06	17.75
2	SSIR-EN	0.06662522	15.49
	RSSIR-Lasso	6.687145e-05	17.04
	RSSIR –EN	1.236344e-06	17.96
3	SSIR-EN	0.06649001	15.41
	RSSIR-Lasso	6.761884e-05	17.02
	RSSIR –EN	1.22211e-06	17.44
4	SSIR-EN	0.06652121	15.56
	RSSIR-Lasso	6.790988e-05	17.05
	RSSIR –EN	1.205502e-06	17.15

Table18:The results of example 1, based on Ave0’s, and MSE when n = 200 and α =0.20, for dist3 and dist4.

Dist	Method	MSE	Ave. 0’s
1	SSIR-EN	9.373769e-05	14.36
	RSSIR-Lasso	8.388876e-05	16.03
	RSSIR –EN	1.026498e-06	17.13
2	SSIR-EN	0.08373724	14.40
	RSSIR-Lasso	8.373449e-05	16.03
	RSSIR –EN	1.063089e-06	17.90
3	SSIR-EN	0.08225067	14.38
	RSSIR-Lasso	8.186602e-05	16.01
	RSSIR -EN	9.963342e-07	17.06
4	SSIR-EN	0.08357264	14.43
	RSSIR-Lasso	8.476293e-05	17.01
	RSSIR -EN	1.009577e-06	17.54

Table19: The results of example 1, based on Ave0's, and MSE when n = 200 and α =0.25, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.336718e-05	14.5
	RSSIR-Lasso	1.265607 e-05	16.06
	RSSIR -EN	1.019899e-06	17.79
2	SSIR-EN	0.1232604	14.37
	RSSIR-Lasso	0.0001305999	16.01
	RSSIR –EN	9.203766e-07	17.63
3	SSIR-EN	0.1220834	15.45
	RSSIR-Lasso	0.0001169491	16.03
	RSSIR –EN	4.841108e-07	17.75
4	SSIR-EN	0.1652626	15.47
	RSSIR-Lasso	0.0001626196	17.03
	RSSIR -EN	5.528127e-07	17.77

Table20:The results of example 1, based on Ave0's, and MSE when n = 200 and α =0.30, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.465006e-06	14.49
	RSSIR-Lasso	1.278697e-06	15.03
	RSSIR –EN	4.784754e-07	16.93
2	SSIR-EN	0.1478462	15.31
	RSSIR-Lasso	0.0001443273	16.05
	RSSIR –EN	7.593916e-07	17.13
3	SSIR-EN	0.1511251	15.41
	RSSIR-Lasso	0.0001445022	16.03
	RSSIR –EN	4.93295e-07	17.24
4	SSIR-EN	0.1426464	15.33
	RSSIR-Lasso	0.0001447042	16.03
	RSSIR -EN	6.6451e-07	18.67

Table21:The results of example 1, based on Ave0’s, and MSE when n = 200 and α =0.35, for dist3 and dist4.

Dist	Method	MSE	Ave. 0’s
1	SSIR-EN	1.341259e-6	15.41
	RSSIR-Lasso	1.259276e-6	16.03
	RSSIR -EN	6.785619e-07	17.48
2	SSIR-EN	0.1341288	15.43
	RSSIR-Lasso	0.0001255179	17.02
	RSSIR -EN	7.097112e-07	18.99
3	SSIR-EN	0.1273917	15.49
	RSSIR-Lasso	0.0001217017	18.01
	RSSIR -EN	8.010773e-07	19.79
4	SSIR-EN	0.1341003	16.51
	RSSIR-Lasso	0.0001287542	18.06
	RSSIR -EN	6.420542e-07	19.41

Table 22: The results of example 2, based on Ave0’s, and MSE when n = 50 and α =0.05, for dist3 and dist4.

Dist	Method	MSE	Ave. 0’s
1	SSIR-EN	4.680351e-05	1.58
	RSSIR-Lasso	3.332191e-05	3.03
	RSSIR -EN	1.236556e-05	3.52
2	SSIR-EN	0.04513449	1.52
	RSSIR-Lasso	6.082133e-05	3.01
	RSSIR -EN	3.071832e-05	5.00
3	SSIR-EN	0.04507638	1.37
	RSSIR-Lasso	5.94675e-05	3.01
	RSSIR -EN	2.922632e-05	6.63
4	SSIR-EN	0.04508374	1.36
	RSSIR-Lasso	5.988378e-05	3.02
	RSSIR -EN	2.946017e-05	6.58

Table 23: The results of example 2, based on Ave0's, and MSE when n = 50 and α =0.10, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	0.487712e-04	2.38
	RSSIR-Lasso	6.151865e-05	5.00
	RSSIR –EN	2.524819e-05	6.59
2	SSIR-EN	0.04876865	2.48
	RSSIR-Lasso	6.131084e-05	5.02
	RSSIR –EN	2.540396e-05	6.82
3	SSIR-EN	0.04860849	2.56
	RSSIR-Lasso	5.698737e-05	5.04
	RSSIR –EN	2.264034e-05	6.13
4	SSIR-EN	0.04864073	2.38
	RSSIR-Lasso	5.727573e-05	5.03
	RSSIR –EN	2.268371e-05	6.08

Table 24:The results of example 2, based on Ave0's, and MSE when n = 50 and α =0.15, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	9.752811e-05	2.61
	RSSIR-Lasso	7.430622e-05	5.02
	RSSIR –EN	1.021396e-05	6.53
2	SSIR-EN	0.0752785	2.58
	RSSIR-Lasso	7.430148e-05	5.02
	RSSIR –EN	1.02832e-05	6.42
3	SSIR-EN	0.0750644	2.51
	RSSIR-Lasso	7.091946e-05	6.03
	RSSIR -EN	9.290021e-06	6.59
4	SSIR-EN	0.07510269	2.38
	RSSIR-Lasso	7.112544e-05	6.01
	RSSIR -EN	9.188532e-06	6.42

Table 25:The results of example 2, based on Ave0's, and MSE when n = 50 and α =0.20, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	9.40284e-05	3.47
	RSSIR-Lasso	8.376686e-05	5.02
	RSSIR –EN	7.197939e-06	5.66
2	SSIR-EN	0.09402875	3.04
	RSSIR-Lasso	8.390795e-05	6.01
	RSSIR –EN	7.339105e-06	7.71
3	SSIR-EN	0.0929563	3.42
	RSSIR-Lasso	8.483299e-05	6.02
	RSSIR –EN	6.209121e-06	8.78
4	SSIR-EN	0.09384626	3.48
	RSSIR-Lasso	8.461874e-05	4.02
	RSSIR -EN	6.327946e-06	8.25

Table 26: The results of example 2, based on Ave0's, and MSE when n = 50 and α =0.25, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.254659e-05	4.51
	RSSIR-Lasso	6.335259e-06	6.03
	RSSIR –EN	1.930139e-06	7.90
2	SSIR-EN	0.1254666	4.44
	RSSIR-Lasso	0.0001351151	6.01
	RSSIR –EN	7.09998e-06	7.56
3	SSIR-EN	0.1202211	4.53
	RSSIR-Lasso	0.0001171884	6.03
	RSSIR –EN	5.199173e-06	7.88
4	SSIR-EN	0.1254129	5.44
	RSSIR-Lasso	0.0001315369	7.02
	RSSIR -EN	5.102739e-06	7.86

Table 27: The results of example 2, based on Ave0's, and MSE when n = 50 and α =0.30, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.293606e-05	5.43
	RSSIR-Lasso	1.300193e-05	6.02
	RSSIR -EN	1.345039e-06	6.96
2	SSIR-EN	0.1293615	5.48
	RSSIR-Lasso	0.0001302165	6.05
	RSSIR -EN	6.393441e-06	7.97
3	SSIR-EN	0.1242821	5.35
	RSSIR-Lasso	0.0001200639	6.02
	RSSIR -EN	4.940732e-06	8.89
4	SSIR-EN	0.1292835	5.46
	RSSIR-Lasso	0.0001247306	7.01
	RSSIR -EN	4.758929e-06	8.51

Table 28:The results of example 2, based on Ave0's, and MSE when n = 50 and α =0.35, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.437443e-05	5.48
	RSSIR-Lasso	1.412027e-06	6.04
	RSSIR -EN	1.118897e-06	7.78
2	SSIR-EN	0.1437446	5.6
	RSSIR-Lasso	0.0001411884	6.03
	RSSIR -EN	6.163878e-06	8.24
3	SSIR-EN	0.1397133	5.51
	RSSIR-Lasso	0.0001352369	7.05
	RSSIR -EN	4.341259e-06	9.48
4	SSIR-EN	0.1436688	6.48
	RSSIR-Lasso	0.0001355744	8.02
	RSSIR -EN	4.339982e-06	9.48

Table 29: The results of example 2, based on Ave0's, and MSE when n = 100 and α =0.05, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	2.190735e-04	7.49
	RSSIR-Lasso	3.586474e-05	8.02
	RSSIR –EN	1.969126e-05	8.62
2	SSIR-EN	0.027081	7.43
	RSSIR-Lasso	4.570905e-05	8.01
	RSSIR –EN	2.274569e-05	9.20
3	SSIR-EN	0.02700096	7.54
	RSSIR-Lasso	4.292843e-05	8.02
	RSSIR –EN	2.086527e-05	10.61
4	SSIR-EN	0.02700723	7.48
	RSSIR-Lasso	4.308358e-05	9.02
	RSSIR –EN	2.086641e-05	10.62

Table 30:The results of example 2, based on Ave0's, and MSE when n = 100 and α =0.10, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	5.491451e-05	7.47
	RSSIR-Lasso	3.618804e-05	9.03
	RSSIR –EN	1.194799e-06	10.53
2	SSIR-EN	0.05491358	8.43
	RSSIR-Lasso	6.639158e-05	9.02
	RSSIR –EN	1.200572e-05	10.34
3	SSIR-EN	0.05476601	8.69
	RSSIR-Lasso	5.890493e-05	10.04
	RSSIR –EN	9.989967e-06	10.09
4	SSIR-EN	0.05478319	8.58
	RSSIR-Lasso	5.934513e-05	10.02
	RSSIR –EN	1.003302e-05	11.70

Table 31:The results of example 2, based on Ave0's, and MSE when n = 100 and α =0.15, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	8.379288e-05	10.55
	RSSIR-Lasso	8.319258e-05	11.02
	RSSIR –EN	4.459176e-06	12.31
2	SSIR-EN	0.08379099	10.57
	RSSIR-Lasso	8.321388e-05	11.02
	RSSIR –EN	4.502566e-06	12.31
3	SSIR-EN	0.08337906	10.50
	RSSIR-Lasso	8.196286e-05	11.02
	RSSIR –EN	3.986228e-06	12.27
4	SSIR-EN	0.08361779	10.52
	RSSIR-Lasso	8.16096e-05	11.02
	RSSIR –EN	3.979309e-06	12.16

Table 32:The results of example 2, based on Ave0's, and MSE when n = 100 and α =0.20, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.198281e-05	10.46
	RSSIR-Lasso	1.138074 e-05	11.02
	RSSIR –EN	1.058396e-06	12.11
2	SSIR-EN	0.1128276	11.47
	RSSIR-Lasso	0.0001142325	12.04
	RSSIR –EN	3.109118e-06	13.10
3	SSIR-EN	0.1104485	11.53
	RSSIR-Lasso	0.0001048415	12.02
	RSSIR –EN	2.526569e-06	13.18
4	SSIR-EN	0.1123821	11.49
	RSSIR-Lasso	0.0001133031	13.04
	RSSIR -EN	2.636017e-06	13.12

Table 33:The results of example 2, based on Ave0's, and MSE when n = 100 and α =0.25, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	2.184619e-05	10.45
	RSSIR-Lasso	1.243701 e-05	12.03
	RSSIR –EN	2.116956e-06	13.03
2	SSIR-EN	0.118463	11.39
	RSSIR-Lasso	0.0001248949	12.02
	RSSIR –EN	2.158885e-06	13.02
3	SSIR-EN	0.1147694	11.55
	RSSIR-Lasso	0.000112617	13.04
	RSSIR –EN	1.510166e-06	13.60
4	SSIR-EN	0.1175648	11.34
	RSSIR-Lasso	0.0001187808	13.03
	RSSIR –EN	1.552647e-06	14.76

Table 34:The results of example 2, based on Ave0's, and MSE when n = 100 and α =0.30, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.429294e-05	11.44
	RSSIR-Lasso	1.400087e-05	13.03
	RSSIR –EN	1.572459e-06	13.10
2	SSIR-EN	0.142932	12.58
	RSSIR-Lasso	0.0001395143	13.02
	RSSIR –EN	1.601245e-06	14.00
3	SSIR-EN	0.1399566	12.60
	RSSIR-Lasso	0.0001361497	13.06
	RSSIR –EN	1.233555e-06	14.01
4	SSIR-EN	0.1428769	12.33
	RSSIR-Lasso	0.000141761	14.05
	RSSIR -EN	1.196548e-06	14.99

Table 35:The results of example 2, based on Ave0's, and MSE when n = 100 and α =0.35, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.998768e-06	11.59
	RSSIR-Lasso	1.47831 e-06	13.02
	RSSIR –EN	1.042574e-06	13.36
2	SSIR-EN	0.1498788	12.62
	RSSIR-Lasso	0.0001485344	13.04
	RSSIR –EN	1.451878e-06	14.53
3	SSIR-EN	0.1497443	12.60
	RSSIR-Lasso	0.0001476943	13.02
	RSSIR –EN	1.140442e-06	14.56
4	SSIR-EN	0.1497785	12.62
	RSSIR-Lasso	0.0001468358	14.04
	RSSIR -EN	1.16248e-06	14.72

Table 36:The results of example 2, based on Ave0's, and MSE when n = 200 and α =0.05, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	3.189274e-04	12.39
	RSSIR-Lasso	4.387594e-05	14.08
	RSSIR –EN	1.165695e-05	14.43
2	SSIR-EN	0.03189871	13.28
	RSSIR-Lasso	4.418828e-05	14.05
	RSSIR –EN	1.174325e-05	16.58
3	SSIR-EN	0.03183543	13.42
	RSSIR-Lasso	4.239408e-05	14.06
	RSSIR –EN	1.065352e-05	16.10
4	SSIR-EN	0.03184223	13.47
	RSSIR-Lasso	4.237561e-05	15.02
	RSSIR -EN	1.071299e-05	16.22

Table 37:The results of example 2, based on Ave0's, and MSE when n = 200 and α =0.10, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	4.590554e-04	13.41
	RSSIR-Lasso	4.548223e-05	15.02
	RSSIR –EN	3.98697e-06	16.03
2	SSIR-EN	0.04591272	14.52
	RSSIR-Lasso	4.534859e-05	15.02
	RSSIR –EN	4.044684e-06	17.92
3	SSIR-EN	0.0457968	14.40
	RSSIR-Lasso	4.362968e-05	16.02
	RSSIR –EN	3.262841e-06	17.93
4	SSIR-EN	0.04581071	14.42
	RSSIR-Lasso	4.390169e-05	16.02
	RSSIR -EN	3.275948e-06	17.85

Table 38: The results of example 2, based on Ave0's, and MSE when n = 200 and α =0.15, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	9.671376e-05	14.42
	RSSIR-Lasso	7.012378e-05	16.02
	RSSIR –EN	2.276617e-06	18.07
2	SSIR-EN	0.06671338	15.52
	RSSIR-Lasso	7.02296e-05	17.04
	RSSIR –EN	2.353524e-06	19.94
3	SSIR-EN	0.06652505	15.29
	RSSIR-Lasso	6.79203e-05	17.02
	RSSIR –EN	2.044871e-06	19.33
4	SSIR-EN	0.06655951	15.59
	RSSIR-Lasso	6.756311e-05	17.03
	RSSIR -EN	1.943729e-06	20.12

Table 39: The results of example 2, based on Ave0's, and MSE when n = 200 and α =0.20, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	8.881025e-05	14.38
	RSSIR-Lasso	8.731137e-05	16.03
	RSSIR –EN	1.861217e-06	19.12
2	SSIR-EN	0.08380834	14.34
	RSSIR-Lasso	8.749299e-05	16.03
	RSSIR –EN	1.930359e-06	20.52
3	SSIR-EN	0.08263936	14.27
	RSSIR-Lasso	8.241334e-05	17.02
	RSSIR –EN	1.575206e-06	20.33
4	SSIR-EN	0.08365682	15.48
	RSSIR-Lasso	8.529328e-05	18.38
	RSSIR -EN	1.592363e-06	20.46

Table 40:The results of example 2, based on Ave0's, and MSE when n = 200 and α =0.25, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.17272e-06	16.38
	RSSIR-Lasso	1.059426e-06	18.08
	RSSIR –EN	9.883827e-07	20.13
2	SSIR-EN	0.103309	17.5
	RSSIR-Lasso	0.0001024433	18.03
	RSSIR –EN	1.628671e-06	21.21
3	SSIR-EN	0.1002983	18.44
	RSSIR-Lasso	9.718615e-05	19.04
	RSSIR –EN	1.107554e-06	21.33
4	SSIR-EN	0.1029608	18.36
	RSSIR-Lasso	0.0001012051	19.02
	RSSIR -EN	1.1501e-06	21.66

Table 41:The results of example 2, based on Ave0's, and MSE when n = 200 and α =0.30, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.195251e-06	18.46
	RSSIR-Lasso	1.13147e-06	19.03
	RSSIR -EN	1.105524e-06	22.43
2	SSIR-EN	0.1195242	18.65
	RSSIR-Lasso	0.0001126962	19.03
	RSSIR -EN	1.173969e-06	22.57
3	SSIR-EN	0.1122663	18.66
	RSSIR-Lasso	0.0001085499	20.04
	RSSIR -EN	8.777419e-07	22.85
4	SSIR-EN	0.119479	18.51
	RSSIR-Lasso	0.000112262	20.02
	RSSIR -EN	7.939724e-07	22.19

Table 42:The results of example 2, based on Ave0's, and MSE when n = 200 and α =0.35, for dist3 and dist4.

Dist	Method	MSE	Ave. 0's
1	SSIR-EN	1.341499e-06	18.96
	RSSIR-Lasso	1.316776e-06	20.04
	RSSIR -EN	1.064537e-06	21.94
2	SSIR-EN	0.1341508	18.47
	RSSIR-Lasso	0.0001321163	20.08
	RSSIR -EN	1.106291e-06	22.94
3	SSIR-EN	0.1288302	18.58
	RSSIR-Lasso	0.0001216745	20.03
	RSSIR -EN	9.462509e-07	23.52
4	SSIR-EN	0.1341038	18.43
	RSSIR-Lasso	0.0001287023	20.04
	RSSIR -EN	8.133561e-07	24.79

From the results of tables 1,2,3,….42, it can be seen that there is a slight outperform for the suggested approach where it has a lower MSE and it has a bigger values based on Ave.0’s. In case of three distributions of x and error, we can note that SIR-EN method was sensitive for the contamination but other methods RSSIR-Lasso and RSSIR-EN were not affected because they have the robustness. Also, we can see that the performance of RSSIR-EN outperformed RSSIR-Lasso method in terms of V.S based on Ave.0’s. For the previous two examples, the MSE values for RSSIR-EN are less than their values for RSSIR-Lasso and SSIR-EN. This means that the suggested RSSIR-EN has the best performance than the rest methods depending on the MSE of simulation studies. It is clear that under various settings, the proposed RSSIR-EN has a good performance in terms of variable selection and estimation accuracy.

3.2. Boston housing data

This data was collected by (Harrison and Rubinfeld, 1978), the data set includes n = 506 observations and p = 14 predictor, where y is medv (median value of owner occupied homes in $ 1000's). X includes 13 predictors. The predictors are : x1 is (rate of crime), x2 is (proportion of residential land zoned), x3 is (proportion of non-retail business acres), x4 is (the Charles river (= 1 if tract bounds river; 0 otherwise)), x5 is (concentration of nitric oxides), x6 is (average of rooms), x7 is (proportion of owner-occupied units), x8 is (weighted mean of distances), x9 (index of accessibility),x10 is (rate of property tax), x11 (pupil – teacher ratio), x12 is (proportion of black population) and x13 is (lower status). The data set is available and public from R package. The predictors and y are standardized separately for ease of explanation. To verify the performance of the proposed RSSIR-EN.

We made a comparison to evaluate the accuracy of the suggested method RSSIR-EN and SSIR-EN, RSSIR-Lasso methods based on the mean squared error(MSE) and number of zero's coefficient

Table1: The results of Boston housing based on number of zero's and MSE

Method	MSE	Number of zero's
SSIR-EN	0.05335893	9
RSSIR-Lasso	0.01069643	10
RSSIR-EN	0.007849862	11

From the result of table 1, it can be seen that there is a slight outperform for the suggested approach where it has a lower MSE and it has a bigger values based on number of zero's coefficients. We can note that SIR-EN method was sensitive for the contamination but other methods RSSIR-Lasso and RSSIR-EN were not affected because they have the robustness. Also, we can see that the performance of RSSIR-EN outperformed RSSIR-Lasso method in terms of V.S based on number of zero's coefficients. For the Boston housing data, the MSE values for RSSIR-EN are less than their values for RSSIR-Lasso and SSIR-EN. This means that the suggested RSSIR-EN has the best performance than the rest methods depending on the MSE. It is clear that under various settings, the proposed RSSIR-EN has a good performance in terms of variable selection and estimation accuracy.

Table2: The results of Boston housing based on beta

SSIR-EN	RSSIR-Lasso	RSSIR-EN
1.735522	0.0000000	0.00000000
0.000000	0.7516025	0.00000000
0.000000	0.0000000	0.00000000
0.000000	0.0000000	0.00000000
1.504926	0.0000000	0.00000000
0.000000	0.0000000	0.00000000
0.000000	0.0000000	0.00000000
0.000000	0.0000000	0.00000000
0.000000	0.0000000	0.00000000
5.473775	0.0000000	7.76415849
2.399465	1.0064451	0.00000000
0.000000	3.1148360	0.00000000
0.000000	0.0000000	1.83860357

From the correlation matrix in table, it is clear that there are high correlations among the variables. High pairwise correlations are found in $(X_9, X_1)(X_{10}, X_1)(X_8, X_2)(X_5, X_3)(X_7, X_3)(X_9, X_3)(X_{10}, X_5)(X_{13}, X_5)$ and others as shown in the following table3;

Table3: The results of Boston housing based on correlation of variables

	crim	zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	b	lstat
crim	1	-0.20047	0.406583	-0.05589	0.420972	-0.21925	0.352734	-0.37967	0.625505	0.582764	0.289946	-0.38506	0.455621
zn	-0.20047	1	-0.53383	-0.0427	-0.5166	0.311991	-0.56954	0.664408	-0.31195	-0.31456	-0.39168	0.17552	-0.41299
indus	0.406583	-0.53383	1	0.062938	0.763651	-0.39168	0.644779	-0.70803	0.595129	0.72076	0.383248	-0.35698	0.6038
chas	-0.05589	-0.0427	0.062938	1	0.091203	0.091251	0.086518	-0.09918	-0.00737	-0.03559	-0.12152	0.048788	-0.05393
nox	0.420972	-0.5166	0.763651	0.091203	1	-0.30219	0.73147	-0.76923	0.611441	0.668023	0.188933	-0.38005	0.590879
rm	-0.21925	0.311991	-0.39168	0.091251	-0.30219	1	-0.24026	0.205246	-0.20985	-0.29205	-0.3555	0.128069	-0.61381
age	0.352734	-0.56954	0.644779	0.086518	0.73147	-0.24026	1	-0.74788	0.456022	0.506456	0.261515	-0.27353	0.602339
dis	-0.37967	0.664408	-0.70803	-0.09918	-0.76923	0.205246	-0.74788	1	-0.49459	-0.53443	-0.23247	0.291512	-0.497
rad	0.625505	-0.31195	0.595129	-0.00737	0.611441	-0.20985	0.456022	-0.49459	1	0.910228	0.464741	-0.44441	0.488676
tax	0.582764	-0.31456	0.72076	-0.03559	0.668023	-0.29205	0.506456	-0.53443	0.910228	1	0.460853	-0.44181	0.543993
ptratio	0.289946	-0.39168	0.383248	-0.12152	0.188933	-0.3555	0.261515	-0.23247	0.464741	0.460853	1	-0.17738	0.374044
b	-0.38506	0.17552	-0.35698	0.048788	-0.38005	0.128069	-0.27353	0.291512	-0.44441	-0.44181	-0.17738	1	-0.36609
lstat	0.455621	-0.41299	0.6038	-0.05393	0.590879	-0.61381	0.602339	-0.497	0.488676	0.543993	0.374044	-0.36609	1

As well as testing the presence of outliers through the method $(mean_{+}^{-} 3\sigma)$ in variables Boston housing data(Lehmann,2013).

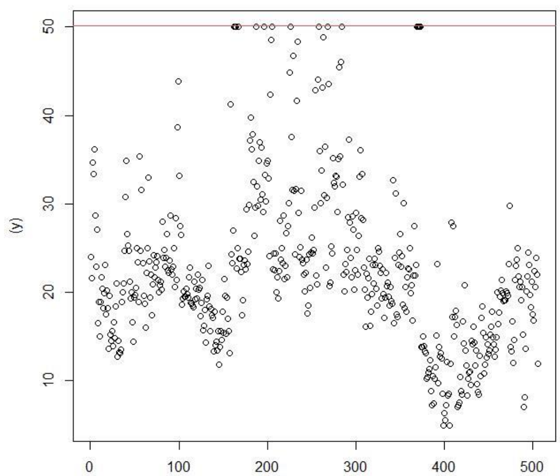

Figure-1: Test for the presence of outliers in Y

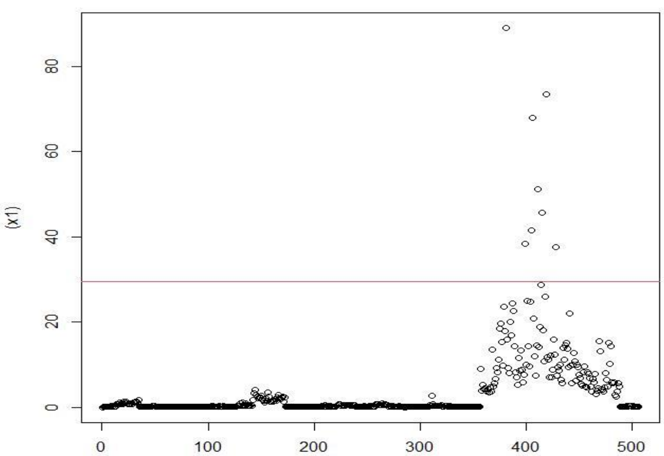

Figure-2: Test for the presence of outliers in X_1

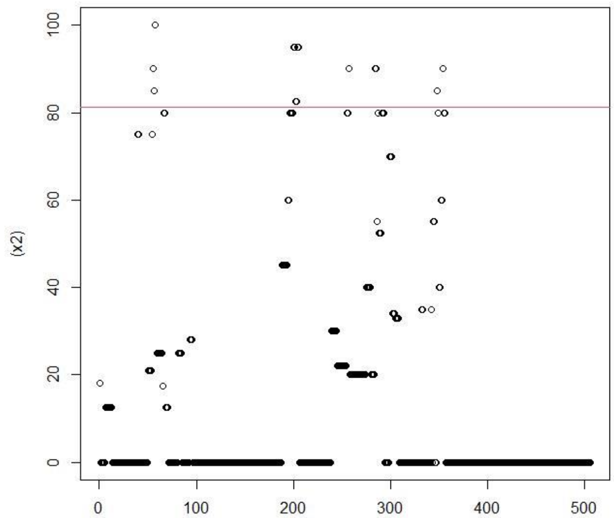

Figure-3: Test for the presence of outliers in X_2

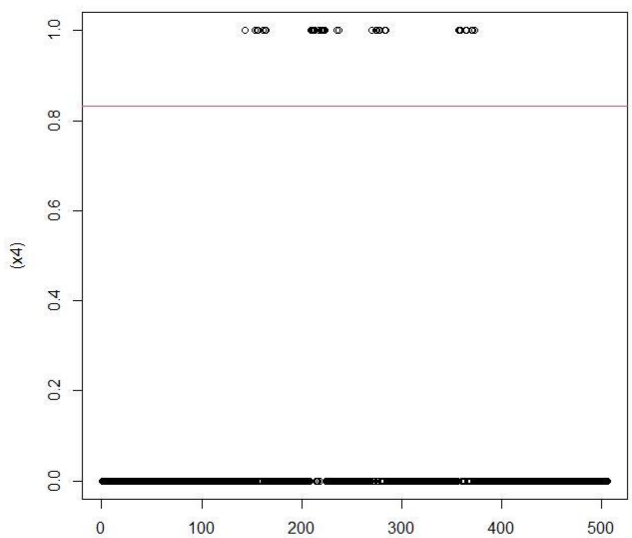

Figure-4: Test for the presence of outliers in X_4

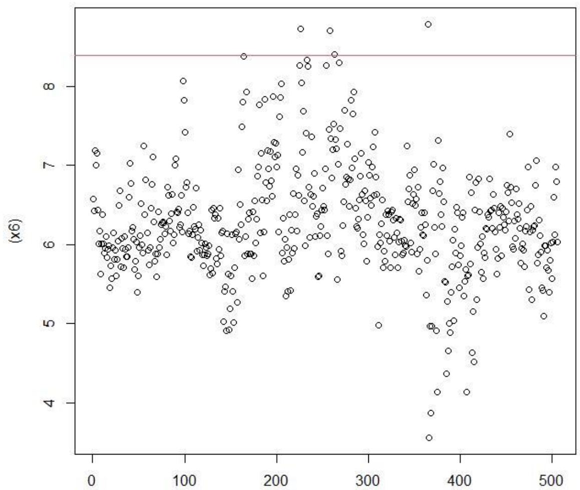

Figure-5: Test for the presence of outliers inX_6

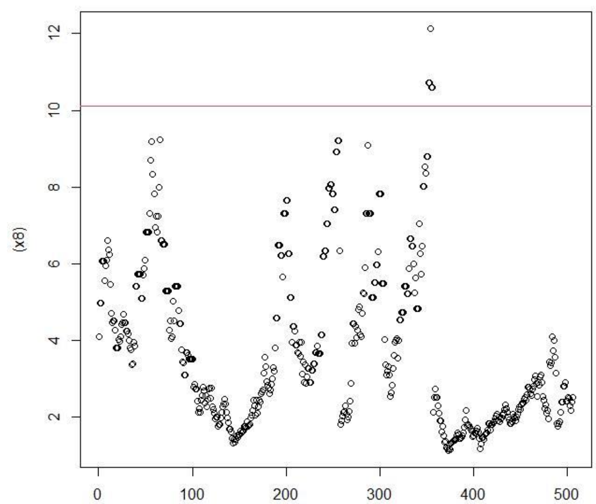

Figure-6: Test for the presence of outliers in X_8

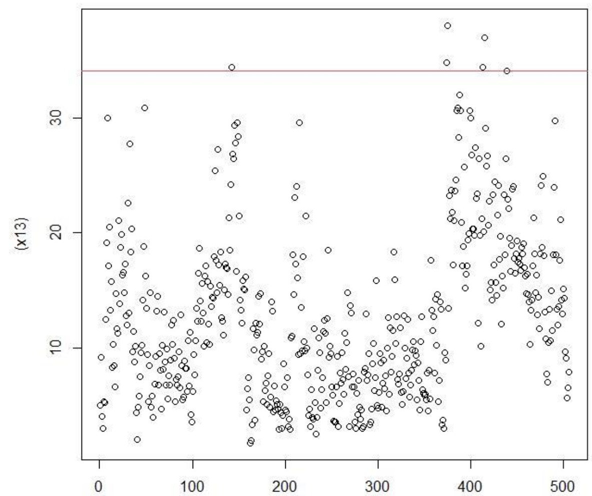

Figure-7: Test for the presence of outliers in X_{13}

3-3-Real data for anemia

In this section, to check the performance of the suggested RSSIR-EN method, we used the SSIR-EN, RSSIR-Lasso and RSSIR-EN methods in analysis anemia data. Data were collected for 200 samples of anemia patients from Thalassemia Specialist Center in Al-Diwaniyah. We assumed the response variable Y is the level of hemoglobin(HB) in blood, also we assumed twenty-one independent variable X as follows;

X_1 is the age.
X_2 is the gender.
X_3 is the blood group.
X_4 is the length.
X_5 is the weight.
X_6 is Academic achievement.
X_7 is living.
X_8 is the income.
X_9 is the nature of food.
X_{10} is having surgeries.
X_{11} is iron percentage.
X_{12} is White blood cells(WBC)
X_{13} is Neutrophils(NE)
X_{14} is Lymphocytes(LY)
X_{15} is Monocytes.(MO)
X_{16} is Eosinophils(EO)
X_{17} is Basophils.(BA)
X_{18} is Platelet count test(PLT)
X_{19} is Mean platelet volume(MPV)
X_{20} is the genetic factor.
X_{21} is the social status.

We made a comparison to evaluate the accuracy of the suggested method RSSIR-EN and SSIR-EN, RSSIR-Lasso methods based on the mean squared error(MSE) and number of zero's coefficient.

Table1: The results of Real data based on number of zero's and MSE

Method	MSE	Number of zero's
SSIR-EN	0.002985756	8
RSSIR-Lasso	0.003162099	13
RSSIR-EN	0.002700182	15

From the result of table1, it can be seen that there is a slight outperform for the suggested approach where it has a lower MSE and it has a bigger values based on number of zero's coefficients. We can note that SIR-EN method was sensitive for the contamination but other methods RSSIR-Lasso and RSSIR-EN were not affected because they have the robustness. Also, we can see that the performance of RSSIR-EN outperformed RSSIR-Lasso method in terms of V.S based on number of zero's coefficients. For the Real data for anemia, the MSE values for RSSIR-EN are less than their values for RSSIR-Lasso and SSIR-EN. This means that the suggested RSSIR-EN has the best performance than the rest methods depending on the MSE. It is clear that under various settings, the proposed RSSIR-EN has a good performance in terms of variable selection and estimation accuracy.

Table2: The results of Real data based on beta

SSIR-EN	RSSIR-Lasso	RSSIR-EN
2.083801	0.0000000	0.0000000
2.941862	0.0000000	2.2890836
1.304397	1.1615607	2.1744716
0.000000	0.0000000	0.0000000
1.331138	0.0000000	0.0000000
0.000000	0.0000000	0.0000000
2.812370	0.0000000	0.0000000
1.558690	0.5661126	0.0000000
0.000000	0.4323028	0.0000000
55.012939	0.0000000	29.9060514
0.000000	0.0000000	0.0000000
1.014341	2.4093509	0.0000000
2.583451	0.0000000	2.8620446
4.161742	1.2082380	0.0000000
2.571519	1.7851408	1.3786562
3.013674	0.9849392	0.0000000
0.000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000
4.640600	0.0000000	5.7933282
0.000000	1.8679699	0.0000000
0.000000	0.0000000	0.0000000

From the correlation matrix in table, it is clear that there are high correlations among the variables. High pairwise correlations are found in $(X_1,X_5)(X_1,X_6)(X_4,X_5)(X_4,X_6)(X_5,X_6)(X_6,X_5)(X_{13},X_{14})(X_6,X_4)(X_5,X_4)$ and others as shown in the following table(3);

Table3: The results of Real data based on correlation of variables

	y	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20	x21
y	1	0.099009	0.03269	-0.08187	0.052947	0.076646	-0.03159	0.015007	-0.04634	-0.02121	0.059264	0.053919	-0.00341	0.076441	0.07494	0.01491	-0.04989	-0.02562	0.00338	0.03594	-0.00717	-0.08832
x1	0.099009	1	-0.22493	0.044117	0.706896	0.836397	0.547873	-0.14271	0.001766	0.30435	0.379591	0.07619	-0.05413	-0.10423	-0.14547	-0.13241	-0.04399	-0.01546	-0.05396	-0.05957	0.09762	-0.7545
x2	0.03269	-0.22493	1	0.118254	-0.26711	-0.23519	-0.26677	0.03561	-0.0943	-0.10544	-0.04189	-0.11941	0.09865	0.093787	0.142302	0.176342	0.043949	0.047061	0.109557	0.118898	-0.07857	0.33493
x3	-0.08187	0.044117	0.118254	1	-0.05436	-0.025	-0.06068	0.064263	-0.04882	-0.04016	0.129762	0.008999	-0.04134	9.64E-05	0.046909	0.075744	0.024732	-0.00566	-0.01007	0.111751	-0.03526	-0.00718
x4	0.052947	0.706896	-0.26711	-0.05436	1	0.8402	0.758062	-0.1437	-0.06192	0.233008	0.284292	0.115462	-0.05504	-0.18104	-0.3221	-0.27742	-0.07297	-0.05116	-0.04867	-0.03098	0.304392	-0.5365
x5	0.076646	0.836397	-0.23519	-0.025	0.8402	1	0.745113	-0.11523	-0.00354	0.240066	0.302868	0.128584	-0.08219	-0.09836	-0.18168	-0.20932	-0.02318	-0.01503	-0.01541	-0.02346	0.191937	-0.65411
x6	-0.03159	0.547873	-0.26677	-0.06068	0.758062	0.745113	1	-0.08608	0.044776	0.07266	0.133924	0.141047	-0.08335	-0.12675	-0.20006	-0.18181	0.033849	0.008288	-0.05525	-0.04786	0.265309	-0.43871
x7	0.015007	-0.14271	0.03561	0.064263	-0.1437	-0.11523	-0.08608	1	-0.16347	-0.10985	-0.21368	-0.07867	-0.02567	-0.09294	-0.06244	0.058824	0.043206	-0.07641	-0.12348	0.029822	-0.1463	0.040043
x8	-0.04634	0.001766	-0.0943	-0.04882	-0.06192	-0.00354	0.044776	-0.16347	1	0.27672	-0.07753	0.051884	-0.01577	0.150038	0.135724	0.065495	0.124934	0.013174	0.164057	0.082776	3.27E-21	-0.02722
x9	-0.02121	0.30435	-0.10544	-0.04016	0.233008	0.240066	0.07266	-0.10985	0.27672	1	-0.02866	0.09432	-0.06751	0.024284	-0.03951	-0.01865	-0.01153	-0.07201	0.082975	0.145892	0.068768	-0.12486
x10	0.059264	0.379591	-0.04189	0.129762	0.284292	0.302868	0.133924	-0.21368	-0.07753	-0.02866	1	-0.03574	0.064821	-0.08102	-0.03682	0.000902	0.023005	0.04549	0.04684	-0.00364	-0.00718	-0.41145
x11	0.053919	0.07619	-0.11941	0.008999	0.115462	0.128584	0.141047	-0.07867	0.051884	0.09432	-0.03574	1	0.017098	-0.0015	0.002937	0.003736	0.08196	0.084707	0.07472	-0.0376	0.078966	-0.0721
x12	-0.00341	-0.05413	0.09865	-0.04134	-0.05504	-0.08219	-0.08335	-0.02567	-0.01577	-0.06751	0.064821	0.017098	1	0.173883	0.22368	0.277487	0.176519	0.126632	0.129048	0.004752	-0.13477	-0.05022
x13	0.076441	-0.10423	0.093787	9.64E-05	-0.18104	-0.09836	-0.12675	-0.09294	0.150038	0.024284	-0.08102	-0.0015	0.173883	1	0.596196	0.450427	0.084997	0.062102	0.336177	0.301521	0.016293	0.034648
x14	0.07494	-0.14547	0.142302	0.046909	-0.3221	-0.18168	-0.20006	-0.06244	0.135724	-0.03951	-0.03682	0.002937	0.22368	0.596196	1	0.466761	0.171442	0.140717	0.218334	0.100602	-0.08771	0.066185
x15	0.01491	-0.13241	0.176342	0.075744	-0.27742	-0.20932	-0.18181	0.058824	0.065495	-0.01865	0.000902	0.003736	0.277487	0.450427	0.466761	1	0.404269	0.197417	0.200366	0.346201	-0.13767	0.083034
x16	-0.04989	-0.04399	0.043949	0.024732	-0.07297	-0.02318	0.033849	0.043206	0.124934	-0.01153	0.023005	0.08196	0.176519	0.084997	0.171442	0.404269	1	0.430241	0.222033	0.187002	-0.09923	-0.04128
x17	-0.02562	-0.01546	0.047061	-0.00566	-0.05116	-0.01503	0.008288	-0.07641	0.013174	-0.07201	0.04549	0.084707	0.126632	0.062102	0.140717	0.197417	0.430241	1	0.053895	0.047705	0.019042	-0.09463
x18	0.00338	-0.05396	0.109557	-0.01007	-0.04867	-0.01541	-0.05525	-0.12348	0.164057	0.082975	0.04684	0.07472	0.129048	0.336177	0.218334	0.200366	0.222033	0.053895	1	0.096304	0.009564	0.021263
x19	0.03594	-0.05957	0.118898	0.111751	-0.03098	-0.02346	-0.04786	0.029822	0.082776	0.145892	-0.00364	-0.0376	0.004752	0.301521	0.100602	0.346201	0.187002	0.047705	0.096304	1	-0.04241	0.037809
x20	-0.00717	0.09762	-0.07857	-0.03526	0.304392	0.191937	0.265309	-0.1463	3.27E-21	0.068768	-0.00718	0.078966	-0.13477	0.016293	-0.08771	-0.13767	-0.09923	0.019042	0.009564	-0.04241	1	0.025198
x21	-0.08832	-0.7545	0.33493	-0.00718	-0.5365	-0.65411	-0.43871	0.040043	-0.02722	-0.12486	-0.41145	-0.0721	-0.05022	0.034648	0.066185	0.083034	-0.04128	-0.09463	0.021263	0.037809	0.025198	1

As well as testing the presence of outliers through the method ($mean_{+}^{-}3\sigma$) in variables real data (Lehmann,2013).

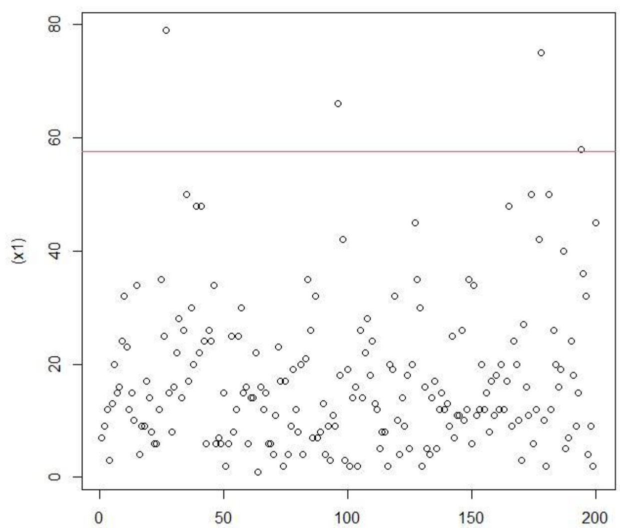

Figure-1: Test for the presence of outliers in X_1

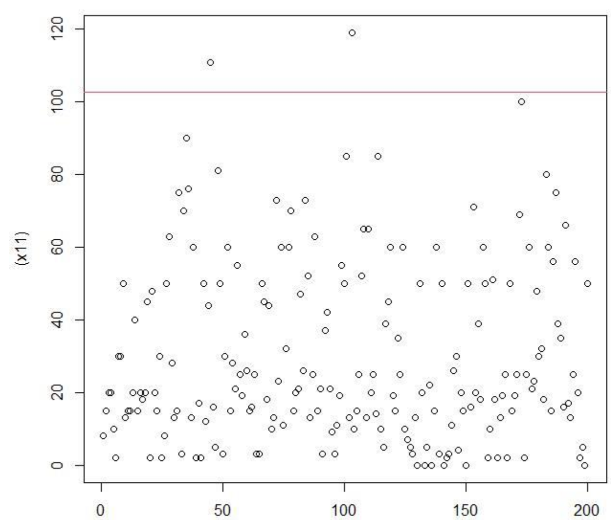

Figure-2: Test for the presence of outliers in X_{11}

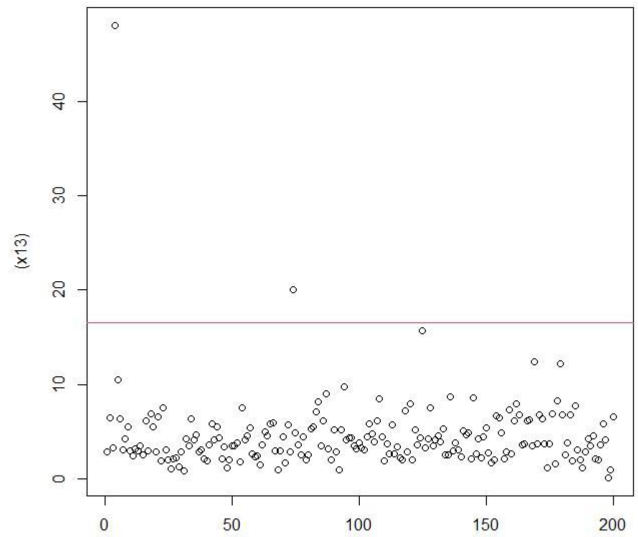

Figure-3: Test for the presence of outliers in X_{13}

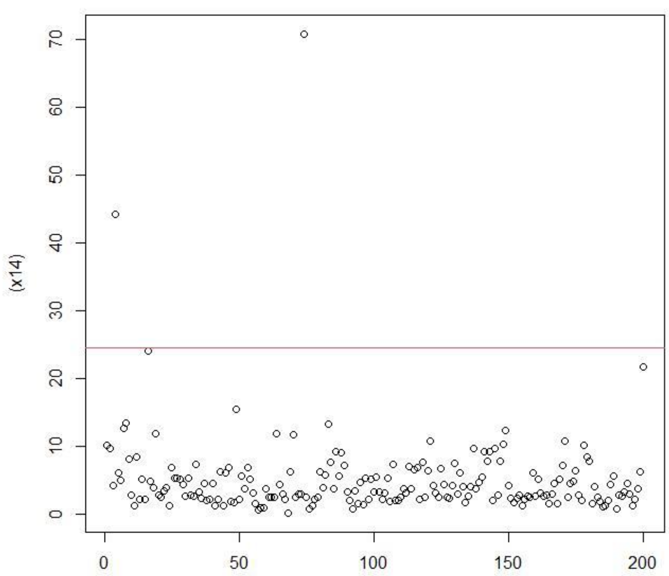

Figure-4: Test for the presence of outliers in X_{14}

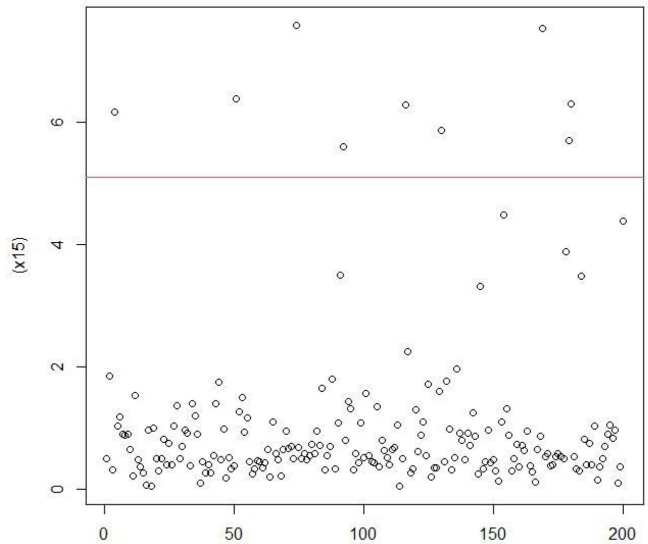

Figure-5: Test for the presence of outliers in X_{15}

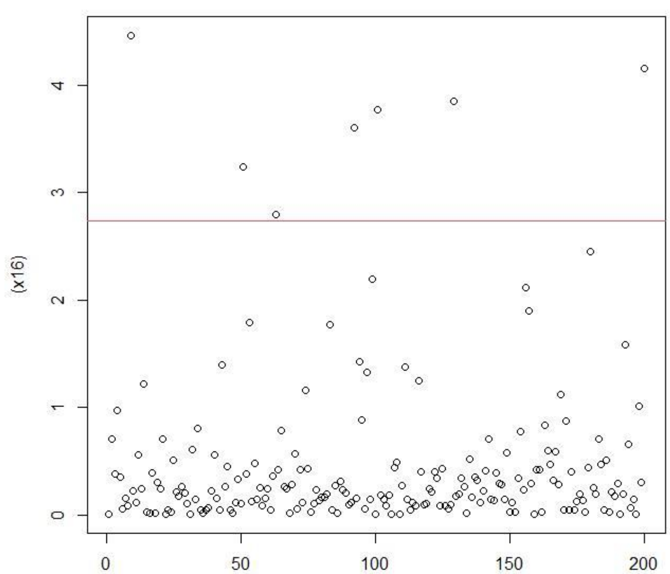

Figure-6: Test for the presence of outliers inX_{16}

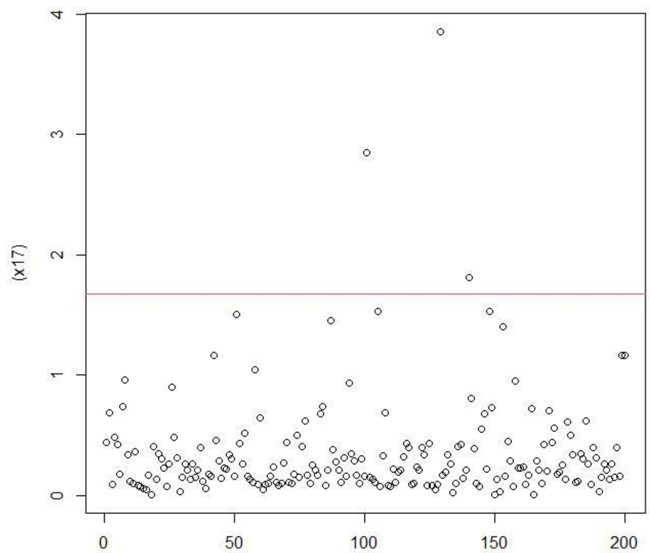

Figure-7: Test for the presence of outliers inX_{17}

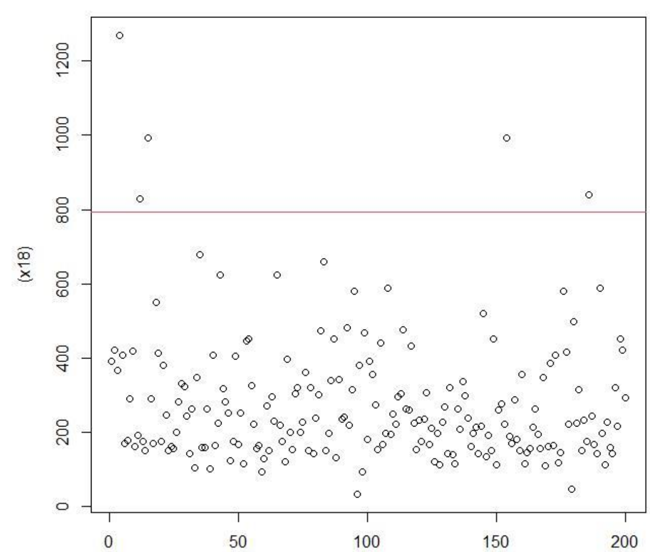

Figure-8: Test for the presence of outliers in X_{18}

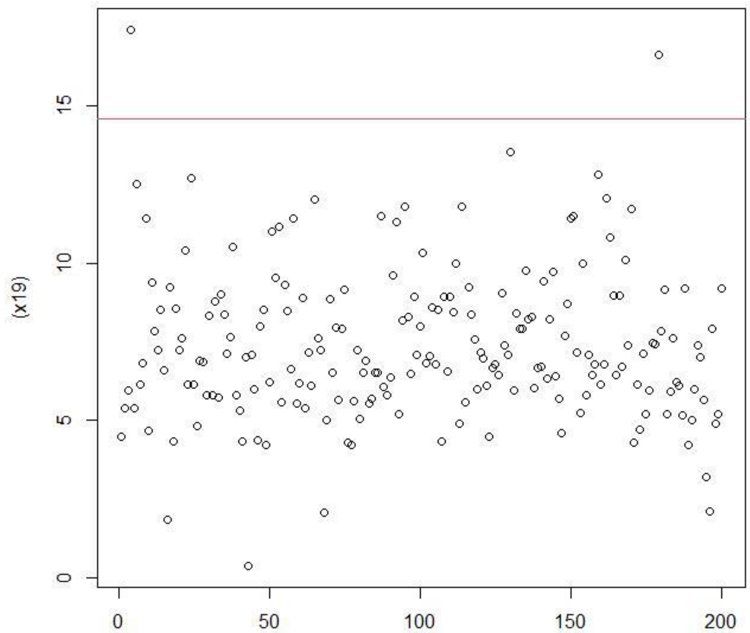

Figure-9: Test for the presence of outliers in X_{19}

Chapter four

(Conclusions and Future work)

4-1- Conclusions

1-We have suggested RSSIR-EN method in this study. It is a robust method to V.S and dimension reduction simultaneously under SDR settings.

2-This method has the efficiency when the predictors are highly correlated.

3-The results of numerical studies for between simulations and real data analysis have shown that the proposed RSSIR-EN has a best behavior in a V.S and estimation accuracy even with the existence outliers in predictors X and response variable Y.

4- The simulation studies demonstrated for different distributions of error and predictors X that the proposed RSSIR-EN outperformed the competitors RSSIR-Lasso and SSIR-EN approaches.

5- In addition, analytic results of anemia data and Boston Housing (B.H) data showed that the proposed method has best and consistent results.

4-2- Future work

We recommend this simple method for analyzing big data. This method can be developed and another version can be suggested robust sparse slice inverse regression with adaptive elastic net(RSSIR-Ad EN).

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المستخلص:

في تطبيقات طريقة الانحدار الشرائحي، يعد الانحدار العكسي الشرائحي المتناثر **(SIR)** طريقة لتقليل الأبعاد دون ان يفقد أي معلومات حول الانحدار الشرائحي. على الرغم من أن **(SIR)** قد تم إثباته كطريقة فعالة للتعامل مع مشاكل الأبعاد العالية، إلا أنه يعاني من حيث انه يعطينا تركيبات خطية التي تحتوي على جميع التنبؤات الاساسية. وقد اقترح العديد من الباحثين طرق مختلفة للتعامل مع هذه المشكلة من خلال الجمع بين طرق اختيار المتغيرات مع **SIR** ،وقد تكونت إحدى هذه الطرق بين **SIR** و**Elastic Net (SIR-EN)** حيث تعتبر **SIR –EN** طريقة فعالة دون افتراض نموذج معلمي، اذ إنه ينتج حلولًا دقيقة ومؤثرة عندما تكون المتنبئات مترابطة بشكل كبير في ظل افتراضات تقليل الأبعاد فيها ومع ذلك، فإن **SSIR- EN** ليست حصينة للقيم الشاذه بسبب وجود دالة خسارة المربعات الصغرى (OLS) والتي تتأثر بالقيم الشاذه في البيانات.

ونتيجة لذلك، اقترحنا **RSSIR-EN** كنسخة حصينة الى **SSIR-EN** للقيم الشاذه في كل من المتغير التابع والمتغيرات المستقلة.

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قسم الاحصاء

الأنحدار
الشرائحي المعكوس المتناثر الحصين مع التطبيق

رسالة مقدمة الى مجلس كلية الادارة والاقتصاد وهي جزء من متطلبات نيل درجة الماجستير في علوم الاحصاء

اعداد الطالبة

ضحى سالم وحيد الكيم

بإشراف

أ.د.علي جواد كاظم الكناني

1444 هـ **2023 م**