Forecasting Volatility Of OPEC Oil prices using EGARCH and ARMA-GARCH models

Prof. Dr. Mohammed H. Al-Sharoot
Department of Statistics
College of Administration and Economics
University Of AL-Qadisiyah-IRAQ
mohammed.AIsharoot@qu.edu.iq

Hanan A. Al-rashide
Department of Statistics
College of Administration and Economics
University Of AL- Qadisiyah-IRAQ
hananaallii7@gmail.com

Abstract

Some time series are characterized by their great volatility over time, especially time series related to the movement of the economy, and those related to the change in stock prices or the movement of financial transactions and stock markets, which are characterized by being non- stationary over time due to the change in the behavior of observations, making them suffer from the problem of Heteroscedasticity . The paper aims to building the best model to predict the future fluctuations in the daily OPEC oil price by applying different number of conditional autoregressive Heteroscedasticity models such as GARCH , EGARCH and ARMA-GARCH models , when errors follow Student's-t distribution, the results shows that the best model for predicting OPEC oil prices fluctuations is EGARCH(1,1), based on the AIC, SIC, and H-QIC .

1- Introduction

Some researchers focus on time series topics because of their importance in their studying the behavior of different phenomena during specific time periods through their analysis and interpretation. The aim of time series analysis is to describe the characteristics of the phenomenon, build a model and predict future based on what happened in the past. Phenomena that are fluctuations with time such as financial time series, it becomes inappropriate to apply linear models because some assumptions about random errors are not fulfilled, such as the mean errors are equal to zero , the variance is fixed with time and the errors are independent . which imposed a new challenge on scientists. so that non-linear models were proposed to take into account the problem of fluctuations in the time series, and to improve the matching of the model to the data and the ability to explain the fluctuations that occur in the different time series. Robert F. Engle in 1982 presented [10] new class of models called Autoregressive Conditional Heteroscedasticity models (ARCH(p)) to treats the problems in ARMA models especially in financial time series which some fluctuations (Volatility) associated with time. ARCH(p) model has been generalized by Bollerslev 1986[5] , who proposed the so-called Generalized Autoregressive Conditional Heteroscedasticity (GARCH). Nelson in 1991[17] proposed which were known an Exponential Generalized Autoregressive Conditional Heteroscedasticity models (EGARCH) to treat Asymmetric time series by developing a formula that differs GARCH family models, through adding equation of positives conditional variance in stacte of placing constraints on the model's parameters, but by formulating conditional variance equation in a way that sets the logarithm of the variance rather than the variance itself. positive. This invalidates a limitation on the model parameters as in the GARCH model, according to the exponential function structure, the range of the function is always positive, that is, the conditional variance is always positive and there is no “positive” or “negative” on its parameters and these models perform a specific function, which is to stabilize the variance, that is, to make the variance permanent and independent on time. For this reason, two types of variance will be exposed. First, conditional variance and the non-conditional variance for the same reason cannot be used for these models in order to predict the future of the time series only after using the models of a mixed of ARMA with GARCH models known as ARMA-GARCH. and this paper aims to build The best model for predicting daily fluctuations in OPEC oil prices for the period from 2/1/2003 to 30/6/2021 by applying a number of conditional autoregressive models of Heteroscedasticity such as GARCH models, EGARCH models, ARMA-GARCH .

2- Autoregressive Conditional Heteroscedasticity models ($ARCH_{(p)}$) [12][1][9][10][4]

This model was first proposed by (Engle, 1982) through his research on the variance of inflation in the United Kingdom. This type of model led to a major transformation in econometrics by filling the gap in the ARMA model, which assumes the stability of variance. The ARCH model has the ability to capture a set of fluctuations in the financial series, so these models can be treats the problem of Heteroscedasticity of random error variance by making it variates in time and it is defined as follows:

mean equation $z_t = \mu + \varepsilon_t$

$= \sigma_t W_t \quad \varepsilon_t \quad \sim \text{iid}N(0,1)\ W_t$

$$\sigma_t^2 = \alpha_\circ + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \alpha_i\varepsilon_{t-p}^2 \ldots\ldots\ldots\ldots(1)$$

$\alpha_\circ > 0$, $\alpha_i \geq 0$, $i = 1,2, \ldots\ldots,p$ (the parameters of the model)
z_t : return series .
μ : the mean of the series of stationary returns.
ε_t : the residuals series is unrelated
W_t : A series of randomly located variables with mean 0 and variance of 1.
σ_t^2: represents the conditional variance .
Equation (1) is known as the volatility equation and can be written as:

$$\sigma_t^2 = \alpha_\circ + \sum_{i=1}^{p} \alpha_i\, \varepsilon_{t-i}^2 \qquad \text{volatility equation} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The unconditional variance of $(\varepsilon_t)$is[20]

$$var\ (\varepsilon_t) = \frac{\alpha_0}{1 - \sum_{i=1}^{p} \alpha_i}$$

The process is stationary if the sum parameters of the autoregressive parameters are positive and less than one that is:

$$\sum_{i=1}^{p} \alpha_i < 1$$

3- Generalized Autoregressive Conditional Heteroscedasticity Model ($GARCH_{(p,q)}$) [2][9][18][12][20][19]

This model was proposed by (Bollerslev, 1986) as an extension of the ARCH model by adding many parameters to describe the volatility process of asset returns, which is known as the generalized ARCH model and is denoted by GARCH (p,q) and defined as follows:

$$z_t = \mu + \varepsilon_t$$
$$\varepsilon_t = \sigma_t \; W_t \qquad W_t \sim iid\, N(0,1)$$
$$\sigma_t^2 = \alpha_\circ + \sum_{i=1}^{p} \alpha_i \; \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \; \sigma_{t-j}^2 \quad \dots\dots\dots (3)$$

whereas :

$\alpha_\circ > 0$, ($\alpha_i \geq 0$, i = 1,2, ….. ,p) , ($\beta_j \geq 0$, j = 1,2, …… , q) and that $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} B_j < 1$

the unconditional variance of (ε_t)[20]

$$var\,(\varepsilon_t) = \frac{\alpha_0}{1 - \sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} B_j} > 0$$

The volatility shocks are more stationary in the case of approaching the correct one, and despite of the importance of ARCH (p) and GARCH (p,q) in modeling financial time series, but they are dissatisfaction for some economic analysts, especially in the matter of diagnosing the relationship between conditional variance and random error square, where this relationship is achieved in the case that the studied variables have the same effect size and the same sign, but in the case where the fluctuations are moving in opposite directions and with amplitude of varying effects, these models do not take into account those fluctuations. The dissatisfaction of the analysts economists led to the emergence of several models, such as Exponential GARCH.

4- Exponential Generalized Autoregressive Conditional Heteroscedasticity Models (EGARCH) [1][6][10][17][12]

This model was proposed by Nelson in (1991) to treats the asymmetry of fluctuations around shocks. This model is a development of the generalized GARCH model presented by (Bollerslev, 1986) that assumes symmetry of oscillations and the positive constraint imposed on the parameters. EGARCH model describes the relationship between the previous values of the random error and the logarithm of conditional variance, there is constraints on the parameters which ensuring that there are no negative effects of the conditional variance allowing to avoid the constraints of positive parameters (β_j, α_i)

the model EGARCH(p, q), (p ≥1) & (q ≥1) can be written as[1]

$$z_t = \mu + \varepsilon_t$$
$$\varepsilon_t = \sigma_t \; W_t \qquad W_t \sim iid\, N(0,1)$$
$$\log(\sigma_t^2) = \alpha_\circ + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^{p} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \sqrt{\frac{2}{\pi}} \right\} + \lambda_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)^{[1]}$$

or

$$log(\sigma_t^2) = \alpha_\circ + \sum_{i=1}^{p} \alpha_i \, g(Z_t) + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) \;\dots\dots\dots\dots\dots\dots (5)^{[10]}$$

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$$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E(|Z_t|)) \qquad \& \qquad Z_t = \varepsilon_t/\sigma_t$$

$$E(\varepsilon_t/\sigma_t) = E\left\{\frac{|\varepsilon_{t-i}|}{\sigma_{t-i}}\right\} = \sqrt{\frac{2}{\pi}}$$

Where $\alpha_\circ$, $(\beta_j, j = 1,2,\ldots..q)$ ،$(\alpha_i, i = 1,2,\ldots..p)$are the parameters not required to be positive, Zt is a standard normal variable $g(Z_t)$ allows the magnitude and the signal (Zt) to be separate effects from fluctuations, and that (Zt) are positive then the function $g(Z_t)$ is linear with parameters (θ+γ) and if (Zt) are negative, then $g(Z_t)$ is linear with the parameters (θ-γ) , This situations allows for asymmetry in the ups and downs of the stock price, which in turn is very useful, especially in the context of securities pricing, where the parameter α represents the volume effect or the symmetric effect of the model and representing the measure of stationary in conditional fluctuations no matter what happens in the market . When β is large, the volatility takes a long time to return to the crisis in the market. And the parameter γ measures the asymmetry or the effect of lifting, and this criterion is so important that the EGARCH model allows to test the asymmetry if γ=0 then the model is symmetric.

5- The Hybrid model ARMA (n, m) - GARCH (p, q) [14][15][16][6][7][8]

We know that ARMA models (n,m) have conditional mean of the prior information and conditional variance of the error. where GARCH models (p,q) have a constant conditional mean of the prior information and non-constant conditional variance of the error. If each of the conditional conditions is dependent on the past (non-constant), the two models will be combined with a model known as the hybrid ARMA (n,m) - GARCH (p,q) model defined follows:

$$y_t = \emptyset_0 + \sum_{i=1}^{n} \emptyset_i y_{t-i} + \varepsilon_t - \sum_{j=1}^{m} \Theta_j \varepsilon_{t-j} \quad \ldots\ldots\ldots..(6)$$

$$\varepsilon_t = \sigma_t \; W_t \quad , \qquad W_t \sim iid(0,1)$$

$$\sigma_t^2 = \Omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

Where :

y_t : ARMA (n,m)
e_t : is white noise with a mean of zero and a variance equal to one .
σ_t : is conditional variance and it is a function of the time difference of (ε_t , W_t) .

So it was mixed ARMA models with GARCH models where ARMA model are used for modeling and fitting conditional mean represents the conditional mean , and use GARCH model for modeling and fitting conditional variance, and it represents conditional variance.

6-Augmented Dickey-Fuller Test [14][2][3]

The Augmented Dickey-Fuller test (ADF) is used to detect the presence of a unit root in the univariate test, i.e. to test whether the time series is strong stationary or not. The ADF test is defined follows:

$$\Delta z_t = \alpha + \beta_t + \Upsilon z_{t-1} + \sum_{j=1}^{k} \delta_j \Delta z_{t-j} + \varepsilon_t \qquad \ldots\ldots \mathbf{(7)}$$

$\Delta z_t = z_t - z_{t-1}$, z_t is represents the time series to be tested .
k: the number of shifts .
$\varepsilon_t \sim$ iid (0, σ^2) and $(\alpha , \lambda , \delta_j, \Upsilon)$ the model parameters.
the hypothesis is :

$H_\circ : \Upsilon = 0$ The time series is non- stationary on mean.
.
$H_1 : \Upsilon \neq 0$ The time series is stationary on mean.

The test statistic is :

$$t = \frac{\widehat{\Upsilon}}{se(\widehat{\Upsilon})} \qquad \dots \dots (8)$$

7 -Ljung - Box Test [2] [13]

The test was proposed by (Ljung & Box) in 1978 is used to test whether the errors of the model fitted a time series are random :

$H_\circ : \rho_1 = \rho_2 = \cdots = \rho_k \dots = \rho_m = 0 \quad ; \qquad k = 1,2, \dots , m$
$H_1 : \rho_k \neq 0$ for some values of k .

Using the following statistics:

$$Q_M = \left(n(n+2) \sum_{k=1}^{m} \frac{\hat{\rho}_k^2}{n-k} \right) \sim \chi^2_{m-P} \dots \dots . . (9)$$

$\boldsymbol{n}$: the sample size (number of time series observations).
m : the number of backshifts for the autocorrelation .
P :The number of parameters estimated in the model .

8**- Lagrange Multiplier - ARCH Test** [2] [3]

proposed by Engle in 1982 to test whether the errors follow ARCH process is based on estimating the equation under study

$$\varepsilon_t^2 = \alpha_\circ + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + . \dots \dots \dots \dots . + \alpha_p \varepsilon_{t-p}^2 \qquad \dots \dots (10)^{[2]}$$

the test statistic as follows:

$$LM = ARCHtest = n \, \hat{R}^2 \sim \chi^2_{(r)} \dots \dots . (11)^{[2]}$$

n : the sample size.
r : The number of parameters estimated in the model
LM : stands for Lagrange multiplier.
$\widehat{R}^2$: the coefficient of determination estimated from $\boldsymbol{\hat{\varepsilon}_{t-1}^2, \hat{\varepsilon}_{t-2}^2, . \dots , \hat{\varepsilon}_{t-p}^2}$.

$$\hat{R}^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \qquad , \qquad 0 \leq \hat{R}^2 \leq 1$$

SSR : Sum of squares of the regression.

SST: Total sum of squares.

9 **- Estimation** [14]

Using the Maximum likelihood Method to estimate GARCH parameters (p, q) as follows:

$$f(\varepsilon_t/F_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{1}{2}\frac{\varepsilon_t^2}{\sigma_t^2}\right) \qquad (12)^{[14]}$$

The natural logarithm (L) function of vector parameters $\vartheta = (\alpha_\circ, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q)'$

We can write as follows:

$$L(\vartheta) = \sum_{t=1}^{n} I_t(\vartheta) \qquad \dots\dots\dots\dots (13)$$

the conditional logarithm of the parameter vector $\boldsymbol{\vartheta}$ is

$$I_t(\vartheta) = \mathrm{Ln}\, f(\varepsilon_t/F_{t-1})$$

$$I_t(\vartheta) = \frac{1}{2}\mathrm{Ln}(2\pi) - \frac{1}{2}\mathrm{Ln}(\sigma_t^2) - \frac{1}{2}\left(\frac{\varepsilon_t^2}{\sigma_t^2}\right) \qquad \dots\dots\dots..(14)$$

The following derivatives are calculated:

$$\frac{\partial I_t}{\partial \vartheta} = \frac{\partial I_t}{\partial \sigma_t^2}\,\frac{\partial \sigma_t^2}{\partial \vartheta}$$

The logarithm of the conditional probability density function is derived for the variable $\alpha_\circ$, α_i , β_j .

10 -Model selection criteria[13]

There are several criterions to choose the best model among the proposed fitted models for the studied data, these criteria was developed to select the most common model as follows :

I - Akaikes Information Criterion (AIC) [13][14]

This criterion was introduced by Akaike in 1974 an information standard known as (AIC) used to evaluate the suitability of time series models, we choose the model that gives the least AIC . The AIC formula can be written as:

$$AIC = n\ln(\hat{\sigma}_e^2) + 2L \qquad \dots\dots\dots (15)$$

n: the sample size.

$$\hat{\sigma}_e^2 = \frac{1}{n-L}\sum_{t=1}^{n}(z_t - \hat{z}_t)^2 \qquad \dots\dots\dots..(16)$$

L: the number of parameters in the model.

II - Schwarz Information Criterion (SIC) [2][14]

In 1978, Schwarz and Akaike proposed another criterion for determining the degree of the model known as the Schwartz Information Criterion, (SIC) and defined as follows:

$$SIC = n\, ln(\hat{\sigma}_e^2) + L\,\mathrm{ln}(n) \qquad \dots\dots\dots (17)$$

n: the sample size.

$$\hat{\sigma}_e^2 = \frac{1}{n-L}\sum_{t=1}^{n}(z_t - \hat{z}_t)^2$$

This criterion addressed the problem of over-estimation in the AIC standard, and make the penalty of the additional parameters stronger than the penalty in the AIC standard .

III - Hannan- Quinn Criterion (HQC)[13][2]

In 1979, this criterion was proposed by (Quinn) and (Hannan) HQC to determine the rank of the model and its formula:

$$HQC = \ln \hat{\sigma}_e^2 + 2L\ C\ ln\left(\frac{ln(n)}{n}\right) \quad , \quad C > 2 \quad \ldots\ldots\ldots (18)$$

As the second limit above decreases as quickly as possible at the stability of the rank due to the repeated logarithm.

11-Forecasting[10][12]

Forecasting is one of the most important goals of model building in time series, as it represents the last stage of time series analysis that cannot be reached without passing all diagnostic tests to validate the model used in forecasting.

Below is the prediction for the GARCH model and in the same way for all extensions of the model and from them EGARCH, ARMA- GARCH

suppose for the case GARCH(p, q) of p = 1, q = 1, GARCH (1,1) and my agencies:

$$\sigma^2{}_t = E\ (\varepsilon^2{}_t\ |I_t) = \hat{\alpha}_\circ + \hat{\alpha}_1\ \varepsilon^2{}_{t-1} + \hat{\beta}_1\ \sigma^2{}_{t-1}$$

Predicting one future value

$$\sigma^2{}_{t+1} = E(\varepsilon^2{}_{t+1}|I_t) = \hat{\alpha}_\circ + \hat{\alpha}_1\ E(\varepsilon^2{}_t|I_t)\ + \hat{\beta}_1\ \sigma^2{}_t$$

$$\sigma^2{}_{t+1} = \hat{\alpha}_\circ + \ \hat{\alpha}_1\ \sigma^2{}_t\ \ + \hat{\beta}_1\ \sigma^2{}_t$$

$$\sigma^2{}_{t+1} = \hat{\alpha}_\circ\ + \left(\hat{\alpha}_1\ + \hat{\beta}_1\right)\sigma^2{}_t$$

Prediction of value L

$$\sigma^2{}_{t+l} = E(\varepsilon^2{}_{t+l}|I_t) = \ \hat{\alpha}_\circ + \ \hat{\alpha}_1 E(\varepsilon^2{}_{t+l-1}|I_t)\ + \hat{\beta}_1\ E\ (\sigma^2{}_{t+l-1}\ |I_t\ )$$

$$\hat{\alpha}_\circ + \ \hat{\alpha}_1\ \sigma^2{}_{t+l-1}\ \ + \hat{\beta}_1\ \sigma^2{}_{t+l-1}\ \ = \ \sigma^2{}_{t+l}$$

$$\sigma^2{}_{t+l} = \hat{\alpha}_\circ\ + \left(\hat{\alpha}_1\ + \hat{\beta}_1\right)\sigma^2{}_{t+l-1}$$

Thus, the general formula for predicting GARCH (p, q) models is as follows:

$$\sigma^2{}_{t+l} = \hat{\alpha}_\circ + \sum_{i=1}^{p}\hat{\alpha}_i\sigma^2{}_{t+l-i} + \sum_{j=1}^{q}\hat{\beta}_j\sigma^2{}_{t+l-j}$$

12 - Forecasting accuracy measures[11]

The following measures are used to measure forecast accuracy , they are very important to know the chosen model.

i- Root Mean Square Error (RMSE) [11][12]

This criterion is defined as the square root of the squared difference between both the real variance and the estimated variance σ_t^2, and since there is no significant real variance the time series observations ε_t^2 are used, thus the RMSE formula is given as:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\sigma_t^2 - \widehat{\sigma^2}_t)^2} \qquad \dots\dots\dots(19)$$

$\hat{\sigma}_t^2$: represents the estimated variance.

σ_t^2 : represents the actual contrast.

ii- Mean Absolute Error (MAE) [11][12]

This measure is defined as the absolute difference between the actual variance and forecast variability measures, and it has the following formula:

$$MAE = \frac{1}{\text{n}}\sum_{t=1}^{\text{n}}\left|\sigma_t^2 - \widehat{\sigma^2}_t\right| \qquad \dots\dots\dots\dots\dots(20)$$

13- **Applied side**

This aspect includes an applied study on the construction and selection appropriate fluctuation models for the daily OPEC oil prices, excluding the stopping days, for the period from (2/01/2003) to (30/06/2021) where the number of observations are 4769, using conditional autoregressive in Heteroscedasticity , ARCH and GARCH.

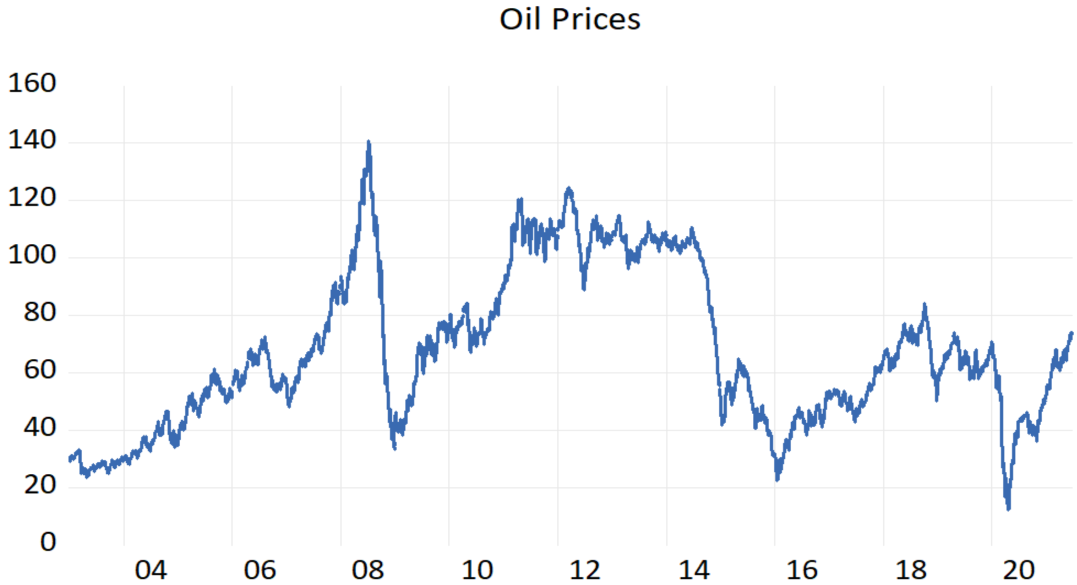


Figure (1) shows the time series of daily oil prices

It is clear from Figure (1) that the time series of oil prices is non -stationary in mean and variance has high volatility, which indicates the presence of fluctuations in the variance . for the purpose of revealing the stationary of the time series of daily oil prices, the Augmented Dickey-Fuller test and the test results were calculated as shown in Table (1)

Table (1) shows the Dickey-Fuller developer Time Series Test

Null Hypothesis: OP has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=31)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.932124	0.3176
Test critical values:	1% level	-3.431540	
	5% level	-2.861951	
	10% level	-2.567031	

*MacKinnon (1996) one-sided p-values.

We observe from Table (1) that the p-value is (0.3176) , we cannot reject the null hypothesis which means that the time series is non-stationary using the Box-Ljung test and through its Q statistic, we get the results as shown in Table (2).

Table (2) shows the autocorrelation, partial autocorrelation and Box-Ljung time series test functions

Date: 03/25/22 Time: 09:19
Sample: 1/02/2003 6/30/2021
Included observations: 4768

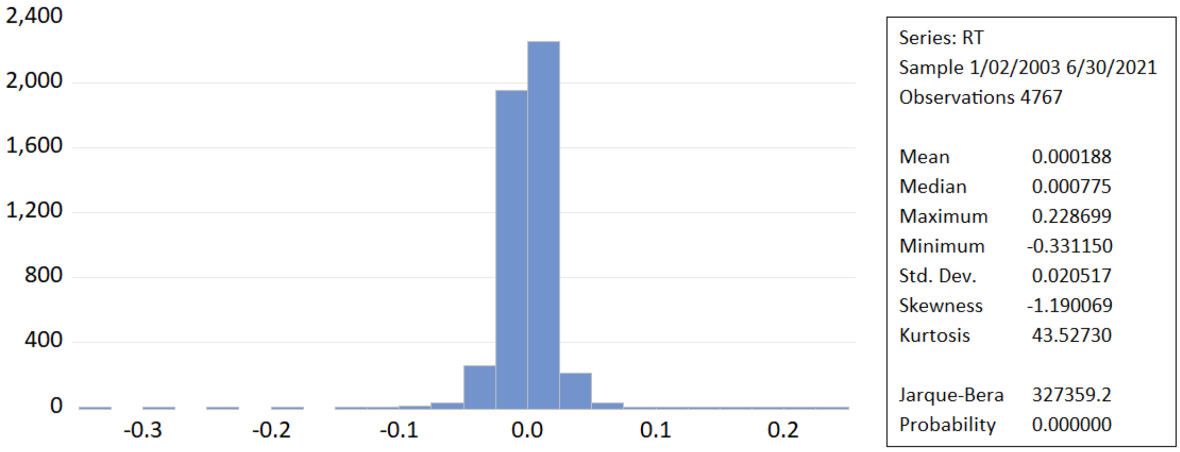
	AC	PAC	Q-Stat	Prob
1	0.999	0.999	4761.4	0.000
2	0.998	-0.180	9510.7	0.000
3	0.996	0.029	14248.	0.000
4	0.995	-0.023	18973.	0.000
5	0.993	-0.024	23685.	0.000
6	0.992	-0.023	28383.	0.000
7	0.990	0.014	33068.	0.000
8	0.989	-0.047	37739.	0.000
9	0.987	0.011	42396.	0.000
10	0.985	-0.014	47038.	0.000
11	0.984	-0.024	51665.	0.000
12	0.982	0.011	56277.	0.000
13	0.980	-0.030	60874.	0.000
14	0.979	-0.004	65455.	0.000
15	0.977	-0.033	70020.	0.000
16	0.975	-0.020	74568.	0.000
17	0.973	-0.005	79099.	0.000
18	0.971	-0.006	83612.	0.000
19	0.969	0.014	88109.	0.000
20	0.967	-0.020	92588.	0.000
21	0.965	-0.016	97049.	0.000
22	0.963	-0.009	101493	0.000
23	0.961	0.000	105918	0.000
24	0.959	-0.012	110325	0.000
25	0.957	-0.024	114713	0.000
26	0.954	-0.031	119082	0.000
27	0.952	-0.017	123431	0.000
28	0.950	0.009	127759	0.000
29	0.947	-0.015	132068	0.000
30	0.945	-0.020	136356	0.000
31	0.943	0.008	140622	0.000
32	0.940	-0.013	144869	0.000
33	0.938	-0.003	149094	0.000
34	0.935	-0.003	153297	0.000
35	0.933	-0.004	157480	0.000
36	0.930	-0.032	161641	0.000

Table (2) shows the significance of all autocorrelation which means rejecting the null hypothesis , and accepting the alternative hypothesis that says there is a sequential autocorrelation between the observations . and So that the increases are equal and independent of stationary in mean and variance , the returns y_t can be determine as follows :

$$y_t = \ln \frac{Z_t}{Z_{t-1}} = (\ln Z_t - \ln Z_{t-1})$$

where y_t is return at time t; In is the natural logarithm ; Z_t is the current daily stock price at time t, and Z_{t-1} is the previous daily stock price at time t-1. table (3) shows a summary for some descriptive measures of the returns series.

Table (3) shows some descriptive measures of the return series



It is clear from Table (3) that the mean of the return series is equal to (0.000188) with a standard deviation (0.020517), and the value of the skewness coefficient is (-1.190069), which indicates that the distribution of the returns series contains a tail to the left, and that the kurtosis coefficient is equal to (43.52730), which It indicates that the series has thick ends and is characterized by flatness and this indicates dispersion and therefore differs from the normal distribution, and this was confirmed by the Jarque-Bera test where the p-value corresponding to the test was (0.000000), which indicates that the data of the returns series do not follow the normal distribution at the level of significant (0.05). The graph of the return series can be illustrated in Figure (2).

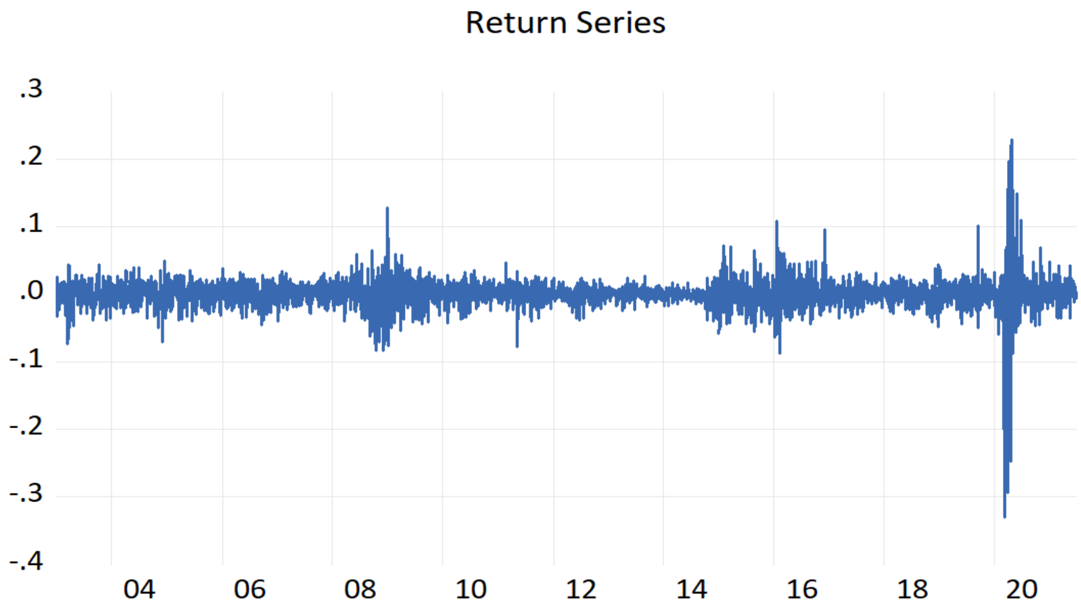


Figure (2) shows the series of returns to daily oil prices

From figure (2) we find that the series contains periods of volatility, followed by periods of relative stagnation in fluctuations, and so on over time.

Table (4) shows the Dickey-Fuller Extended Return Series Test

Null Hypothesis: RT has a unit root
Exogenous: Constant
Lag Length: 13 (Automatic - based on SIC, maxlag=31)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-15.31331	0.0000
Test critical values:	1% level	-3.431543	
	5% level	-2.861952	
	10% level	-2.567032	

*MacKinnon (1996) one-sided p-values.

It is clear from Table (4) that the (p-value) is equal to (0.0000) Which we rejecting the null hypothesis and accepting the alternative hypothesis that the return series is stationary.

To find out if the series has autocorrelation or not , we will calculate its Q statistic and the functions of autocorrelation and partial autocorrelation, as shown in the following table (5):

Table (5) shows the autocorrelation, partial autocorrelation and Box-Ljung test functions for the Returns Series

Date: 03/25/22 Time: 10:23
Sample (adjusted): 1/02/2003 6/29/2021
Included observations: 4767 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.134	0.134	85.155	0.000
		2	0.017	-0.000	86.600	0.000
		3	0.032	0.030	91.423	0.000
		4	0.053	0.046	105.03	0.000
		5	0.071	0.058	128.82	0.000
		6	-0.088	-0.109	166.13	0.000
		7	0.076	0.103	193.82	0.000
		8	-0.000	-0.031	193.82	0.000
		9	0.034	0.039	199.50	0.000
		10	0.034	0.024	204.90	0.000
		11	-0.035	-0.038	210.81	0.000
		12	0.055	0.044	225.45	0.000
		13	0.004	0.006	225.54	0.000
		14	0.096	0.082	269.55	0.000
		15	-0.004	-0.026	269.64	0.000
		16	0.019	0.027	271.36	0.000
		17	0.062	0.033	289.83	0.000
		18	0.009	0.005	290.18	0.000
		19	-0.017	-0.044	291.58	0.000
		20	-0.033	-0.005	296.85	0.000
		21	0.068	0.051	318.80	0.000
		22	0.003	-0.017	318.85	0.000
		23	-0.004	0.008	318.91	0.000
		24	0.042	0.032	327.27	0.000
		25	0.022	0.012	329.57	0.000
		26	0.072	0.050	354.46	0.000
		27	-0.041	-0.047	362.42	0.000
		28	0.025	0.020	365.41	0.000
		29	0.022	0.011	367.73	0.000
		30	-0.016	-0.027	368.91	0.000
		31	0.038	0.029	375.85	0.000
		32	-0.055	-0.048	390.23	0.000
		33	-0.003	-0.013	390.28	0.000
		34	-0.020	-0.009	392.24	0.000
		35	0.028	0.024	395.99	0.000
		36	-0.013	-0.027	396.81	0.000

It is clear from the values of the p-value column in Table (5) that the null hypothesis which states (there is no sequential autocorrelation of the return series) and the acceptance of the alternative hypothesis which states that (there is sequential autocorrelation of the return series) is below the level of significance (0.05), which indicates that the existence of a sequential autocorrelation between the observations of the returns Series.

To check whether the studied series is linear or not, the Tsay- test was used. The test results were as follows:

Table (6) shows the results of the nonlinearity test for the studied time series

Type of Test	Statistic	p-value
Tsay test	2.459697	0.0004

And through the (p-value) mentioned in Table (6) are less than (0.05) this indicates that the time series is a non-linear series.and using the Box-Jenkins methodology to build the model and forecast on the returns series observations, where a set of ARMA models were reconciled and their parameters were estimated by using maximum likelihood estimation (MLE), which are shown in Table (7)

Table (7) shows the compatibility of a group of models with some ARMA model selection criteria

Model	AIC*	BIC	HQ
ARMA(2,2)	**-4.958823**	**-4.950683**	**-4.955963**
ARMA(1,2)	-4.956094	-4.949311	-4.953711
ARMA(2,1)	-4.955371	-4.948588	-4.952988
ARMA(1,0)	-4.952462	-4.948392	-4.951032
ARMA(0,1)	-4.952279	-4.948209	-4.950849
ARMA(1,1)	-4.952043	-4.946616	-4.950136
ARMA(2,0)	-4.952043	-4.946616	-4.950136
ARMA(0,2)	-4.951980	-4.946553	-4.950073

It is clear from Table (7) that the best model is ARMA(2,2) because it has the lowest differentiation criteria (AIC, SIC, HQ) .

Using the greatest possibility method, the parameters of model were estimated, which are shown in Table (8).

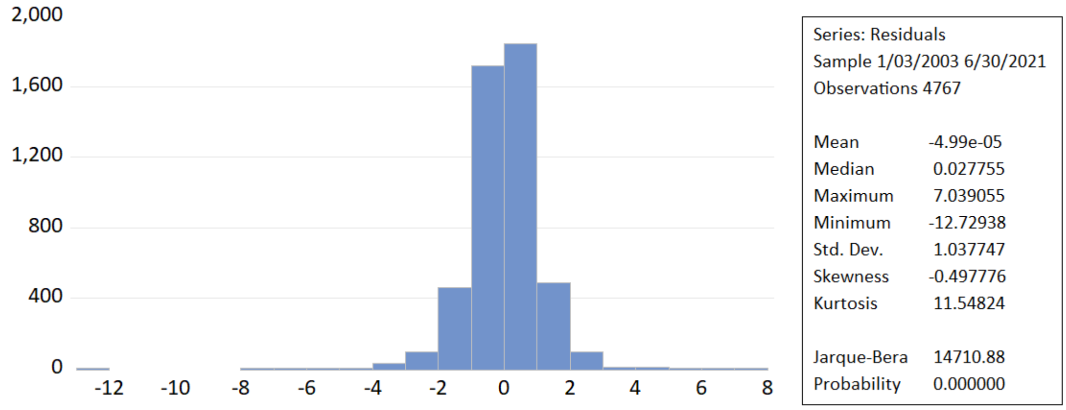
Table (8) shows the estimated values of ARMA(2,2) model parameters.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000191	0.000545	0.349781	0.7265
AR(1)	0.198827	0.022434	8.862736	0.0000
AR(2)	0.668094	0.014312	46.68044	0.0000
MA(1)	-0.084249	0.022929	-3.674279	0.0002
MA(2)	-0.708605	0.015163	-46.73272	0.0000
SIGMASQ	0.000410	1.83E-06	224.5593	0.0000

R-squared	0.025320	Mean dependent var	0.000189
Adjusted R-squared	0.024297	S.D. dependent var	0.020513
S.E. of regression	0.020262	Akaike info criterion	-4.958823
Sum squared resid	1.955526	Schwarz criterion	-4.950683
Log likelihood	11830.31	Hannan-Quinn criter.	-4.955963
F-statistic	24.74667	Durbin-Watson stat	1.960524
Prob(F-statistic)	0.000000		

From Table (8) we notice that all parameters of the model are significant. Residual analysis is an essential part and an important stage to know the validity of the model under study. This process is carried out either using statistical tests or using graphs, as shown in Figure (3).

Figure (3) shows the frequency chart and the overall descriptive statistics for the series of residuals resulting from matching the ARMA model (2, 2)



We notice from Figure (3) that the kurtosis coefficient was equal to (11.5428), so it is greater than 3, which confirms that the residuals are not distributed according to the normal distribution with negative skew modulus equal to (-0.49777). Thus, the residuals lost the state of a normal distribution. Also , in order to determine the nature of the distribution of the observations of the residual series, the (Kolmogorov-Smirnov) test was applied and the test results were as follows:

Table (8) Kolmogorov-Smirnov test for series of residuals

Distributions	D	p-value
Normal	0.52114	3.6e-15
Student's-t	0.03222	0.14430

Through the table (8) , it can be seen that the probability values of the (student's-t) distribution test statistics is greater than the probability (0.05), while the probability value of the Normal distribution is less then (0.05) , this indicates that the residual series follows the (student's-t) distribution.

ARCH and Ljung-Box test for ARMA residuals

To detect the presence of the effect of ARCH on the series of residues, the Ljung-Box test was used, and the results were as shown in Table (10).

table (10) Ljung-Box Test and Arch-LM Test for ARMA Residues

Ljung-Box test			Arch-LM test		
Lag	Q-Statistic	p-value	Lag	Obs*R-squared	p-value
5	21.176	0.0000	5	641.9976	0.0000
10	94.867	0.0000	10	755.5626	0.0000

15	174.93	0.0000	15	893.8865	0.0000
20	198.44	0.0000	20	1237.231	0.0000
25	221.84	0.0000	25	1291.860	0.0000

From the table (10), it can be seen that all p-values are less than 0.05 which mean that the null hypothesis that says of residuals has effect of Arch is not accepted, and we also notice the results of the Ljung-Box test that the (p-values) of the residual series values and its squares are less than (0.05), which indicates the existence of a autocorrelation, and that the series of residuals is characterized by the property of Heteroscedasticity

Estimation

i- Estimation of the GARCH model

By studying the autocorrelation and partial functions, and depending on the tests used in diagnosing the degree of models described in the previous paragraphs, four models can be suggested when the random error follows the Student's-t distribution , shown in Table (11) where the described models and parameters were estimated and criteria were calculated choose models as follows:

Table (11) shows the estimation of GARCH models using Student's t distribution of errors

MODEL / Estimate		GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH(2,2)
Mean Equation	μ	0.000490	0.000494	0.000492	0.000491
Variance Equation	Ω	2.00E-06	1.93E-06	2.12E-06	2.78E-06
	α1	0.088228	0.098376	0.094049	0.089671
	α2		-0.012393		0.032914
	β1	0.908275	0.910665	0.828768	0.514804

	β2			0.073490	0.357757
AIC		-5.617901	-5.617536	-5.617510	-5.617061
SIC		-5.609757	-5.608034	-5.608008	-5.606202
H-Q		-5.615040	-5.614198	-5.614172	-5.613246

From Table (11) we find that the best model according to AIC, SIC and H-QIC selection criteria is GARCH(1,1) shows estimated equation is

$$z_t = 0.000490 + \sqrt{2.00E^{-06} + 0.088228\ \varepsilon_{t-1}^2 + 0.908275\ \sigma_{t-1}^2} * w_t$$

ii - Estimation of the EGARCH model

Table (12) shows the estimation of the model parameters, as well as the criteria for choosing the appropriate model, as follows:

Table (12) shows estimating EGARCH models using Student's t distribution of errors

MODEL / Estimate		EGARCH(1,1)	EGARCH(1,2)	EGARCH(2,1)	EGARCH(2,2)
Mean Equation	μ	0.000201	0.000206	0.000204	0.000232

Variance Equation	Ω	-0.199167	-0.192197	-0.229505	-0.344574
	α1	0.141058	0.176869	0.163420	0.132769
	α2			-0.068685	0.114223
	β1	-0.058456	-0.039854	0.811150	-0.096683
	β2		-0.058322		0.240562
	λ	0.989331	0.989793	0.176640	0.741265
AIC		-5.628635	-5.628502	-5.628542	-5.628494
SIC		-5.619134	-5.617643	-5.617683	-5.616278
H-Q		-5.625297	-5.624687	-5.624727	-5.624202

It is noted from Table (12) that the best studied model within the EGARCH models, which will be relied upon is EGARCH(1,1), according to the criteria for choosing the model AIC, SIC, H-Q
The form can be written as follows:

$$z_t = 0.000201 + \sqrt{e^{-0.199167 - 0.058456\log(\sigma_{t-1}^2) + 0.141058\left|\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}} - \sqrt{\frac{2}{\pi}}\right| + 0.989331\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}} -}} * \mathrm{w}_t$$

iii- Estimation of ARMA-GARCH
The ARMA-GARCH model was applied, model parameters were estimated, and criteria for choosing the best model were calculated. As shown in Table 13.

Table (13) shows estimating ARMA-GARCH models using Student's t distribution for errors

MODEL / Estimate		ARMA(1,1)-GARCH(1,1)	ARMA(1,1)-GARCH(1,2)	ARMA(1,1)-GARCH(2,1)	ARMA(1,1)-GARCH(2,2)
Mean Equation	μ	0.000652	0.000656	0.000654	0.000666
Variance Equation	Ω	0.020154	0.019525	0.021232	0.009369
	α1	0.088159	0.096713	0.093407	0.104598

	α2		**-0.010384**		**-0.062598**
	β1	**0.908299**	**0.910248**	**0.837716**	**1.402374**
	β2			**0.065150**	**-0.446029**
AIC		**-5.621180**	**-5.620811**	**-5.620765**	**-5.620518**
SIC		**-5.611680**	**-5.609954**	**-5.609908**	**-5.608304**
H-Q		**-5.617842**	**-5.616997**	**-5.616951**	**-5.616227**

From the results of table (13) we find that the best model will be ARMA(1,1)-GARCH(1,1) according to the selection criteria and the model can be written as follows :

z_t =0.000652 $\varepsilon_{t-1} + \sqrt{0.020154 + (0.088159)\varepsilon^2_{t-1} + 0.908299\sigma^2_{t-1}} * \mathrm{w_t}$

Choose the appropriate model

Table (14) illustrates a comparison between the criteria for choosing the appropriate model

Table (14) shows the comparison of GARCH, EGARCH and . models GARCH-ARMA

Models	AIC	SIC	HQ
GARCH(1,1)	-5.61504	-5.609757	-5.617901
EGARCH(1,1)	-5.625297	-5.619134	-5.628635
ARMA(1,1) – GARCH(1,1)	-5.617842	-5.61168	-5.62118

We conclude from Table (14) that the EGARCH(1,1) model is superior model according to the criteria AIC, SIC, and H-Q.

Check the fit of the model.

Using Ljung-Box to find out the extent of residual correlation and the LM ARCH test to check the stability of variance for this model as in Tables (15) and (16).

Table (15) shows the Ljung-Box test for series of residuals

Lag	Q-Stat	Prob*
5	5.8658	0.319
10	10.106	0.431
15	13.059	0.598
20	31.605	0.048
25	39.152	0.036
30	50.359	0.011
35	54.466	0.019

We show from table (15) that the probability value in the lags (5,10,15) was greater than (0.05), which indicates the acceptance of the null hypothesis which states that there is no autocorrelation between the squares of the residuals, while in lags (20,25,30, 35) the probability value is less than (0.05), which means that the null hypothesis is rejected and the alternative hypothesis is accepted, that is, there is which autocorrelation between the residual squares.

Table (16) showing the LM ARCH residual test

Heteroskedasticity Test: ARCH

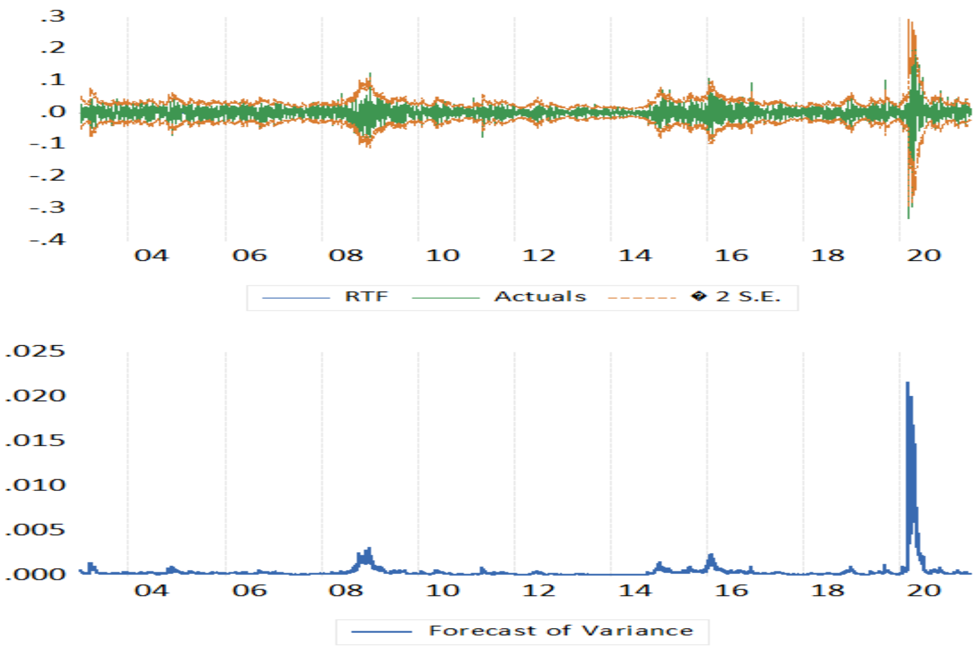
F-statistic	0.916081	Prob. F(1,4764)	0.3386
Obs*R-squared	0.916290	Prob. Chi-Square(1)	0.3385

From table (16), we note that the probability value corresponding to the test, which is equal to (0.3385) is greater than (0.05), which indicates that we accept the null hypothesis which states that the variance is homogeneous for the error and therefore there is no ARCH effect.

Forecasting future volatility

After determining the appropriate model ، EGARCH(1,1) is using to predict the fluctuations as shown in Figure (4)

Figure (4) shows the graph of the series of returns, the predicted values, and the prediction of volatility (variance)



Volatility forecast performance

The purpose of the prediction within the sample is to test the predictive power of the model, so it is not necessary that the chosen model be the one that gives the best prediction. absolute error (MAE) as table (17) shows the results of these criteria

Table (17) shows the comparison between each of the models GARCH(1,1) and EGARCH(1,1) and ARMA(1,1) - GARCH(1.1) based on accuracy criteria

MODEL	RMSE	MAE
GARCH (1,1)	0.97464	0.82401
EGARCH (1,1)	0.29843	0.15622
ARMA(1,1) - GARCH (1,1)	0.65540	0.50104

It is clear from the table (17) that we note the superiority of the EGARCH model over the rest of the models according to the accuracy criteria, the root mean squares error and the mean absolute error, which in turn indicates that the model is very accurate and therefore is the best model for predicting daily oil price fluctuations.

Conclusions and Recommendations

Conclusions

1- The series of oil prices is non-stationary on mean and variance.
2. The series of return for oil prices does not follow the normal distribution.
3. The series of returns to oil prices contains periods of fluctuation, followed by periods of relative stagnation over time.
4. Oil prices series of returns is stationary on mean .
5. The advantage of the EGARCH (1.1) model over the GARCH(1,1) and ARMA(1,1) - GARCH(1,1) models for forecasting the future volatility of OPEC oil prices through standards AIC, SIC, H-Q and precision scales RMSE, MAE
6. The conditional Autoregressive Heteroscedasticity models are more efficient in predicting the volatility.

Recommendations

1. Use other comparison models such as GJR-GARCH, IGARCH and NGARCH.
2. Use other methods to estimate model parameters such as QMLE.
3. Use of GARCH family models to predict other financial time series to estimate and study the behavior of these series because they have the ability to explain the behavior of these series that is characterized by Heteroscedasticity of variance.

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