Iraqi Exchange pricing Analysis with Stochastic Delay Differential Equations

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Abstract: *In this paper, I mostly focused on the application of stochastic delay differential equations (SDDE) to Iraqi exchange price. I used the stochastic delay differential equation theory to solve the parallel dollar exchange pricing problem in the Iraqi Central Bank of which underlying prices can be described by standard Geometric Brownian Motion. This paper discusses the Black-Scholes Process as a model of drift and diffusion parameters. Moreover, the application of the ordinary Black-Scholes, delay in drift term delay in diffusion term, and delay in both of drift and diffusion terms delay stochastic differential equations. I figured out that the SDDE can model the Iraqi exchange prices in a more explained and analytical way.*

Keywords - *Black-Scholes, Iraqi exchange price, stochastic delay differential equation, Geometric Brownian Motion.*

I. Introduction:

In the field of finance, things changed randomly and unexpected events. Along with this, exchange price phenomenon motivates many authors to study and analyses its behaviors by the tools of mathematical finance. Black and Scholes are two experts in the field of field of finance developed what is called Black-Scholes model in ١٩٧٣, see Black and Scholes (١٩٧٣a,١٩٧٣b), in ١٩٧٩ cox et al. produced the Binomial model, and in ١٩٧٧ Boyle produced the Monte Carlo model. Black –Scholes model motivate many authors to developed more finance models, such that, martingale price model that proposed by Harrison and Kreps (١٩٧٠), stochastic volatility model that produced by Hull and White (١٩٨٧), and Levy process model that introduced by chan (١٩٩٩).In this paper, we introduces the Black –Scholes model to study and analyses behavior of exchange price. In financial market, the modeling of the asset prices is based on stochastic fluctuations. So, suppose that the stochastic differential equation that model the asset price satisfy,

$$ds_t = \mu s_t dt + \sigma s_t dw_t \dots\dots\dots\dots..(١)$$

The equation (١) modeled with initial solution $S_. > ٠$ at $t = ٠$, Where μ is the drift parameter that measures the average growth rate of price, σ is the diffusion parameter that measures the strength of price fluctuation (dispersion), and W_t is the Brownian Motion Process, see Aladagli (٢٠١٧), Zheng (٢٠١٥) for further information. The solution of process (١) is defined as follows.

$$s(t) = s(٠)e^{\left(\mu-\frac{\sigma^٢}{٢}\right)t+\sigma w(t)} \dots.(٢)$$

Where the expected value and variance of solution (١) are as follows

$Es(t) = s(٠)e^{\mu t},$

$$V(s(t) = s^٢_{(٠)}e^{٢\mu t}\left[e^{\sigma^٢ t} - ١\right].$$

The equation (١) considered as stochastic differential equation (sde). The sde have some drawbacks, such as, sde cannot use the historical information (past information) about the interested phenomenon. It is well known that the past information is very important to understand and to analyze the future movements of the underlying market. This past (historical) information is called delay or the memory. By denote the delay parameter by λ, and adding this parameter will make the stochastic differential equation more better and realistic model. Hence, the stochastic delay differential equation (sdde) in both of drift and diffusion terms is defined by

$$ds(t) = f\big(t, s(t), s(t-\lambda)\big)dt + g\big(t, s(t), s(t-\lambda)\big)dw(t)$$

Where f and g are function defined as

$R^{+} \times R \times R \rightarrow R$

and s(t) is the stock price. See Arriojas et al. (٢٠٠٧).

II. Existence and uniqueness solution of SDDE:

In this section, first we will give the general formulation of stochastic delay differential equations. Second, some definitions and theorem about the existence and uniqueness for SDDEs will be discussed. These definitions are necessary to understand the conditions clearly. Now, suppose that we have the following SDDE,

$$\begin{cases} dx(t) = f(t, x(t), x(t-\lambda))dt + g(t, x(t), x(t-\lambda))dw(t) \;; ٠ \leq t \leq T \\ x(t) = \emptyset(t) \qquad -\lambda \leq t \leq ٠ \quad , T > ٠ \end{cases} \qquad \ldots \quad (٣)$$

Where λ is the delay parameter which is a positive fixed number, and $\phi(t)$ is a measurable random variable satisfy

$$[E(\sup|\emptyset(t)|^{p}]^{\frac{١}{p}} < \infty$$

The equation (٣) defined based on a complete probability space (Ω, F, P) with filtration$[F_t]_{t\geq ٠}$. Buckwar (٢٠٠٢).

Definition (١): Let $(\Omega\,, F, P)$ be a probability space filtered with $[F_t]_{t\geq ٠}$, and $W(t)$ is the Brownian motion. Then the solution x(t) of the SDDE in (٣) with the initial value $\emptyset(٠)$ can be written as the following Itô from

$$x(t) = \emptyset(٠) + \int_{٠}^{t} g(s, x(s)\,, x(s-\lambda)ds + \int_{٠}^{t} g(s, x(s)\,, x(s-\lambda)dw(s).$$

Definition (٢): The two functions f and g in equation (٣) are said to satisfy the local Lipchitz condition if there is a positive constant C satisfies:

$$|f(t, x_١, y_١) - f(t, x_٢, y_٢)| \vee |g(t, x_١, y_١) - g(t, x_٢, y_٢)| \leq \mathbf{c}(|x_١ - x_٢| + |y_١ - y_٢|)$$

for $x_١, y_١, x_٢, y_٢ \in R\,, t \in R^{+}$

Definition (٣): The two functions f and g in equation (٣) satisfy the linear growth conditions if k satisfies

$$|f(t, x, y)|^{٢} \vee |g(t, x, y)|^{٢} \leq K(١ + |x|^{٢} + |y|^{٢})$$

for all $(t, x, y) \in R^{+} \times R \times R$.

Theorem (١): The two functions f and g in equation (٣) satisfies the definition (١) and (٣), and then there exists unique strong solution to equation (٣). Hence, the solution salsifies,

$$\mathrm{E}\left[sup_{t\in[-\lambda,T]}|x(t)|^{٢}\right] < \infty \quad ,T > ٠$$

The numerical solution for SDDE can be obtained through Euler–Maruyama (E-M) scheme which is one of widely used numerical tool which provides the strong solution for SDDE. Euler–Maruyama used the stochastic calculus through Ito process to find the numerical solution of SDDE in (٣), see Lamberton and Lapeyre (٢٠٠٧), and Oksendal (٢٠١٣) for the prove.

Theorem (١): Suppose that an SDDE is defined as follows,

$$\mathrm{dx(t)} = \mathrm{g}\big(\mathrm{t}, \mathrm{x(t)}, \mathrm{x}(\mathrm{t}-\lambda)\big)\mathrm{dt} + \mathrm{u}\big(\mathrm{t}, \mathrm{x(t)}, \mathrm{x}(\mathrm{t}-\lambda)\big)\mathrm{dw(t)} \quad ; \mathrm{t} \in [\mathrm{a}١, \mathrm{a}٢]$$

$$\mathrm{x}(\mathrm{t}_٠) = \psi_٠(\mathrm{t}) \qquad ; \mathrm{t}_٠ - \lambda \leq \mathrm{t} \leq \mathrm{t}_٠$$

Partition of the interval $[\mathrm{a}١, \mathrm{a}٢]$ as follows

$$٠ = \mathrm{t}_٠ < \mathrm{t}_١ < \cdots < \mathrm{t}_\mathrm{N} = \mathrm{t}$$

$$\Delta\mathrm{t}_{\mathrm{n}+١} = \mathrm{t}_{\mathrm{n}+١} - \mathrm{t}_\mathrm{n} = \mathrm{z}$$

Here $\Delta\mathrm{t}_{\mathrm{n}+١}$ are the time increments and z is uniform step size defined as follows $\mathrm{z} = \frac{\mathrm{a}١-\mathrm{a}٢}{\mathrm{N}}$, and $\Delta\mathrm{w}_n = \mathrm{w}(\mathrm{t}_{\mathrm{n}+١}) - \mathrm{w}(\mathrm{t}_\mathrm{n})$ represents the standard Brownian process with $\mathrm{n} = ٠, ١, \ldots, \mathrm{N} - ١$, and $\mathrm{t}_\mathrm{n} = \mathrm{a}١ + \mathrm{nz}$

Then, the sequence of numerical solutions given as follows

$$\mathrm{x}_{\mathrm{n}+١} = \mathrm{x}_\mathrm{n} + \mathrm{g}(\mathrm{t}_\mathrm{n}, \mathrm{x}_\mathrm{n}, \psi_{\mathrm{n}-\lambda})\mathrm{h} + \mathrm{u}(\mathrm{t}_\mathrm{n}, \mathrm{x}_\mathrm{n}, \psi_{\mathrm{n}-\lambda})\Delta\mathrm{w}_\mathrm{n} \quad ; ٠ \leq \mathrm{n} \leq \mathrm{N} - ١$$

See Jassim (٢٠٠٦) for more details.

III. Real Data Analysis :

In this section, we provide an application of SDDEs in order to explain and examine the concepts that are discussed in the last section and see the effect of time delay in the SDDE. In this application, we consider the market of the Iraqi exchange price (the parallel price of US. Dollar). So that Iraqi exchange price usually depends on the past information. Therefore, in this application we suppose that the model of Iraqi exchange price is expressed as a function of delay term and the behavior of delay term is examined in drift and/or diffusion term. The data are collected on a daily basis from ٢٠١٤-٢٠١٩. The Iraqi exchange price responds to the information that is obtained at the previous time point λ. Based on that assumption, this feedback process is following SDDE while implementing a linear delay into the geometric Brownian motion for Iraqi exchange price in Black-Scholes model. The formulation of this model is defined as follows:

$$\mathrm{ds}_\mathrm{t} = (\mu\mathrm{s}_\mathrm{t} + ٢\mathrm{e}^{-١}(\mathrm{s}_\mathrm{t} - \lambda) + ٣.٢\mathrm{e}^{-١})\mathrm{dt} + \sigma^{٢}(\mathrm{s}_\mathrm{t} + ٣(\mathrm{s}_\mathrm{t} - \lambda))\mathrm{dwt}$$

We considered three scenarios for the above SDDE, the first one is that the delay parameter is only in drift term, the second one is that the delay parameter is only in diffusion term, and the third one is that the delay parameter is in both of drift and diffusion terms. The following figure illustrates the Iraqi exchange prices different pathways.

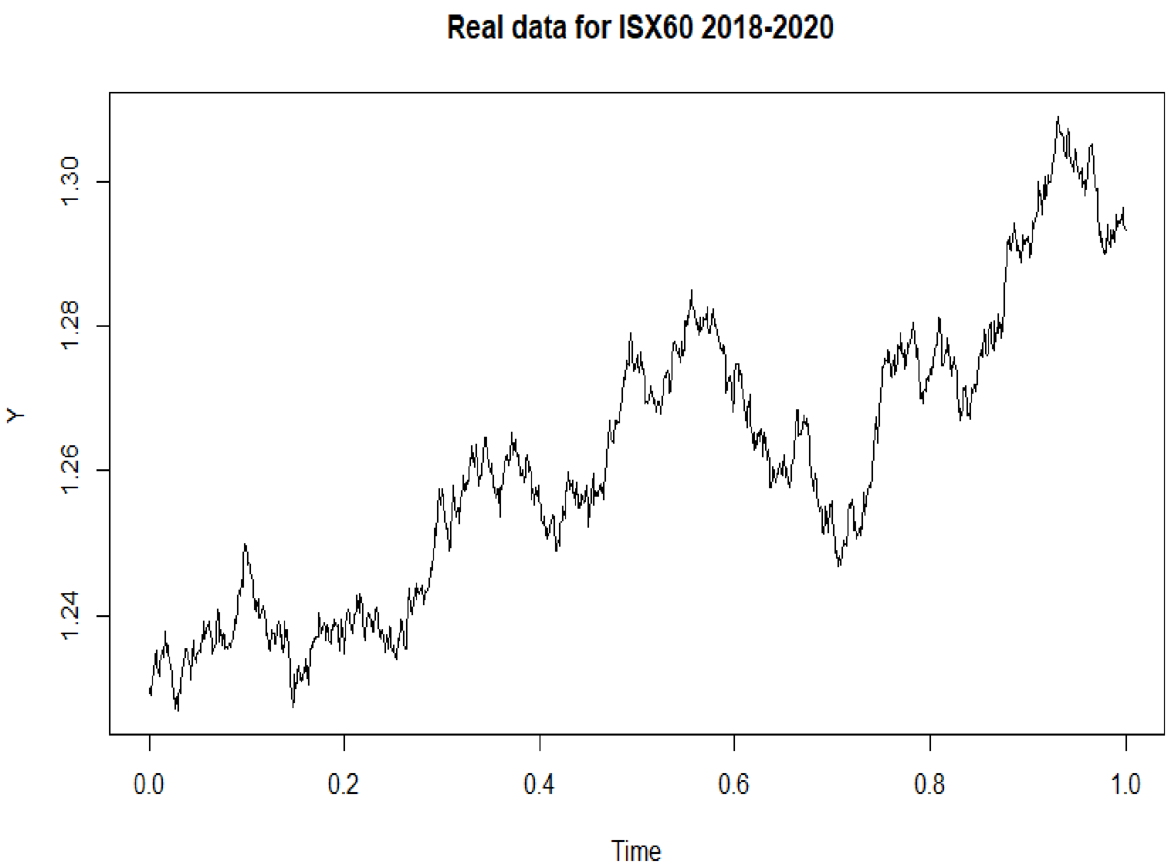

Figure ١: Sample path for SDDE

The Figure (١) displays one sample path with λ = ٠.٦, μ = ٠.٠٠٣, σ = ٠.٠٥. With time t = ٠, the sample pathway decreases while pathway with time t = ١ increases. We observed that graph shows increases values for the sample paths between t = ٠ and t = ١.

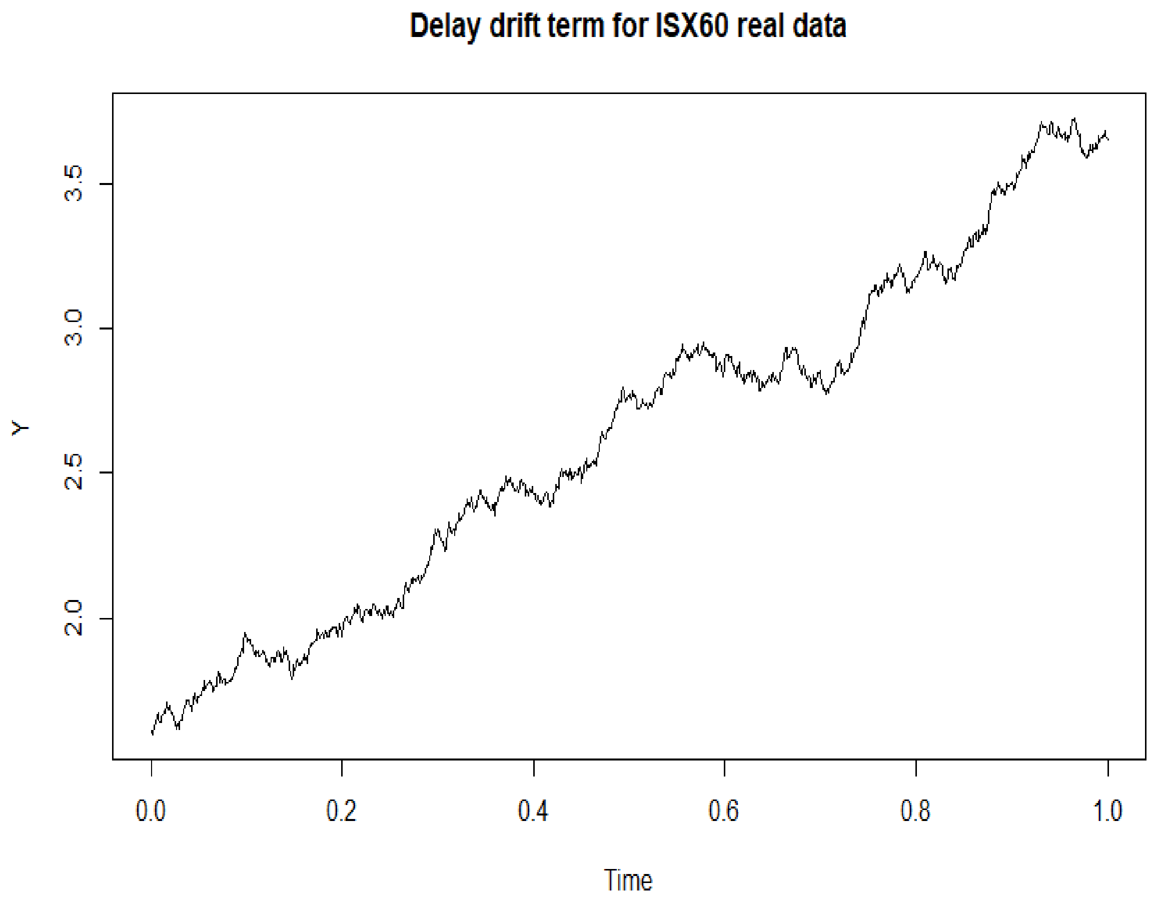

Figure ٢: Sample path for SDDE in drift term

The Figure (٢) displays one sample path withλ in only in drift term where $\lambda = ٠.٦, \mu = ٠.٠٠٣, \sigma = ٠.٠٥$. With time t = ٠, the sample pathway decreases while pathway with time t =١ increases. We observed that graph shows increases values for the sample paths between t = ٠ and t = ١. The following table explains some important values for the results of Black-Scholes model.

Table (١) some important statistics

Min	$١^{st}.Qu$	Median	Mean	$٣^{rd}.Qu$	Max
١.٥٩٨	٢.٠٧٣	٢.٧٤٢	٢.٦٣٠	٣.٠٩١	٣.٧٣٠

The table (١) shows that ٪٥٠ of the values between ٢.٠٧٣ and ٣.٠٩١, in addition, ٪٧٥ of values lower than ٣.٠٩١١ and ٪٥٠ have results above ٢.٧٤٢.

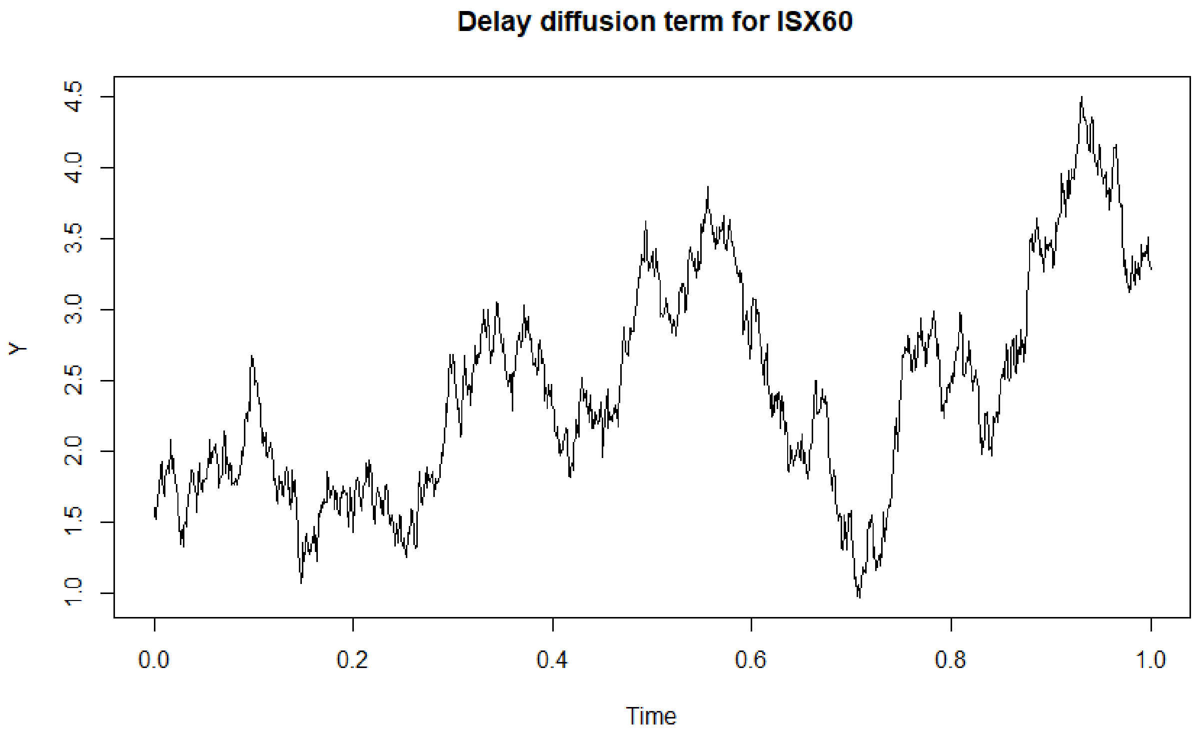

Figure ٣: Sample path for SDDE in diffusion term

The Figure (٣) displays one sample path with λ in only in diffusion term, where $\lambda = ٠.٦, \mu = ٠.٠٠٣, \sigma = ٠.٠٥$. With time t = ٠, the sample pathway decreases while pathway with time t =١ increases. We observed that graph shows increases values for the sample paths between t = ٠ and t = ١. The following table explains some important values for the Black-Scholes model results.

Table (٢) some important statistics

Min	$١^{st}.Qu$	Median	Mean	$٣^{rd}.Qu$	Max
٠.٩٧٤	١.٨٠٩	٢.٣٥٤	٢.٤٣٩	٢.٩٣٠	٤.٥٠٢

The table (٢) shows that ٪٥٠ of the values between ١.٨٠٩ and ٢.٩٣٠, in addition, ٪٧٥ values lower than ٢.٩٣٠ and ٪٥٠ have results above ١.٨٠٩.

IV. Conclusions:

In this paper, we focus on the application of stochastic delay differential equations to Iraqi exchange pricing. We use the geometric Brownian motion to model the price of underlying exchange. With the assumption of Black-Scholes model used to analyze the Iraqi exchange price. We can notice that the Black-Scholes model is very applicable and feasible for the Iraqi exchange price. We have discussed some definitions and theorem about the existence and uniqueness of SDDE. We used Euler Maruyam scheme for SDDEs and its convergence pathway analysis. In order to illustrate the impact of delay, we substitute the delay term (λ=). The influence of delay is analyzed in some figures and tables.

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