Weighted M-Huber in the Presence of High Leverage Point

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Abstract: When the regression data contains an outlier, a serious problem leads to a breakdown of the least squares estimator, where robust regression methods should be recommended. It is well known that the M-Huber method has a 0.50 high breakdown point, but it breaks down when a single leverage point exists in the data. This article suggests a weighted M-Huber estimate by assigning a specific weight for each leverage point present in the design matrix of X. The performance of the proposed method has been tested with the original M-Huber by using real data and simulation. The results show that our weighted M-Huber method is more efficient and reliable than M-Huber.

Keywords: M-Huber, Robust regression, outliers, Breakdown point, and Leverage point.

Introduction

It is well known that the estimates of the Least Squares(LS) method are the best linear unbiased estimates when their assumptions are met. Unfortunately, it is very hard to satisfy all of these assumptions (Uraibi, 2009). For instance, the presence of outliers violates the normality assumption of random errors, and therefore, robust regression methods are recommended. There are two types of outliers in regression data, one in the y-direction or in the regression residuals, which are so-called outliers, and leverage points that are present in the X-direction. Hence, the random error distribution F_ε is approximately normal and can be formalized as follows

$$F_\varepsilon = (1-\varepsilon)N + \varepsilon H, \qquad (1)$$

where N is normal with zero mean and constant variance, H may be another distribution and $\varepsilon \in [0,0.5]$. Sometimes, H is also normal distribution, but with different parameters and in case Eq. (1) is considered normal mixture distribution. However, the parameters of H would determine the shape of the distribution, probably thin-tailed or heavy-tailed (thinner or heavier than exponential distribution). Moreover, Huber and Ronchetti (1981) introduced M-estimate that is an iterated and re-weighted LS method to obtain robust regression coefficients. Also, Rousseeuw (1984) introduced Least Median Squares (LMS), which is ordering the squared of residuals from lower to upper values. Then, the estimation of regression coefficients is based on the half of data that analog the lower values of squared residuals. In addition, Rousseeuw and Leroy (1987) considered dealing with 50% of data means losing a lot of information about the studied phenomena. Therefore, they suggested Least Trimmed Squares (LTS), which looks for the clean subset of data after trimming the proportion of outliers. Then, the regression coefficients should be estimated using LS. The Least Absolute Deviation (Huber, 1987) was put forward to minimize the sum of absolute values of errors. The breakdown point of these methods is 1/n when the single leverage point is present in the dataset set (Croux et al., 2003).

The classical Mahalanobis Distance (MD) measure was the most familiar choice to identify the leverage points. However, Rousseeuw and Zomeren (1990) said that MD suffers from masking phenomena effects; thus, it is a non-robust measure (see: Midi et al. (2020), Uraibi and Midi (2020), Uraibi and Alhussieny (2021)). Some statistics practitioners have believed the best alternative choice is hat matrix (Hoagline and Welsch, 1978). However, they were not aware it had proportional relation with the classical squared Mahalanobis distance (Chatterjee and Hadi, 2015). Hence, it is known as a non-robust measure too. Apart from that, Ellis and Morgenthaler (1992), Hubert and Rousseeuw (1997), Giloni et al. (2006a, 2006b), and Arslan (2012) reported that down-weighting the leverage point can improve the conditional breakdown point of the estimator. Therefore, they identified leverage points based on the values of robust Mahalanobis distances and then derivative weight function to increase the value of the breakdown point. In addition, Uraibi (2019) used a new weighted function to overcome the problem of high leverage points. His procedure is to employ the Re-weighted Fast and Consistent High breakdown estimator (RFCH) that was introduced by Olive and Hawkins (2020) (see: Uraibi et al. (2015), Uraibi et al. (2017) and Uraibi et al. (2019)). This paper adopts Giloni et al. (2006a) weighted function to identify and assign downing weights to the leverage points.

The rest of the paper is organized as follows: Section 2 discusses Weighted M-estimate, while Section 3 discusses Rela data. Then, in Section 4, the simulation study has been done to assess the performance of the Weighted M estimate. Moreover, the result of real data has been discussed in Section 5. Finally, a brief conclusion of this research follows Section 6.

2. Weighted M-estimate

Consider the linear regression model,

$$y_i = \mathbf{x}_i^{\mathrm{T}}\beta + \varepsilon_i, \qquad i = 1,2,\ldots,n, \tag{2}$$

where $\mathbf{x}_i$ is the p dimensional of independent variables, which may include an intercept, β is a p −vector of unknown regression coefficients, ε_i is the random errors with mean equals to zero and constant variance. By taking the expected value of Eq. (2) would result in

$$\hat{y}_i = \mathbf{x}_i^{\mathrm{T}}\hat{\beta}, \tag{3}$$

where $\hat{\beta}$ estimates are the best linear unbiased estimates that minimize the objective function of sum squared residuals.

$$\hat{\beta} \quad = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^{n} \varepsilon_i^2, \tag{4}$$

in which $\varepsilon_i = y_i - \mathbf{x}_i^{\mathrm{T}}\hat{\beta}$.

Suppose that the data set has a leverage point in the x-direction and let the ε_i terms follow the distribution of contaminated model Eq. (1). In this case, LS is not a practical choice, where robust methods are recommended. One of the familiar robust methods is M-estimate, which is resistant to outliers, but it is sensitive to leverage points with zero breakdowns. Our proposed method takes into account weighted M-estimate (WM-estimate) to increase the breakdown point. WM-estimate can be described into the following steps:

Step **1.** Weighted design matrix $\mathbf{x}$.

The MCD (Rousseeuw and Van Driessen, 1999) location and scatter estimators have to be computed in this step. Then, the vector of Robust Mahalanobis Distance RMD^2 is calculated as follows

$$RMD^2 = (\mathbf{x} - \hat{\mu}_{MCD})' C_{MCD}^{-1} (\mathbf{x} - \hat{\mu}_{MCD}). \quad (5)$$

It is obvious that when the i^{th} observation is leverage point, the i^{th} RMD^2 would be a large value. Thus, assigning low weight for leverage point requires inversely proportional of the i^{th} RMD_i^2 with the clean subset (Giloni et al., 2006a),

$$\omega_i = \min\left\{1, \frac{\chi^2_{(0.05,p)}}{RMD_i^2}\right\}. \quad (6)$$

Here, the new weighted design matrix can be written as $\mathbf{x}_\omega = \omega . \mathbf{x}$, and the estimates of LS with $\mathbf{x}_\omega$ can be formalized as follows

$$\hat{\beta}_\omega = (\mathbf{x}'_\omega \mathbf{x}_\omega)^{-1} \mathbf{x}_\omega y, \quad (7)$$

$$\hat{y}_\omega = \mathbf{x}_\omega \hat{\beta}_\omega,$$

$$\hat{\varepsilon}_\omega = y - \mathbf{x}_\omega \hat{\beta}_\omega.$$

Step 2. Iteratively re-weighted least squares (IRLS) for $(\mathbf{x}_\omega, y)$.

The $\hat{\beta}^M$ estimates are obtained by minimizing an objective function ρ expressed as

$$\hat{\beta}^M = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^{n} \rho(\hat{\varepsilon}_{\omega i}), \quad (8)$$

where ρ is a symmetric function with a unique minimum at zero. Taking the partial derivative with respect to β and setting them equal to zero produces a system of normal equations that can solve this minimization problem. Thus, by letting $\psi = \rho'$, we obtain

$$\sum \psi(\hat{\varepsilon}_{\omega i}) x_{\omega i} = 0. \tag{9}$$

Several choices of ρ and ψ functions are available. In this paper, we used bisquare functions (Tukey, 1964) as follows

$$\rho(\hat{\varepsilon}_{\omega i}) = \begin{cases} \left\{1 - \left[1 - \left(\frac{\hat{\varepsilon}_{\omega i}}{k}\right)^2\right]^3\right\} & \text{if} \quad |\hat{\varepsilon}_{\omega i}| \le k \\ 1 & \text{if} \quad |\hat{\varepsilon}_{\omega i}| > k \end{cases}, \tag{10}$$

$$\psi(\hat{\varepsilon}_{\omega i}) = \hat{\varepsilon}_{\omega i} \left[1 - \left(\frac{\hat{\varepsilon}_{\omega i}}{k}\right)^2\right]^2 \qquad I(|\hat{\varepsilon}_{\omega i}| \le k), \tag{11}$$

$$w(\hat{\varepsilon}_{\omega i}) = \frac{\psi(\hat{\varepsilon}_{\omega i})}{\hat{\varepsilon}_{\omega i}} = \begin{cases} \left[1 - \left(\frac{\varepsilon}{k}\right)^2\right]^2 & \text{if} \quad |\hat{\varepsilon}_{\omega i}| \le k \\ 0 & \text{if} \quad |\hat{\varepsilon}_{\omega i}| > k \end{cases}, \tag{12}$$

where $I(.)$ stands for indicator function, that is

$$I(\hat{\varepsilon}_{\omega i}) = \begin{cases} 1 & \text{if} \quad \hat{\varepsilon}_{\omega i} > 0 \\ 0 & \text{if} \quad \hat{\varepsilon}_{\omega i} < 0 \end{cases}.$$

Consequently, the estimation equation may be written as

$$\sum w(\hat{\varepsilon}_{\omega i}) x'_{\omega} = 0. \tag{13}$$

These estimating equations require minimizing $\sum w_i^2 (\hat{\varepsilon}_{\omega i})^2$ by using iteratively re-weighted least-squares (IRLS).

$$\hat{\beta}_{\omega}^{M(j)} = \left(\mathbf{x}'_{\omega} w^{(j-1)} \mathbf{x}_{\omega}\right)^{-1} \mathbf{x}'_{\omega} w^{(j-1)} y. \tag{14}$$

In IRLS, the initial fit is calculated, where a new set of weights is calculated based on the results of the initial fit. The iterations are continued until a convergence criterion is satisfied.

3. Simulation

The simulation studies have been done to know the performance of the WM-Huber method compared with the M-Huber method. The design matrix $X_{n\times 6}$ of the six independent variables is generated randomly from a multivariate normal distribution with zero means and $\rho^{|i-j|}$ variance and covariance matrix, $\rho = 0.20$. The maximum value of X_1, which are replaced by the value generated from $\chi^2_{(0.05,50)}$ to create the high leverage point and $m = \alpha \times n$ observations of $\{X_3, X_4, X_6\}$, are contaminated by using the previous contamination mechanism to create another leverage point. Here, α is the percentage of the outlying observation. The first m of random errors $\varepsilon_{m\times 1}$ vector is generated from a chi-square distribution with 50 degrees of freedom. Moreover, the remaining $\varepsilon_{(n-m)\times 1}$ are generated from random normal distribution $N(0,2)$. The maximum value in X_1 are times by a random value of $\chi^2_{(0.05,50)}$ to create a high leverage point. Suppose that the population regression coefficient, which is denoted as $\beta_{7\times 1}$ is known, and $\beta_{7\times 1} = (0.001,1,0,0,1,1,1)$, the response variable $y_{n\times 1}$ can be computed as follows

$$y = X_{n\times 7} \;\; \beta_{7\times 1} + \varepsilon_{n\times 1}. \tag{15}$$

This simulation scenario has been considered when α = 0.05 for $n = \{45,65,85,100\}$, where n is the number of samples. This simulation study is designed to have three non-zero coefficients and three zero coefficients. The best method is the one that diagnoses the correct significant and non-significant coefficients as well as the lower average of robust residuals standard error. For this purpose, both methods were used to get the results of 5000 datasets. The average of coefficients $\hat{\beta}$ $, SD(\hat{\beta}), t, \bar{P}$.value and R.S.Error is computed for both methods' overall datasets. The decision-making about rejecting or accepting the null hypothesis that assumes the regression coefficients are equal to zero would be based on $\bar{P}$.value. Here, the null hypothesis will get rejected when the ($\bar{P}$.value< 0.05). Meanwhile, when the alternative hypothesis of a specific regression coefficient is accepted, that means it is different from zero. The method will recognize the significant coefficient, which is denoted by three stars (***). The best method is the one that diagnostic the correct significant and non-significant coefficients. On the other hand, the best model is the one that has lower R,S.Error than the others.

Table 1. The simulation result of the M-Huber method when $n = 45, \alpha = 0.05$.

	$\hat{\beta}^M$	$SD(\hat{\beta}^M)$	t	$\bar{P}$.value	Sig	R.S.Error
Intercept	0.555	1.137	0.090	0.474		6.007
X1	5.213	0.156	20.672	0.284		
X2	-0.575	1.146	-0.047	0.511		
X3	0.110	1.044	0.104	0.471		
X4	0.798	1.044	2.259	0.105		
X5	4.155	1.144	8.614	0.024	***	
X6	4.080	0.953	10.405	0.026	***	

The results in tables (1,2,3,4,5,6,7,8) show the simulation result when $n = \{45,65,85,100\}$ were contaminated by 0.05 outliers and leverage points. It is clear that all $\bar{P}$.values of intercept, X1,X2,X3 coefficients in tables$\{1,3,5,7\}$ are greater than 0.05. In other words, the null hypothesis testing that claim $\hat{\beta}_0^M = \hat{\beta}_1^M = \hat{\beta}_2^M = \hat{\beta}_3^M = 0$ is accepted. Indeed, from our simulation setting $\hat{\beta}_1^M \neq 0$, this case is considered a type II error. Although the null hypothesis testing claim that $\hat{\beta}_4^M = \hat{\beta}_5^M = \hat{\beta}_6^M = 0$ is rejected, M-Huber was doing well with these variables. However, in general, the robustness of this method is not sufficient to deal high leverage point that occurs in the data of X1. On the other hand, the robust performances of the WM-Huber method are presented in tables (2,4,6,8). It is notable that no type II errors nor type I errors are robust against all types of outliers and leverage points. Both methods are consistent, where the sample size is increasing, but WM-Huber is more robust and stable than the M-Huber method as all tables have shown that the R.S.Error of WM-Huber is lower than M-Huber R.S.Error. Moreover, the R.S.Error of WM-Huber looks constant, while its counterpart with M-Huber is much affected in the presence of high leverage points.

Table 2. The simulation result of the WM-Huber method where $n = 45, \alpha = 0.05$.

	$\hat{\beta}^M$	$SD(\hat{\beta}^M)$	t	$\bar{P}$.value	Sig	R.S.Error
Intercept	0.555	0.311	-0.046	0.516		1.712
X1	5.213	0.348	2.973	0.041	***	
X2	-0.575	0.375	-0.019	0.508		
X3	0.110	0.355	0.048	0.480		
X4	0.798	0.357	2.938	0.030	***	
X5	4.155	0.373	11.199	0.000	***	
X6	4.080	0.340	12.196	0.000	***	

Table 3. The simulation result of the M-Huber method where $n = 65, \alpha = 0.05$.

	$\hat{\beta}^M$	$SD(\hat{\beta}^M)$	t	$\bar{P}$.value	Sig	R,S.Error
Intercept	0.850	0.850	-0.004	0.505		5.945
X1	4.742	0.143	22.706	0.254		
X2	-0.742	0.854	-0.178	0.547		
X3	0.008	0.779	-0.078	0.525		
X4	0.995	0.795	2.965	0.060	***	
X5	4.053	0.842	10.666	0.024	***	
X6	4.000	0.698	13.172	0.020	***	

Table 4. The simulation result of the WM-Huber method where $n = 65, \alpha = 0.05$.

	$\hat{\beta}^M$	$SD(\hat{\beta}^M)$	t	$\bar{P}$.value	Sig	R,S.Error
Intercept	0.239	0.254	-0.033	0.506		1.717
X1	4.742	0.284	3.568	0.017	***	
X2	-0.742	0.302	-0.063	0.516		
X3	0.008	0.283	-0.050	0.518		
X4	0.995	0.284	3.664	0.010	***	
X5	4.053	0.299	13.645	0.000	***	
X6	4.000	0.270	15.259	0.000	***	

Table 5. The simulation result of the M-Huber method where $n = 85, \alpha = 0.05$.

	$\hat{\beta}^M$	$SD(\hat{\beta}^M)$	t	$\bar{P}$.value	Sig	R,S.Error
Intercept	0.063	0.411	-0.049	0.511		3.681
X1	2.306	0.078	20.484	0.254		
X2	-0.279	0.412	-0.043	0.504		
X3	-0.032	0.368	-0.018	0.509		

X4	1.013	0.375	4.130	0.018	***	
X5	3.956	0.409	15.018	0.010	***	
X6	4.019	0.335	18.249	0.005	***	

Table 6. The simulation result of the WM-Huber method where $n = 85, \alpha = 0.05$.

	$\hat{\beta}^M$	$SD(\hat{\beta}^M)$	t	$\bar{P}$.value	Sig	R,S.Error
Intercept	0.092	0.221	-0.084	0.518		1.773
X1	2.487	0.246	4.129	0.004	***	
X2	-0.263	0.257	0.081	0.478		
X3	0.018	0.245	-0.030	0.506		
X4	0.956	0.245	4.180	0.003	***	
X5	4.048	0.254	16.040	0.000	***	
X6	4.024	0.233	17.451	0.000	***	

Table 7. The simulation result of the M-Huber method where $n = 100, \alpha = 0.05$.

	$\hat{\beta}^M$	$SD(\hat{\beta}^M)$	t	$\bar{P}$.value	Sig	R,S.Error
Intercept	0.092	0.477	-0.058	0.508		3.962
X1	2.487	0.086	20.879	0.257		
X2	-0.263	0.476	-0.052	0.514		
X3	0.018	0.435	-0.002	0.505		
X4	0.956	0.436	3.605	0.033	***	
X5	4.048	0.469	13.327	0.006	***	
X6	4.024	0.393	16.165	0.008	***	

Table 8. The simulation result of the WM-Huber method where $n = 100, \alpha = 0.05$.

	$\hat{\beta}^M$	$SD(\hat{\beta}^M)$	t	$\bar{P}$.value	Sig	R,S.Error
Intercept	0.063	0.203	-0.016	0.505		1.786
X1	2.306	0.224	4.476	0.002	***	
X2	-0.279	0.235	0.053	0.484		
X3	-0.032	0.224	-0.033	0.509		
X4	1.013	0.224	4.515	0.001	***	
X5	3.956	0.233	17.446	0.000	***	
X6	4.019	0.215	18.806	0.000	***	

4. The Modified Market value of Iraq's Trade Banks

The data are collected from the official website of the Iraqi Market Exchange for nine local trade banks, which are the most traded than others for the period (2011-2015). In addition, the researchers considered six official websites (Trading Rate (X1), Earning per share (EPS) (X2), share turnover ratio (X3), Annual Average price (X4), the Assets (X5), and Undistributed earnings (X6)).

We modified this data by replacing the 5th observation of X1 and 15th of X4 with random observations that have been generated from $\chi^2_{(0.05,50)}$ distribution to contaminate both variables using leverage points. The y_{30} and y_{45} observations are replaced with two values from $\chi^2_{(0.05,50)}$ distribution too in order to contaminate the response variable. Here, y is the banks market value that we expect is affected with these variables according to the multiple linear regression model that can be described as follows:

$$y_{(45\times1)} = X_{(45\times7)}\beta_{(7\times1)} + \varepsilon_{(45\times1)}.$$

Table 9. The estimate of the M-Huber method for the modified market value of Iraq's trade Banks data.

	Value	$SD(\beta)$	t	P. value	Sig	R,S.Error
Intercept	-0.247	0.040	-6.251	0.999		0.25
X1	-0.006	0.007	-0.877	0.807		
X2	-0.269	0.071	-3.807	0.999		
X3	0.003	0.040	0.080	0.468		
X4	-0.007	0.006	-1.248	0.890		
X5	0.403	0.047	8.558	1.07E-10	**	
X6	0.344	0.076	4.555	2.640E-05	**	

It is clear that only the coefficients of X5 and X6 are significant, with 0.25 robust residual standard errors of the regression model. We note that although X2 and X3 do not have leverage points, both are non-significant, while X1 and X4, which have leverage points, are non-significant too. The results presented in Table 10 shows that the method of the WM-Huber considered that the coefficients of X1 and X4 are significant to confirm its robustness against the leverage points. On the other hand, both M-Huber and WM-Huber agreed that (X5, X6) variables are non-zero coefficients while (X2, X3) are non-significant. The robust residual standard errors of the new model are 0.09, which is lower than the value of M-Huber model standard errors.

It is noticed in Table 9 that three of the regression coefficients are negative, which are the intercept of the Trading Rate, Earning per share and Annual Average price. It indicates that these variables have inverse relationships with the market value. In other words, the higher the market value, for instance, results in lower (Trading Rate, Earning per share and Annual Average price) variables, and vice versa. As for the rest of the variables, they maintained a positive relationship with the market value of the banks. On the other hand, Table 10 presents that Trading Rate positively relates to banks market values, such as others, except for earnings per share and Annual Average price variables.

Table 10: The estimates of the WM-Huber for the modified market value of Iraq's trade banks data.

	Value	$SD(\beta)$	t	P. value	Sig	R,S.Error
Intercept	-0.138	0.021	-6.656	1.000		0.09
X1	0.088	0.023	3.742	0.0003	***	
X2	-0.143	0.072	-1.977	0.972		
X3	-0.008	0.036	0.228	0.410		
X4	0.216	0.036	6.001	2.79E-07	***	
X5	0.567	0.067	8.465	1.4E-10	***	
X6	0.181	0.052	3.506	0.0005	***	

5. Conclusion

This paper suggests the weighted M-Huber method to tackle the problem of leverage points present in the design matrix X. Due to the M-Huber being resistant to outliers in regression residuals, the WM-Huber is resistant to outliers and leverage points. Furthermore, based on the results of real data and simulation, the evidence points are almost exclusive to the high performance and robustness of the WM-Huber. At the same time, the M-Huber method has been unreliable when the high leverage points are present in the dataset. These findings encourage us to recommend using the WM-Huber in scientific applications.

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