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Ideals and permuting revers tri – derivations of prime and semi prime rings

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بسم الله الرحمن الرحيم

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صدق الله العلي العظيم

سورة المجادلة – الآية (11)

الاهداء

الى من انعش قلب طفولتي

الى تلك الشيبة المقدسة

حبيبي و عالمي و مسندي ابي

لولاك لما كنت حيث امكث الآن..

الى حبيبتي امي

الى اخوتي و اصدقائي

الى اصحاب الدماء الطاهرة

فخر وطني و سوره الاعظم

الى من اختنقوا حتى نتنفس

جيشآ و حشدا

الى من قال عنهم النبي محمد ص : ((فضل العالم على العابد كفضل القمر ليلة البدر على سائر الكواكب)) الى كل من تعلمت حرفا تحت اشعته اساتذتي الاعزاء واخص بالذكر د.مازن عمران كريم الذي لم يمل من تساؤلاتي ولم يزعجه فضول علمي الى كل من ساعدني في انجاز هذا العمل اهدي بحثي هذا متمنيا من الله ان تكون نقطة بداية في طريق الحياة..

الى اللا نهاية لنجاحاتي ان شاء الله.

شكر و تقدير

بعد الحمد والشكر لله رب العالمين الذي منَ عليَ بفضله وكرمه والصلاة والسلام على الصادق الامين محمد صلى الله عليه وسلم وآل بيته الطيبين الطاهرين وانطلاقآ من قوله صل الله عليه وسلم: (من لا يشكر الناس لا يشكر الله). وفي مستهل هذا البحث وعرفاناً مني بالجميل أتقدم بجزيل شكري وفائق تقديري الى اساتذتي الأفاضل في قسم الرياضيات واخص بالذكر منهم : د.مازن عمران كريم الذي تكرم واشرف على هذا البحث بكل مسؤولية وفي تسهيل مهمتي وانضاج تجربة البحث العلمي وكان له الفضل الكبير في مساعدتي. وأتوجه لكل من مد لي يد العون بالشكر والامتنان , فجزاهم الله عني خير الجزاء .

وختاما اسال الله العلي القدير ان يكون هذا العمل خالصا لوجهه , وان يجعله علما نافعا , ويسهل لي به طريقا الى الجنة .

Contents

Abstract

The concept of a permuting tri – derivation has been stated by Duran õzden and Mehmet Ali õztārk . In young – soo . Jung & Kyoo – Hong park has prove some results concerning symmetric bi – derivation on prime and semi prime rings . C . Jaya Subba Reddy & M. Ramakrishna Naik . has studies reverse derivation and permuting reverse tri – derivations on prime rings C. Jaya Subba Raddy ,V. Vijaya Kumar & K. Hemavathi studied few results ideals and permuting tri – derivations of prime and semiprime rings . I this research we give some results on ideals and permuting reverse tri – derivations of prime and semiprime rings .

Introduction

Throughout this research R , will be denoted an associative ring with center Z (R) . Recall that a ring R is called prime if for any a , b ∈ R , a R b = (0) implies that either a = 0 or b = 0 for any x , y ∈ R . A ring R is said to be semi prime if a R a = 0 with a ∈ R implies a = 0 . we shall write [x , y] for xy – yx .

A mapping D (. , . , .) : R x R x R → R is called permuting if D (x , y , z) = D (x , z , y) = D (y , x , z) = D (y , z , x) = D (z , x , y) = D (z , y , x) for all x , y , z ∈ R . A mapping d : R → R define by d (x) = D (x , x , x) is called the trace of D , where D (. , . , .) : R x R x R → R is a permuting mapping . It obvious that , if D (. , . , .) : R x R x R → R is a permuting mapping which is also tri – additive then the trace d of D satisfies the relation d (x + y) = d (x) + d (y) + 3 D (x , x , y) + 3 D (x , y , y) for all z , y , ∈ R .

Let R be a ring and I be a non – zero right(resp . lefer) ideal of R . An additive mapping d : R → R is said to be a derivation if d (x y) = d (x) y + x d (y) for all x , y ∈ R . An additive mapping d : R → R is said to be a reverse derivation if d (xy) = d (y) x + y d (x) , for all x , y ∈ R . if D (. , . , .) is tri – additive and satisfies the identities D (xw , y , z) = D (x , y , z) w + x D (w , y , z) , D (x , yw , z) = D (x , y , z) w + y D (x , w , z) and D (x , y , zw) = D (x , y , z) w + z D (x , y , w) then D (. , . , .) is called a permuting tri – derivation . If D (. , . , .) is reverse tri – additive and satisfies the identity $D(xw, y, z) = D(w, y, z)x + wD(x, y, z)$, $D(x, yw, z) = D(x, w, z)y + wD(x, y, z)$, $D(x, y, zw) = D(x, y, w)z + wD(x, y, z)$ for all $x, y, z, w \in R$. then $D(., ., .)$ is called a permuting reverse tri – derivation we shall make use of commutator identities, $[x, yz] = [x, y]z + y[x, z]$ and $[xy, z] = [x, z]y + x[y, z]$ for all $x, y, z \in R$.

Chapter One

BASIC DEFINITIONS

Definition (1-1) :

Let R be a non – empty set and Let + , . be two binary operation define on R then (R , + , .) is a ring if :

1- (R , +) commutative group .

2- (R , .) Semi group .

3- a . (b + c) = a.b + a.c

(b + c) . a = b.a + c.a for all a , b , c $\in$ R .

Example (1-2) :

Let $\mathbb{Z}$ be the set of integers and Let + , . be two binary operation defined on $\mathbb{Z}$ then ($\mathbb{Z}$, + , .) is a ring .

Definition (1-3) :

Let (R , + , .) be a ring and S be a non – empty sub set of R then (S , + , .) is called a Subring of (R , + , .) if (S , + , .) it self-ring .

Definition (1-4) :

Let (R , + , .) be a ring and $\emptyset \neq S \subseteq R$ then (S , + , .) is a Subring of the ring (R , + , .) if and only if :

1- a – b $\in$ S $\forall a , b \in S$

2- a . b $\in$ S $\forall a , b \in S$

Example (1-5) :

Let ($\mathbb{Z}$, + , .) be the ring of integers and $\emptyset \neq \mathbb{Z}_e \subseteq \mathbb{Z}$ then $(\mathbb{Z}_e , + , .)$ is a Subring .

Definition (1-6) :

A ring (R , + , .) is called Commutative ring if a.b = b.a for all a , b ∈ R .

Definition (1-7) :

Let (R , + , .) be a ring and I non – empty sub set of R then (I , + , .) is called an ideal of (R , + , .) if the following conditions are satisfying:

1. a – b ∈ I

2. r.a ∈ I and a.r ∈ I for all a , b ∈ I , r ∈ R .

Example (1-8) :

($\mathbb{Z}$e , + , .) be an ideal of ($\mathbb{Z}$, + , .)

Definition (1-9) :

A ring R is called prime if for any a , b ∈ R , a R b = {0} implies a = 0 or b = 0 .

Definition (1-10) :

A ring R is called Semi prime if for any $a \in R$, $a R a = \{0\}$ implies $a = 0$.

Remark (1-11) :

Every prime ring is Semi prime but the convers is not true in general . The following example show this fact :

Example(1-12) :

A ring $(Z_{35} , +_{35} , ._{35})$ is Semi prime ring since For all $a \in Z_{35}$ if $a Z_{35} a = \{0\}$ implies $a = 0$ but we note that $(Z_{35} , +_{35} , ._{35})$ is not prime ring since $5 Z_{35} 7 = \{0\}$ but $5 \neq 0$ and $7 \neq 0$.

Definition (1-13) :

An additive mapping $D: R \rightarrow R$ is called a derivation of R if

$D (xy) = D (x)y + x D (y)$ for all $x , y \in R$.

Definition (1-14) :

An additive mapping $D: R \rightarrow R$ is Said to be reverse derivation if

$D (x y) = D (y) x + y D (x)$ for all $x , y \in R$.

Definition (1-15) :

A mappingD: $R \times R \times R \to R$ is said to be permuting if :

$D(a,b,c) = D(a,c,b) = D(b,a,c) = D(b,c,a) = D(c,a,b) = D(c,b,a)$.

Definition (1-16):

A mapping d : R define by d (x) = D (x, x ,x) is called the trace of

D , where D (. , . , .) : R x R x R → R is a permuting mapping

Definition (1-17):

A mapping D (. , . , .) : R x R x R → *R* is called tri – additive if :

1- $D(x + w, y, z) = D(x, y, z) + D(w, y, z)$

2- $D(x, y + w, z) = D(x, y, z) + D(x, w, z)$

3- $D(x, y, z + w) = D(x, y, z) + D(x, y, w)$

for all $x,y,z,w \in R$.

Definition (1-18):

A tri – additive mapping D(. , . , .) : R x R x R → R is called permuting if :

$D(x,y,z) = D(x,z,y) = D(y,x,z) = D(y,z,x) = D(z,x,y) = D(z,y,x)$ for all $x, y, z \in R$.

Remark (1-19) :

It is obvious that , if $D(.,.,.) : R \times R \times R \rightarrow R$ a permuting tri – additive mapping then the trace of $D(.,.,.)$ satisfies relation:

$d(x + y) = d(x) + d(y) + 3D(x, x, y) + 3D(x, y, y)$ for all $x, y \in R$.

Definition (1-20) :

A permuting tri – additive mapping $D(.,.,.) : R \times R \times R \rightarrow R$ is called permuting tri – derivation if :

1- $D(xw, y, z) = D(x, y, z)w + xD(w, y, z)$
2- $D(x, yw, z) = D(x, y, z)w + yD(x, w, z)$
3- $D(x, y, zw) = D(x, y, z)w + zD(x, y, w)$

for all $x, y, z, w \in R$.

Definition (1-21) :

A permuting tri- additive mapping $D(.,.,.)$: $R\times R\times R \rightarrow R$ is called reverse permuting tri-derivation if :

1- $D(xw, y, z)=D(w, y, z)x + wD(x, y, z)$

2- $D(x, yw, z)=D(x, w, z)y + wD(x, y, z)$

3- $D(x, y, zw)=D(x, y, w)z + wD(x, y, z)$ for all $x,y,z,w \in R$

Definition(1-22) :

Let (R , + , .) be a ring , define [a , b] as [a , b] = ab – ba is called commutator of a , b .

Definition (1-23) :

A permuting tri – additive mapping D (. , . , .) : R x R x R → R is called a permuting left tri – dervation if :

1- D (xy, z ,w)= x D (y , z , w) + yD (x , z , w)

2- D (x, yz ,w) = y D (x , z , w) + zD (x , y , w)

3- D (x, y , zw) = z D (x , y , w) + wD (x , y , z)   for all x,y,z,w ∈ R

Definition (1-24) :

Let (R, +, .) be a ring then annihilator R (I) is {r ∈ R | x r $= 0$, $\forall x \in$ R}

Example (1-25) :

In (Z_8 ,$+_8$, $._8$) , I = {0 , 4} the anih Z_8 (I)={0 , 2 , 4 , 6}and left anhi Z_8(I)=right anhi Z_8(I) Since Z_8 is commutative .

Remark (1-26) :

A maping D (. , . , .) : R x R x R → R acts as a right (resp. left) R homomorphism on I if :

1- D (rx,y,z)= D(x,y,z) r

D (x,ry,z) = D(x,y,z)r

D (x,y,rz) = D (x,y,z) r .

2- (D (xr , y , z) = rD (x , y , z)

D (x , yr , z) = rD (x , y , z)

D (x , y , zr) = rD (x , y , z)**)**

Chapter Two

Permuting reverse tri – derivations on prime and semi prime rings

Introduction

In this chapter we introduce some important lemmas to solve and then we prove the main result that is if R is a prime ring of char R $\neq$ 2,3 and I a non-zero left(or right) ideal of R. Let D (. , . , .) :R×R×R $\rightarrow$ R be a prime ring reverse tri-derivation and the trace of D suppose that d(x)=0 , for all x $\in$ I , then d=0 ,that D=0.

Lemma (2-1) :

Let : R→R be a reverse derivation of a prime ring R and I a non – zero ideal of R . Suppose that either

(i) aD (x) = 0 , for all x $\in$ I or

(ii)D(x) a= 0 , for all x $\in$ I holds . Then a = 0 or D = 0 .

Proof :

We have aD (x) = 0 , for all x $\in$ I (1)

Replacing x by yx in a bove equation then , we get :

aD (y x) = 0

a (D (x) y + x D (y)) = 0
aD (x) y + a x D (y) = 0

using equation (1) then , we get

$a\,x\,D(y) = 0$, For all $y \in I$

$a\,R\,D(y) = 0$

Since R is prime which implies that either $a = 0$ or $D = 0$.

(ii) we have $D(x)\,a = 0$, For all $x \in I$ (2)

Replacing x by yx in above equation then , we get :

$D(y\,x)\,a = 0$

$(D(x)\,y + x\,D(y))\,a = 0$

$D(x)\,ya + x\,D(y)\,a = 0$

Using equation (2) then , We get :

$D(x)\,ya = 0$, For all $x , y \in I$

$D(x)\,R\,a = 0$

Since R is a prime which implies that either $a = 0$ or $D = 0$

Lemma (2-2) :

Let R be a prime ring and I a non – zero right ideal of R . If I is a commutative , then R is a commutative .

Lemma (2-3) :

Let R be a prime ring of char R $\neq$ 2 and I a non – zero ideal of R. Let a,b be a fixed elements of R . If axb + bxa = 0 is fulfilled for all x $\in$ I ,

either a = 0 or b = 0 .

Proof :

We have axb + bxa = 0 , for all x $\in$ R .

axb = - bxa , for all x $\in$ R ………… (1)

We replace x by xbrax in (1) , we get

a (xbrax) b = - b (xbraz) a

axbraxb = - (b (xbr) a) xa

By using (1) in the above equation , we get :

axbraxb = (a (xbr) b) xa

(axb) r (axb) = (axb) r (bxa)

Again by using (1) in the above equation , we get :

(axb) r (axb) = (axb) r (- axb)

(axb) r (axb) = - (axb) r (axb)

(axb) r (axb) + (axb) r (axb) = 0

a (axb) r (axb) = 0 , for all r $\in$ R .

Since R is prime ring of char $\neq$ 2 , We get axb = 0 , for all x $\in$ R and hence a = 0 or b = 0 .

Theorem (2-4) :

Let R be a prime ring of char R $\neq$ 2 , 3 and I a non – zero left (or right) ideal of R . Let D (. , . , .) : R x R x R $\rightarrow$ R be a prime ring reverse tri – derivation and d the trace of D suppose that

d (x) = 0 , for all x $\in$ I , then d = 0 , that is D = 0 .

Proof :

We have d (x) = 0 , for all x $\in$ I ……… (1)

the linearization of equation (1) then , we get d (x + y) = 0

d (x) + d (y) + 3D (x , x , y) + 3D (x , y , y) = 0 , for all x , y $\in$ I since

d (x) = d (y) = 0 , and char R $\neq$ 3 , then

D (x , x , y) + D (x , y , y) = 0 ……….. (2)

D (x , x , y) = - D (x , y , y) ……….. (3)

Replacing y by yr (r $\in$ R) in equation (2) and using equation (2) , we get :

D (x , x , yr) + D (x , yr , yr) = 0

D (x , x , r) y + r D (x , x , y) + D (x , r , yr) y + rD (x , y , yr) = 0

D (x , x , r) y + rD (x , x , y) + D (x , r , r) y^2 + rD (x , r , y) y + rD (x , y , r) y + r^2D (x , y , y) = 0

D (x , x , r) y + rD (x , x , y) + D (x , r , r) y^2 + rD (x , y , r) y + rD (x , y , r) y + r^2D (x , y , y) = 0 …………….. (4)

Replacing y^2 by y and r^2 by r , we get :

$D(x, x, r)y + rD(x, x, y) + D(x, r, r)y + 2rD(x, y, r)y + rD(x, y, y) = 0$ ………..(5)

Since $D(x, x, y) = -D(x, y, y)$, then (5) become :

$D(x, x, r)y + D(x, r, r)y + 2rD(x, r, y)y = 0$

replace $D(x, x, r)y = -D(x, r, r)y$, then

$2rD(x, r, y)y = 0$, Since char $R \neq 2$, we get :

$rD(x, r, y)y = 0$ ……………. (6)

replace y by yr in (6) , we get :

$rD(x, r, yr)y = 0$

$rD(x, r, r)y^2 + r^2D(x, r, y)y = 0$

replace y^2 by y and r^2 by r , we get :

$rD(x, r, r)y + rD(x, r, y)y = 0$

from (6) , we get $rD(x, r, r)y = 0$

Since a left annihilator of a non – zero left ideal is zero , So that

$rD(x, r, r) = 0$ ………. (7)

replace x by xr in (7) , we get :

$rD(xr, r, r) = 0$

$rD(r, r, r)x + r^2D(x, r, r) = 0$

$rD(r, r, r)x + rD(x, r, r) = 0$

from (7) , we get :

$rD(r, r, r)x = 0$

$rd(r)x = 0$

Since a left annihilator of a non – zero left ideal is Zero , So that $rd(r) = 0$

Hence $d(r)$ is an element the right annihilator of I

then $d(r) = 0$, for all $r \in R$.

References :

1- Anwar K. Faraj , Sabreen J . Shareen J , Sharref Faraj and shareef “Genaralized permuting 3 – Derivations on prime rings” . Iraqi Journal of Science , 2016 , vol . 57 , No . 3 C , pp 2312 – 2317 .

2- C. Jaya Subba Reddy & B. Rammorthy Reddy . “Orthogonal symmetric reverse Bi – Derivations on semiprime ring” . open Journal of Applied & theoretical Mathematics . Vol. 2 No . 4 , 2016 , Pp 917 – 923 .

3- C. Jaya Subba Reddy & M, Ramakrisha Naik . “Symmetric reverse Bi – Derivations in Semiprime rings” . International Journal of lnnovative Reasersh in Science . Vol . 6 , No . 13 , July 2017 , pp 64 – 67 .

4- C.Jaya Subba Reddy,V.Vijaya Kumar & K.Hemavathi. "Prime Ring with Symmetric Skew 3-Reverse derivation" International Journal Of Mathematics and Computer Applications Research. Vol. 4,Issue 6, Dec 2014, 69-74.

5- Duran õzden and Mehmet Ali õztãrk "permuting g tri-derivation in prime and semi prime rings" KyN G Pook math .J. Vol. 46 , No .2 (2006)pp. 153-167.

6- H. Durna and S. õguz “Permuting tri – derivation in prime and semi – prime rings” , International Journal of Algebra and statistics , Vol . 5 , No. 1 , 2016 , pp. 52 – 58 .

7- Yong – soo . Jung & Kyoo – Hon park . “On prime and seniprime rings with permuting 3 – derivatiions” Bull . Korean math . Soc . Vol . 44 No . 4 (2007) pp. 789 – 794 .