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Orthogonal Generalized permuting higher tri – derivations on semiprime rings

بحث تقدمت به الطالبة

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بإشراف

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بسم الله الرحمن الرحيم

ويسئلونك عن الروح قل
الروح من امر ربي وما
أتيتم من العلم الا قليلا

صدق الله العلي العظيم

سورة الاسراء– الآية (85)

ا لإ هـ د ا ء

- إلى الائمة الاطهار سراج النور في الأرض ....

- إلى من علمني حرفاً ......

- إلى والدي الحبيب كنت خير عوناً لي أدامك الله لي ...

- إلى رمز الحنان ومن الجنان تحت قدميها أمي رعاكِ الله ...

- وإلى جميع الأهل والأصدقاء ممن ساعدني وكان لي عوناً ...

- وإلى استاذي الذي طالما ساعدني في إنجاز هذا البحث الدكتور مازن المحترم المشرف على البحث وفقك الله لكل خير .

- وإلى اساتذتي في قسم الرياضيات

أهدي لهم هذا الجهد المتواضع ...

شكـر وتـقـديـر

(ربِ اوزعني أن أشكر نعمتك التي انعمت علي وعلى والدي وأن أعمل صالحاً ترضاه وأدخلني برحمتك في عبادك الصالحين) .

الحمد لله المنان الرحمن الرحيم الذي خلق الانسان وعلمه البيان وانطق لسانه بأي الذكر والقرآن ، وأصلي واسلم على مبعوث العناية الإلهية والهداية الربانية النبي الامي الذي علم المتعلمين وقاد سفينة العالم الحائرة في خضم المحيط إلى ملكوت رب العالمين .

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وأشكر كل من كان سبباً في وصولي لهذه المرحلة . . . عائلتي . . .

Abstract:

In this paper aring R is presented. We will study the concept of orthogonal generalized permuting higher tri – derivations on aring .

We prove that if R is a 2 - torsion free Semi prime ring , Dn and Gn are orthogonal generalized permuting higher tri- derivations associated with permuting higher tri- derivations dn and gn respectively for all n∈ N . Then the following relations are hold for all x, y, z, w ∈ R and n∈ N

1. D_n (x , y, z) G_n (y, z, w) = G_n (y, z, w) = 0
   hence D_n (x, y, z) G_n (y, z, w) + G_n (x, y, z) D_n (y, z, w)=0
2. D_n and G_n are orthogonal and d_n (x, y, z) G_n (y, z, w) = G_n (x, y, z) d_n (y, z, w) =0
3. G_n and D_n are orthogonal and g_n (x, y, z) D_n (y, z, w)= D_n (x, y, z) g_n (y, z, w) = 0
4. D_n and g_n are orthogonal Permuting higher tri- derivations.
5. d_n G_n = G_n d_n = 0 and gn D_n = D_n g_n = 0
6. G_n D_n = D_n G_n = 0

CHAPTER ONE

Definition 1 . 1 “Ring”

Let R be a non – empty set an let $*$, $\odot$ be two binary operation then the triple (R , $*$, $\odot$) is called ring if its satisfy the following :

1- (R , $*$) is a belian group .

2- (R , $\odot$) is semi group .

3- $\odot$ is distributative over $*$.

4- a $\odot$ (b $*$ c) = a $\odot$ b $*$ a $\odot$ c

Properties of Rings :

1- $\forall a \in R$, 0.a = 0

2- 0 $\neq$ 1

3- a . (- b) = - a.b

4- (- a) . (- b) = a . b

5- (-1) . a = - a

Example : The mathematical system $(\mathbb{Z}, +, .)$ **is a ring .**

Definition 1.2 "Subring"

Let $(R, +, .)$ be a ring and $\emptyset \neq S \subseteq R$ then $(S, +, .)$ is a subring of the ring $(R, +, .)$ if and only if :

1- a – b ∈ S $\forall a, b \in S$

2- a . b ∈ S $\forall a, b \in S$

Example : $(\mathbb{Z}_e, +, .)$ is Subring of $(\mathbb{Z}, +, .)$

Solution :

Since 0 = 2.0 ⟹ 0 ∈ $\mathbb{Z}_e$

So that $\emptyset \neq \mathbb{Z}_e \subset \mathbb{Z}$

Now let a , b ∈ $\mathbb{Z}_e \Rightarrow \exists x, y \in \mathbb{Z}$

a = 2x , b = 2 y   So

a – b = 2x – 2y = 2 (x – y) ∈ $\mathbb{Z}_e$ Since x – y ∈ $\mathbb{Z}$, ∀ x , y ∈ $\mathbb{Z}$

also

a.b = 2x . 2y = 2 . (2x . y) = 2 . (2 (x . y)) ∈ $\mathbb{Z}_e$

Since x.y $\in \mathbb{Z}$ $\forall\ x, y \in \mathbb{Z}$

$\therefore$ ($\mathbb{Z}_e$, + , .) is a subring of ($\mathbb{Z}$, + , .)

Definition 1 . 3 "Prime ring"

A ring R is called prime if for all a , b ∈ R , when a R b = 0 this implies that either a = 0 or b = 0 .

Definition 1.4 "Semi prime ring"

A ring R is called semi prime if for all a , b ∈ R , when a R b = 0 this implies that a = 0

Definition 1.5 "additive mapping"

A mapping f : R → R is called additive mapping if f (x + y) = f(x) + f (y) Hold for all x , y ∈ R where R is an arbitrary ring .

Definition 1.6 "Derivation"

An additive mapping D : R → R where R is un arbitrary ring is called derivation if D (xy) = D (x) y + xD (y) holds for all pairs x , y ∈ R .

Definition 1.7 "generalized derivation"

An additive mapping $F : R \to R$ is called generalized derivation if there exist aderivation $D : R \to R$ such that $f(x y) = f(x) y + x D(y)$ hold for all $x, y \in R$.

Definition 1.8 "orthogonal"

Let D and G be two generalized derivations on R is said to be orthogonal if $D(x) R G(y) = 0 = G(y) R D(x)$ for all $x, y \in R$.

Definition 1.9

A mapping $D : R \times R \times R \to R$ is said to be permuting if :

$D(a, b, c) = D(a, c, b) = D(b, a, c) = D(b, c, a) = D(c, a, b) = D(c, b, a)$

Definition 1.10

A permuting tri – additive mapping $D(., ., .) : R \times R \times R \to R$ is called permuting tri – derivation if :

1- $D(xw, y, z) = D(x, y, z) w + x D(w, y, z)$

2- $D(x, y w, z) = D(x, y, z) w + y D(x, w, z)$

3- $D(x, y, zw) = D(x, y, z) w + z D(x, y, w)$

hold for all $x, y, z, w \in R$.

Definition 1.11 "higher bi – derivation"

Let D = $(di)_{i \in N}$ be a family of tri – additive mapping on RxR into R is said to be higher bi – derivation if :

$$d_n(xy\,,zw) = \sum_{i+j=n} d_i\,(x\,,z)d_j\,(y\,,w)\ \ for\ all\ x\,,y\,,z\,,w\ \in R$$

Let D = $(D_i)_{i \in N}$ and G = $(G_i)_{i \in N}$ are two generalized permating higher tri – derivation on R , then D_n and G_n are said to be orthogonal if for every x , y , z , w ∈ R , n ∈ N : D_n (x , y , z) R G_n (y , z , w) = 0 = G_n (y , z , w) R D_n (x , y , z) where D_n (x , y , z) R G_n (y , z , w) = $\sum_{i=1}^{n} Di\,(x\,,y\,,z)r\ Gi\,(y\,,z\,,w) = 0\ for\ all\ r\ \in R$.

1.12 "higher tri – derivation"

Let D= (di) iϵ N be a family of tri –additive mapping on RX RX R into R is Said to be higher tri –derivation if :

d_n (xy, zw, rt)= $\sum_{i+j=n} di$ (x, z, r) d j (y , w, t) for all x, y, z, w, r, t ∈ R.

1.13 "Jordan higher tri –derivation"

Let D= (di) iϵN be a family of tri –additive mapping on RX RX R into R is Said to be Jordan higher tri –derivation if :

d_n (x^2, y^2, z^2)= $\sum_{i+j=n} di$ (x, y, z) d j (x , y, z) ∀x, y, z ∈ R.

Definition 1.14 "higher generalized tri – derivation"

Let $(D_i)_{i \in N}$ = D_n be a family of additive mapping we say that D_n is higher generalized tri – derivation if there exist d_n = $(d_i)_{i \in N}$ a higher tri – derivation such that

D_n (xy , z , w) = $\sum_{i=1}^{n} D_i\ (x\ ,z\ ,w) d_i\ (y\ ,z\ ,w)$

D_n (x , z , w) = $\sum_{i=1}^{n} D_i\ (x\ ,z\ ,w) d_i\ (x\ ,y\ ,w)$

D_n (x , z , yw) = $\sum_{i=1}^{n} D_i\ (x\ ,z\ ,y) d_i\ (x\ ,z\ ,w)$

CHAPTER TWO

Lemma (2 – 1) :

Let R be a 2 - torsion free semiprime and a , b $\in$ R then the following conditions are equivalent :

1- a R b = 0

2- b R a = 0

3- a R b + b R a = 0

If one of these conditions is fulfilled then a R b = b R a = 0 .

Lemma (2 – 2) :

Let R be a 2 – torsion free semiprime and a , b the elements of R such that a R b + b R a = 0 then a R b = b R a = 0 .

Lemma (2 – 3) :

Let R be a semiprime ring suppose that D_n and G_n are tri – additive mapping satisfies D_n (x , y , z) R G_n (x , y , z) = 0 for all z , y , z $\in$ R and n $\in \mathbb{N}$. Then D_n (x , y , z) R G_n (y , z , w) = 0 for all x , y , z , w $\in$ R , n $\in$ $\mathbb{N}$.

Proof : suppose that D_n (x , y , z) R G_n (x , y , z) = 0

D_n (x , y , z) R G_n (x , y , z)= $\sum_{i=1}^{n} D_i$ (x , y , z) r G_i (x , y , z)= 0 .. (1)

Replace x by x + w in (1) for all z ∈ R we get :-

$D_n (x + w , y , z) R G_n (x + w , y , z) = 0$

$(D_n (x , y , z) + D_n (w , y , z)) R (G_n (x , y , z) + G_n (w , y , z) = 0$

$\sum_{i=1}^{n}(D_i (x , y , z) + D_i (w , y , z)) r (G_i (x , y , z)+ G_i (w , y , z))= 0$

$\sum_{i=1}^{n}(D_i (x , y , z) + r G_i (w , y , z) + D_i (x , y , z) r G_i (w , y , z)$

$+ D_i (w , y , z) r G_i (x , y , z) + D_i (w , y , z) r G_i (w , y , z) = 0$

from equation (1) we get :

$\sum_{i=1}^{n} D_i (x , y , z) r G_i (w , y , z) + \sum_{i=1}^{n} D_i (w , y , z) r G_i (x , y , z) = 0$

$\Rightarrow \sum_{i=1}^{n} D_i (x , y , z) r G_i (w , y , z) = - \sum_{i=1}^{n} D_i (w , y , z) r G_i (x , y , z)$ ….. (2)

Multiplicate (2) by t $\sum Di (x , y , z) r G_i (w , y , z)$

$\sum_{i=1}^{n} D_i(x , y , z) r G_i (w , y , z) t D_i (x , y , z) r G_i (w , y , z) = - \sum_{i=1}^{n} D_i (w , y , z) r G_i (x , y , z) t D_i (x , y , z) r G_i (w , y , z)$

$\sum_{i=1}^{n} D_i (x , y , z) r G_i (w , y , z) t D_i (x , y , z) r G_i (w , y , z) = 0$

Since R is semiprime

$\sum_{i=1}^{n} D_i(x , y , z) r G_i (w , y , z) = 0$ ………….. (3)

$\therefore D_n (x , y , z) R G_n (y , z , w) = 0$.

Lemma (2 – 4) :

Let R be a2 – torsion free semiprime two generalized permuting higher tri – derivations D_n and G_n associated with two permuting higher tri – derivations dn and gn respectively for all n ∈ N . The Dn and G_n are orthogonal if and only if D_n (x , y , z) G_n (y , z , w) + G_n (x , y , z) D_n (y , z , w) = 0 for all x , y , z , w ∈ R .

Proof : Suppose that

D_n (x , y , z) G_n (y , z , w) + G_n (x , y , z) D_n (y , z , w) = 0

$\sum_{i=1}^{n} D_i$(x , y , z) G_i (y , z , w) + G_i (x , y , z) D_i (y , z , w) = 0 …. (1)

Replace x by xf in (1) for all f ∈ R we get :

$\sum_{i=1}^{n} D_i$(xf , y , z) G_i (y , z , w) + G_i (xf , y , z) D_i (y , z , w) = 0

$\sum_{i=1}^{n} D_i$(x , y , z) d_i (f , y , z) G_i (y , z , w) + G_i (x , y , z) g_i (f , y , z) Di (y , z , w) = 0 ……. (2)

Replace d_i (f , y , z) by g_i (f , y , z) in (2) we get :

$\sum_{i=1}^{n} D_i$(x , y , z) g_i (f , y , z) G_i (y , z , w) + G_i (x , y , z) g_i (f , y , z) D_i (y , z , w) = 0

By lemma (2 – 2) we get :-

$\sum_{i=1}^{n} D_i$(x , y , z) g_i (f , y , z) G_i (y , z , w) =

$\sum_{i=1}^{n} G_i$ (x , t , z) g_i (f , y , z) D_i (y , z , w) = 0 ……. (3)

Replace g_i (f , y , z) by r in (3) for all r ∈ R we get :

$D_i (x , y , z) r G_i (y , z , w) = \sum_{i=1}^{n} G_i (x , y , z) r D_i (y , z , w) = 0$

$D_n (x , y , z) R G_n (y , z , w) = G_n (x , y , z) R D_n (y , z , w) = 0$

Thus D_n and G_n are orthogonal .

Conversely :

Suppose that D_n and G_n are orthogonal :

$D_n (x , y , z) R G_n (y , z , w) = 0 = G_n (x , y , z) R D_n (y , z , w)$

$\sum_{i=1}^{n} D_i(x , y , z) r G_i (y , z , w) = 0 = \sum_{i=1}^{n} G_i (x , y , z) r D_i (y , z , w)$

$\sum_{i=1}^{n} D_i (x , y , z) r G_i (y , z , w) + G_i (x , y , z) r D_i (y , x , w) = 0$

By lemma (2 – 1) we get :

$\sum_{i=1}^{n} D_i(x , y , z) G_i (y , z , w) = \sum_{i=1}^{n} Gi (x , y , z) D_i (y , z , w) = 0$

$\sum_{i=1}^{n} D_i(x , y , z) G_i (y , z , w) + G_i (x , y , z) D_i (y , z , w) = 0$

Hence $D_n (x , y , z) G_n (y , z , w) + G_n (x , y , z) D_n (y , z , w) = 0$

Lemma (2 - 5) :

Let R be a 2 – torsion free semi prime two generalized permuting higher tri – derivations D_n and G_n associated with two symmetric higher tri – derivations d_n and g_n respectively for $n \in N$. Then D_n and G_n are orthogonal if and only if :

D_n (x , y , z) G_n (y , z , w) = 0 or G_n (x , y , z) D_n (y , z , w) = 0 for all x , y , x ∈ R , n ∈ ℕ .

Proof : - Suppose that D_n (x , y , z) G_n (y , z , w) = 0

D_n (x , y , z) G_n (y , z , w) = $\sum_{i=1}^{n} D_i$ (x , y , z) G_i (y , z , w) = 0 .. (1)

Replace x by xw in (1) for all w ∈ R we get :

D_n (xw , y , z) G_n (y , z , w) = 0 = $\sum_{i=1}^{n} D_i$ (xw , y , z) G_i (y , z , w)

$\sum_{i=1}^{n} D_i$ (x , y , z) d_i (w , y , z) G_i (y , z , w) = 0 ….. (2)

Replace d_i (w , y , z) by r , ∀ r ∈ R

$\sum_{i=1}^{n} D_i$ (x , y , z) r G_i (y , z , w) = 0

Hence we get the require result .

Similarly way if G_n (x , y , z) D_n (y , z , w) = 0 , we get D_n and G_n are orthogonal .

Conversely , Suppose that D_n and G_n are orthogonal

D_n (x , y , z) R G_n (y , z , w) = (0)

$\sum_{i=1}^{n} D_i$(x , y , z) r G_i (y , z , w) = 0 ∀ r ∈ R .

By lemma (2 – 1) we get

$\sum_{i=1}^{n} D_i$(x , y , z) G_i (y , z , w) = 0

Hence D_n (x , y , z) G_n (y , z , w) = 0

And by G_n (x , y , z) R G_n (y , z , w) = (0)

$\sum_{i=1}^{n} G_i$ (x , y , z) r D_i (y , z , w) = 0

By lemma (2 – 1) we get :

$\sum_{i=1}^{n} G_i$ (x , y , z) D_i (y , z , w) = 0

Thus G_n (x , y , z) D_n (y , z , w) = 0

Lemma (2 – 6) :

Let R be a2 – torsion free semi prime two generalized permuting higher tri – derivations D_n and G_n associated with two symmetric higher tri – derivations d_n and g_n respectively for n ∈ N . Then D_n and G_n are orthogonal iff :

D_n (x , y , z) g_n (y , z , w) = 0 or d_n (x , y , z) G_n (y , z , w) = 0 for all x , y , z , w ∈ R and n ∈ N .

Proof : Suppose that

Dn (x , y , z) g_n (y , z , w) = 0

D_n (x , y , z) g_n (y , z , w) = $\sum_{i=1}^{n} Di$ (x , y , z) g_i (y , z , w) = 0 …. (1)

Replace w by wt in (1) for all t ∈ R we get :

$\sum_{i=1}^{n} D_i$(x , y , z) g_i (y , z , tw) = 0

$\sum_{i=1}^{n} D_i$ (x ,y , z) g_i (y , z , t) g_i (y , z , w) = 0 ….. (2)

Replace g_i (y , z , w) by G_i (y , z , w) in (2) we get :

$\sum_{i=1}^{n} D_i$ (x , y , z) g_i (y , z , t) G_i (y , z , w) = 0 by lemma (2 – 1) :

$\sum D_i$ (x , y , z) G_i (y , z , w) = 0

∴ D_n (x , y , z) G_n (y , z , w) = 0 ⟹ D_n and G_n are orthogonal .

Conversely , Suppose that D_n and G_n are orthogonal :

By (x , y , z) G_n (y , z , w) = 0

$\sum_{i=1}^{n} D_i$ (x , y , z) G_i (y , z , w) = 0 …….. (3)

Replace w by tw in (3) for all t ∈ R we get :

$\sum_{i=1}^{n} D_i$ (x , y , z) G_i (y , z , tw) = 0

$\sum_{i=1}^{n} D_i$ (x , y , z) G_i (y , z , t) g_i (y , z , w) = 0

By lemma (2 – 1) we get :

$\sum_{i=1}^{n} D_i$ (x , y , z) g_i (y , z , w) = 0

Hence D_n (x , y , z) g_n (y , z , w) = 0

And replace x by fx in (3) we get :

$\sum_{i=1}^{n} D_i$ (fx , y , z) G_i (y , z , w) = 0

$\sum_{i=1}^{n} D_i$ (f , y , z) d_i (x , y , z) G_i (y , z , w) = 0 ….. (4)

Multiplication (4) by d_i (x , y , z) G_i (y , z , w)

$\sum_{i=1}^{n} d_i$ (x , y , z) G_i (y , z , w) D_i (f , y , z) d_i (x , y , z) G_i (y , z , w)= 0

Since R is semiprime we get :

$\sum_{i=1}^{n} d_i$ (x , y , z) G_i (y , z , w) = 0

Hence d_n (x , y , z) G_n (y , z , w) = 0

Lemma (2 – 7)

Let R be a2 – torsion free semiprime two generalized permuting higher tri – derivations D_n and G_n associated with two symmetric higher tri – derivations d_n and g_n respectively for all n ∈ N . Then D_n and G_n are orthogonal if and only if :

D_n (x , y , z) G_n (y , z , w) = d_n (x , y , z) G_n (y , z , w) = 0

for all x , y , z , w ∈ R and n ∈ ℕ .

Proof :

Suppose that D_n and G_n are orthogonal By lemma (2 – 5) we get:

D_n (x , y , z) G_n (y , z , w) = 0 …. (1)

And by lemma (2 – 8) we get :

d_n (x , y , z) G_n (y , z , w) = 0 …… (2)

from (1) and (2) we get :

D_n (x , y , z) G_n (y , z , w) = d_n (x , y , z) G_n (y , z , w) = 0

Conversely suppose that :

D_n (x , y , z) G_n (y , z , w) = 0

By Theorem (2 – 5) we get :

Hence D_n and G_n are orthogonal .

Now if d_n (x , y , z) G_n (y , z , w) = 0

By Theorem (2 – 6) we get : D_n and G_n are orthogonal .

Theorem (3 – 1) :

Let R is a2 – torsion free semiprime , D_n and G_n are orthogonal generalized permuting higher tri – derivations associated with symmetric higher tri – derivations d_n and g_n respectively for all n ∈ N . Then the following relations are hold for all x , y , z , w ∈ R and n ∈ N.

Solution :

(1) D_n (x , y , z) G_n (y , z , w) = G_n (x , y , z) D_n (y , z , w) = 0

(2) d_n and G_n are orthogonal and d_n (x , y , z) G_n (y , z , w)

= G_n (x , y , z) d_n (y , z , w) = 0

(3) g_n and D_n are orthogonal and g_n (x , y , z) D_n (y , z , w)

= D_n (x , y , z) g_n (y , z ,) = 0

(4) d_n and g_n are orthogonal symmetric higher tri – derivations .

(5) $d_nG_n = G_nd_n = 0$ and $g_nD_n = D_ng_n = 0$

(6) $G_nD_n = D_nG_n = 0$

(1) D_n (x , y , z) G_n (y , z , w) = G_n (x , y , z) D_n (y , z , w) = 0

hence D_n (x , y , z) G_n (y , z , w) + G_n (x , y , z) D_n (y , z , w) = 0

Proof : Suppose that D_n and G_n are orthogonal , By lemma (2 – 5) we get :

D_n (x , y , z) G_n (y , z , w) = 0 and G_n (x , y , z) D_n (y , z , w) = 0

D_n (x , y , z) G_n (y , z , w) = G_n (x , y , z) D_n (y , z , w) = 0

Hence : D_n (x , y , z) G_n (y , z , w) + G_n (x , y , z) D_n (y , z , w) = 0 .

(2) d_n and G_n are orthogonal and d_n (x , y , z) G_n (y , z , w) = G_n (x , y , z) d_n (y , z , w) = 0

Proof : Suppose that D_n and G_n are orthogonal By lemma (2 – 6) we get :

d_n (x , y , z) G_n (y , z , w) = 0 ……. (1)

$\sum_{i=1}^{n} d_i$ (x , y , z) G_i (y , z , w) = 0 ……. (2)

Replace x by xt in (2) , t ∈ R we get :

$\sum_{i=1}^{n} d_i$ (xt , y , z) G_i (y , z , w) = 0

$\sum_{i=1}^{n} d_i$ (x , y , z) d_i (t , y , z) G_i (y , z , w) = 0 ……… (3)

Replace d_i (t , y , z) by m in (3) m ∈ R we get :

$\sum_{i=1}^{n} d_i$ (x , y , z) m G_i (y , z , w) = 0 …….. (4)

from (1) G_n (x , y , z) D_n (y , z , w) = 0

$\sum_{i=1}^{n} D_i$ (x , y , z) G_i (y , z , w) = 0 …….. (5)

Replace w by fw in (5) we get :

$\sum_{i=1}^{n} G_i$ (x , y , z) D_i (y , z , fw) = 0

$\sum_{i=1}^{n} G_i$ (x , y , z) D_i (y , z , f) d_i (y , z , w) = 0

By lemma (2 – 1) we get :

$\sum_{i=1}^{n} G_i$ (x , y , z) d_i (y , z , w) = 0

G_n (x , y , z) d_n (y , z , w) = 0 …….. (6)

And by $\sum_{i=1}^{n} G_i$ (x , y , z) d_i (y , z , w) = 0 replace w by fw we get :

$\sum_{i=1}^{n} G_i$ (x , y , z) d_i (y , z , fw) = 0

$\sum_{i=1}^{n} G_i$ (x , y , z) d_i (y , z , f) d_i (y , z , w) = 0 ………. (7)

Replace d_i (y , z , f) by d_i (f , y , z) in (7) we get :

$\sum_{i=1}^{n} G_i$ (x , y , z) d_i (f , y , z) d_i (y , z , w) = 0 ……… (8)

Replace d_i (f , y , z) by r in (8) we get :

$\sum_{i=1}^{n} G_i$ (x , y , z) r d_i (y , z, w) = 0 …….. (9)

From (4) and (9) we get D_n and G_n are orthogonal

From (1) and (6) we get :

G_n (x , y , z) d_n (y , z , w) = d_n (x , y , z) G_n (y , z , w) = 0

(3) g_n and D_n are orthogonal and g_n (x , y , z) D_n (y , z , w) = D_n (x , y , z) g_n (y , z , w) = 0

Proof : Similarly way used in the proof of (2) .

(4) d_n and g_n are orthogonal symmetric higher tri – derivations .

Proof : from (1) D_n (x , y , z) G_n (y , z , w) = 0

$\sum_{i=1}^{n} D_i$ (x , y , z) G_i (y , z , w) = 0 …… (1)

Replacing x by fx and w by fw in (1)

$\sum_{i=1}^{n} D_i$ (fx , y , z) G_i (y , z , fw) = 0

$\sum_{i=1}^{n} D_i$ (f , y , z) d_i (x , y , z) G_i (y , z , f) g_i (y , z , w) = 0 ……… (2)

Replace G_i (y , z , f) by m in (2) for all m ∈ R

$\sum_{i=1}^{n} D_i$ (f , y , z) d_i (x , y , z) m g_i (y , z , w) = 0 ……. (3)

Multiplication (3) by d_i (x , y , z) m g_i (y , z , w) we get :

$\sum_{i=1}^{n} d_i$ (x , y , z) m g_i (y , z , w) D_i (f , y , z) d_i (x , y , z) m g_i (y , z, w) = 0

Since R is semiprime we get :

$\sum_{i=1}^{n} d_i$ (x , y , z) m g_i (y , z , w) = 0

d_n (x , y , z) R g_n (y , z , w) = 0

Hence d_n and g_n are orthogonal symmetric higher tri – derivations .

(5) d_n G_n = G_n d_n = 0 and g_n D_n = D_n g_n = 0

Proof : Since by (2) d_n (x , y , z) G_n (y , z , w) = 0

G_n (d_n (x , y , z) G_n (y , z , w) , r_1 , r_2) = 0 $\forall$ r_1 , r_2 $\in$ R

$\sum_{i=1}^{n} G_i$ (d_i (x , y , z) G_i (y , z , w) , r_1 , r_2) = 0 …… (1)

Replace x by xf in (1) for all f ∈ R

$\sum_{i=1}^{n} G_i$ (d_i (xf , y , z) G_i (y , z , w) , r_1 , r_2) = 0

$\sum_{i=1}^{n} G_i$ (d_i (x , y , z) d_i (f , y , z) G_i (y , z , w) , r_1 , r_2) = 0

$\sum_{i=1}^{n} G_i$ (d_i (x , y , z) , r_1 , r_2) g_i (d_i (f , y , z) , r_1 , r_2) g_i (G_i (y , z ,w) , r_1 , r_2)= 0 …..(2)

Replace g_i (G_i (y , x , w) , r_1 , r_2) by G_i (d_i (x , y , z) , r_1 , r_2) in (2) we get :

$\sum_{i=1}^{n} G_i$ (d_i (x , y , z) , r_1 , r_2) g_i (d_i (f , y , z) , r_1 , r_2) G_i (d_i (x , y , z) , r_1 , r_2) = 0

Since R is semiprime we get :

$\sum_{i=1}^{n} G_i$ (di (x , y , z) , r_1 , r_2) = 0

Thus G_n d_n = 0 …….. (3)

And by (2) G_n (x , y , z) d_n (y , z , w) = 0

d_n (G_n (x , y , z) d_n (y , z , w) , r_1 , r_2) = 0

$\sum_{i=1}^{n} d_i$ (G_i (x , y , z) d_i (y , z , w) , r_1 , r_2) = 0 …… (4)

Replace x by xf in (4) we get :

$\sum_{i=1}^{n} d_i$ (G_i (xf , y , z) d_i (y , z , w) , r_1 , r_2) = 0

$\sum_{i=1}^{n} d_i$ (G_i (x , y , z) g_i (f , y , z) d_i (y , z , w) , r_1 , r_2) = 0

$\sum_{i=1}^{n} d_i$ (G_i (x , y , z) , r_1 , r_2) d_i (g_i (f , y , z) , r_1 , r_2) d_i (d_i (y , z , w) , r_1 , r_2) = 0 … (5)

Replace d_i (d_i (y , z , w) , r_1 , r_2) by d_i (G_i (x , y , z) , r_1 , r_2) in (5) we get :

$\sum_{i=1}^{n} d_i$ (G_i (x , y , z) , r_1 , r_2) d_i (g_i (f , y , z) , r_1 , r_2) d_i (G_i (x , y , z) , r_1 , r_2) = 0

Since R is semiprime we get :

$\sum_{i=1}^{n} d_i$ (G_i (x , y , z) , r_1 , r_2) = 0

Thus d_n G_n = 0 …….. (6)

From (3) and (6) we get :

$G_n\, d_n = d_n\, G_n = 0$

Similarly way to prove that : $D_n\, g_n = g_n\, D_n = 0$

(6) $G_n\, D_n = D_n\, G_n = 0$

Proof : Since D_n and G_n are orthogonal

$D_n\,(x , y , z)\, R\, G_n\,(y , z , w) = 0$

$G_n\,(D_n\,(x , y , z)\, R\, G_n\,(y , z , w) , r_1 , r_2) = 0 \quad \forall\ r_1 , r_2 \in R$

$\sum_{i=1}^{n} G_i\,(D_i\,(x , y , z)\, r\, G_i\,(y , z , w) , r_1 , r_2) = 0$

$\sum_{i=1}^{n} G_i\,(D_i\,(x , y , z) , r_1 , r_2)\, g_i\,(r , r_1 , r_2)\, g_i\,(G_i\,(y , z , w) , r_1 , r_2) = 0$ ….. (1)

Replace $g_i\,(G_i\,(y , z , w) , r_1 , r_2)$ by $G_i\,(D_i\,(x , y , z) , r_1 , r_2)$ we get :

$\sum_{i=1}^{n} G_i\,(D_i\,(x , y , z) , r_1 , r_2)\, g_i\,(r , r_1 , r_2)\, G_i\,(D_i\,(x , y , z) , r_1 , r_2) = 0$

Since R is semiprime we get :

$\sum_{i=1}^{n} G_i\,(D_i\,(x , y , z) , r_1 , r_2) = 0$

Thuse $G_n\, D_n = 0$ …… (2)

And by $G_n\,(x , y , z)\, R\, D_n\,(y , z , w) = 0$

$D_n\,(G_n\,(x , y , z)\, R\, D_n\,(y , z , w) , r_1 , r_2) = 0$

$\sum_{i=1}^{n} D_i\,(G_i\,(x , y , z)\, r\, D_i\,(y , z , w) , r_1 , r_2) = 0$

$\sum_{i=1}^{n} D_i$ (G_i (x , y , z) , r_1 , r_2) d_i (r , r_1 , r_2) d_i (D_i (y , z , w) , r_1 , r_2) = 0 …… (3)

Replace d_i (D_i (y , z , w) , r_1 , r_2) by D_i (G_i (x , y , z) , r_1 , r_2) in (3) we get :

$\sum_{i=1}^{n} D_i$ (G_i (x , y , z) , r_1 , r_2) d_i (r , r_1 , r_2) Di (G_i (x , y , z) , r_1 , r_2) = 0

Since R is semiprime we get :

$\sum_{i=1}^{n} D_i$ (G_i (x , y , z) , r_1 , r_2) = 0

Thuse D_n G_n = 0 …….. (4)

From (2) and (4) we get : G_n D_n = D_n G_n = 0

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