Republic of Iraq
Ministry Of higher education
and Scientific research
University of AL-Qadisiyah
College of Education

Some Types of Fuzzy Separation Axioms in Fuzzy Topological Space

A research

University as a partial fulfillment requirement for the degree of Bachelor of science in mathematics

By

Ameer Faisal Abd AL_Rehman

Supervisor

Ass.L.Firas Jawad Obaid

2019

بسم الله الرحمن الرحيم

﴿يؤتي الحكمة من يشاء ومن يؤت الحكمة فقد أوتي خيرا كثيرا وما يذكر إلا أولو الألباب﴾

صدق الله العلي العظيم

سورة البقرة الآية (٢٦٩)

الإهداء

إلى من جرع الكأس فارغاً ليسقيني قطرة حب

إلى من كلّت أنامله ليقدم لنا لحظة سعادة إلى من حصد

الأشواك عن دربي ليمهد لي طريق العلم

إلى القلب الكبير (والدي العزيز)

إلى من أرضعتني الحب والحنان

إلى رمز الحب وبلسم الشفاء

إلى القلب الناصع بالبياض (والدتي الحبيبة)

إلى القلوب الطاهرة الرقيقة والنفوس البريئة إلى رياحين حياتي (إخوتي)

الآن تفتح الأشرعة وترفع المرساة لتنطلق السفينة في عرض بحر واسع مظلم هو بحر الحياة وفي هذه الظلمة لا

يضيء إلا قنديل الذكريات ذكريات الأخوة البعيدة إلى الذين أحببتهم وأحبوني (أصدقائي)

شكر وتقدير

والصلاة والسلام } لئن شكرتم لأزيدنكم {الحمد لله يقول الله في محكم كتابه على اشرف خلق الله سيدنا محمد (صلى الله عليه واله وسلم) القائل: من لم يشكر المخلوق لم يشكر الخالق.

بداية اشكر الله عز وجل الذي ساعدني على اتمام بحثي وتفضل علينا بإتمام هذا العمل.. وبعد

شكرا وتقديرا لحضرة الاستاذ الفاضل فراس جواد اليساري على ما بذله من سعة صدر وكرم طبعه ورحابة خاطره وارشاد وتوجيه وتسديد لأفكاري

فجزاه الله خير جزاء المحسنين

الباحث

Abstract

The aim of this paper to introduce and study fuzzy δ-open set and the relations of some other class of fuzzy open sets like (R-open set,θ-open set, γ-open set, Δ-open set), introduce and study some types of fuzzy

δ-separation axioms in fuzzy topological space on fuzzy sets and study the relations between of themand study some properties and theorems on this subject

Table Of Contents

Introduction

The concept of fuzzy set was introduced by Zedeh in his classical paper [1] in 1965.The fuzzy topological space was introduced by Chang [2] in 1968. Zahran [3] has introduced the concepts of fuzzy δ-open sets, fuzzy δ-closed sets, fuzzy regular open sets, fuzzy regular closed sets. And LuayA.Al.Swidi,AmedS.A.Oon.

In chapter one section one we introduce definions, examples and remark of topological space .

In chapter one section two we introduce definitions and theorems of some properties of topological space

In chapter two section one preview fuzzy topological space on fuzzy set also , in chapter two section two introduce propositions of some types of fuzzy

In chapter two section three introduce definitionÃ, proposition ,Ãremark m examples , theorems of fuzzy separation axioms . In [15] introduced the notion of γ-open set, fuzzy γ-closed set and studied some of its properties. N.V.Velicko[9] introduced the concept of fuzzy θ-open set, fuzzy θ-closed set,the fuzzy separation axioms was defined bySinha[10],And Ismail Ibedou[7]introduced anewsetting of fuzzy separation axioms. The purpose of the present paper is to introduce and study the concepts of fuzzy δ-open sets and some types of fuzzy open set and relationships between of them and study some types of fuzzy δ-separation axioms in fuzzy topological space on fuzzy sets and study the relationships between of them and we examine the validity of the standard results

CHAPTER

ONE

SOME PROPERTIES OF TOPOLOGICAL SPACE

1.1 Basic of Topological space.

1.1.1 Definition :

Let X be a set. A topology on X is a collection $T \subseteq P(X)$ of subsets of X satisfying

1. T contains $\emptyset$ and X;
2. T is closed under arbitrary unions, i.e. if $Ui \in T$ for $i \in I$ then $\bigcup_{i \in I} Ui \in T$;
3. T is closed under finite intersections, i.e. if $U_1, U_2 \in T$ then $U_1 \cap U_2 \in T$.

1.1.2 Definition :

A topological space (X, T) is a set X together with a topology T on it. The elements of T are called open subsets of X. A subset $F \subseteq X$ is called closed if its complement $X \setminus F$ is open. A subset N containing a point $x \in X$ is called a neighborhoods of x if there exists U open with $x \in U \subseteq N$. Thus an open neighbourhood of x is simply an open subset containg x.

Normally we denote the topological space by X instead of (X, T).

1.1.3 Definition :

Let $A \subseteq X$ be a subset of a topological space X. The interior of A is the biggest open subset contained in A. One has $A^{\circ} = \bigcup_{A \supseteq U}$ open U. Dually the closure of A is the smallest closed subset containing A. One has $\overline{A} = \bigcap_{A \subseteq F}$ closed F.

1.1.4 Example:

Consider the following set consisting of 3 points; $X = \{a, b, c\}$ and determine if the set $T = \{\emptyset, X, \{a\}, \{b\}\}$ satisfies the requirements for a topology.

This is, in fact, not a topology because the union of the two sets $\{a\}$ and $\{b\}$ is the set $\{a, b\}$, which is not in the set τ

1.1.5 Example:

Find all possible topologies on $X = \{a, b\}$

1. $\emptyset, \{a, b\}$
2. $\emptyset, \{a\}, \{a, b\}$
3. $\emptyset, \{b\}, \{a, b\}$
4. $\emptyset, \{a\},\{b\},\{a,b\}$

1.1.6 Example:

When X is a set and τ is a topology on X, we say that the sets in τ are open. Therefore, if X does have a metric (a notion of distance), then τ ={all open sets as defined with the ball above} is indeed a topology.

We call this topology the Euclidean topology. It is also referred to as the usual or ordinary topology.

1.1.7 Example:

If $Y \subseteq X$ and τx is a topology on X, one can define the Induced topology as $\tau y = \{O \cap Y \,|\, O \in \tau x\}$.

This last example gives one reason why we must only take finitely many intersections when defining a topology.

1.1.8 Remark:

As promised, we can now generalize our definition for a closed set to one in terms of open sets alone which removes the need for limit points and metrics

1.1.9 Definition:

A set C is closed if $X - C$ is open.

Now that we have a new definition of a closed set, we can prove what used to be definition 1.3.3 as a theorem: *A* set *C* is a closed set if and only if it contains all of its limit points.

Proof: Suppose a set *A* is closed. If it has no limit points, there is nothing to check as it trivially contains its limit points. Now suppose *z* is a limit point of *A*. Then if $z \in A$, it contains this limit point. So suppose for the sake of contradiction that *z* is a limit point and *z* is not in *A*. Now we have assumed *A* was closed, so its complement is open. Since *z* is not in *A*, it is in the complement of *A*, which is open; which means there is an open set *U* containing *z* contained in the complement of *A*. This contradicts that z is a limit point because a limit point is, by definition, a point such that every open set about z meets *A*

Conversely: if A contains all its limit points, then its complement is open. Suppose x is in the complement of A. Then it can not be a limit point (by the assumption that A contains all of its limit points). So x is not a limit point which means we can find some open set around x that doesn't meet A. This proves the complement is open, i.e. every point in the complement has an open set around it that avoids A .

1.1.10 Remark:

Since we know the empty set is open, X must be closed.

1.1.11 Remark:

Since we know that X is open, the empty set must be closed.

Therefore, both the empty set and X and open and closed.

1.1.12 Example :

When X is a set and τ is a topology on X, we say that the sets in τ are open. Therefore, if X does have a metric (a notion of distance), then τ ={all open sets as defined with the ball above} is indeed a topology. We call this topology the Euclidean topology. It is also referred to as the usual or ordinary topology.

1.1.13 Definition:

A subset S of topological space (x, T) is said clopen if it is both open and closed subset of X.

1.2 Some properties of Topological space.

1.2.1 Continuity

In topology a continuous function is often called a function. There are 2 different ideas we can use on the idea of continuous functions.

Calculus Style

1.2.2 Definition:

$f : R\ n \rightarrow R\ m$ is continuous if for every $\in > 0$ there exists $\delta > 0$ such that when $|x - x0| < \delta$ then $|f(x) - f(x0)| < \in$.

The map is continuos if for any small distance in the pre-image an equally small distance is apart in the image. That is to say the image does not jump

Topology Style. In topology it is necessary to generalize down the definition of continuity, because the notion of distance does not always exist or is different than our intuitive idea of distance.

1.2.3 Definition :

A function $f : X \rightarrow Y$ is continuous if and only if the pre-image of any open set in Y is open in X. If for whatever reason you prefer closed sets to open sets, you can use the following equivalent definition:

1.2.4 Definition :

A function $f : X \rightarrow Y$ is continuous if and only if the pre-image of any closed set in Y is closed in X.

1.2.5 Definition :

Given a point x of X, we call a subset N of X a neighborhood of X if we can find an open set O such that $x \in O \subseteq N$.

1. A function $f : X \rightarrow Y$ is continuous if for any neighborhood V of Y there is a neighborhood U of X such that $f(U) \subseteq V$.

2. A composition of 2 continuous functions is continuous

1.2.6 Definition :

A function $f:X \rightarrow Y$ between two topological spaces is called continuous if every $U \subseteq Y$ open in Y the inverse image $f^{-1}(U)$ is open in X.

1.2.7 Proposition :

The identity function is continuous. A composition of two continuous maps is continuous. Thus topological spaces and continuous maps between them form a category, the category of topological spaces.

1.2.8 Definition :(Homeomorphisms)

A homeomorphism is a function $f : X \rightarrow Y$ between two topological spaces X and Y that

1.is a continuous and bijection;
2.has a continuous inverse function f^{-1} .

Another equivalent definition of homeomorphism is as follows.

1.2.9 Definition :

Two topological spaces X and Y are said to be homeomorphic if there are continuous function $f : X \to Y$ and $g : Y \to X$ such that $f \circ g = I_Y$ and $g \circ f = I_X$.

Moreover, the functions f and g are homeomorphisms and are inverses of each other, so we may write f^{-1} in place of g and g^{-1} in place of f.

Here, I_X and I_Y denote the identity maps .

1.2.10 Definition:

Let $\mathfrak{J}$ and $\mathfrak{J}^*$ be two topologies on a given set X. If $\mathfrak{J}^* \supseteq \mathfrak{J}$ then $\mathfrak{J}$ is coarser than $\mathfrak{J}^*$.

1.2.11 Definition :

a topological space (X,T) is said to be completely regular space iff every closed subset F of X and every point $x \in X\text{-}F$ there exist a continuous function $f{:}X \to [0,1]$ such that $f(x)=0$, $f(F)=\{1\}$

1.2.12 Definition :(tychonoff)

a tychonoff space or space is completely regular $T_{1\text{-space}}$

1.2.13 Definition :

Say that a family of sets A is linked if for every $A, B \in A, A \cap B = \emptyset$.

1.2.14 Definition :(pathwise)

Let X be a topological space, and $x, y \in X$. A continuous function $p : I \to X$ such that $p(0) = x$ and $p(1) = y$ is called a path from x to y. X is called pathwise.

1.2.15 Definition :

A collection U of open subsets of a topological space X is called an (open) cover if its union is the whole of X, *i.e.* $\bigcup i \in I \ \ U = \cup \ U_0 \in U \ \ U =$

X. A subcollection $U_0 \subseteq U$ is called a sub-cover if it is itself a cover.

1.2.16 Definition :

A topological space X is called compact if every open cover admits a finite sub-cover

1.2.17 Definition :(locally compact)

A topological space is locally compact if every point $x \in X$ has a compact neighborhood.

1.2.18 Example 1.2. Any compact space is locally compact

1.2.19 Definition :

Product topology Given two topological spaces (X, T) and (Y, T'), we define the product topology on X × Y as the collection of all unions $\bigcup_i U_i \times V_i$, where each Ui is open in X and each Vi is open in Y .

1.2.20 Theorem.

Projection maps are continuous Let (X, T) and (Y, T') be topological spaces. If $X \times Y$ is equipped with the product topology, then the projection map $p_{1=} : X \times Y \to X$ defined by $p_1(x, y) = x$ is continuous. Moreover, the same is true for the projection map $p_2 : X \times Y \to Y$ defined by $p_2(x, y) = y$ □

CHAPTER

TWO

On some types of fuzzy separation axioms

2.1 .fuzzy topological space on fuzzy set

Let X be a non-empty set, a fuzzy set Ã in X is characterized bya function $\mu : X \rightarrow I$, where

I= [0,1] which is written as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$, the collection of all fuzzy sets in

X will be denoted by I^X,that is $I^X = \{\tilde{A} : \tilde{A}$ is a fuzzy sets in X$\}$ where μ is called the membership function .

2.1.2 Proposition

Let Ã and B̃ be two fuzzy sets in X with membership functionsμ

1. $\tilde{A} \subseteq \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$.

2. $\tilde{A} = \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$.

3. $\tilde{C} = \tilde{A} \cap \tilde{B} \leftrightarrow C(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$.

4. $\tilde{D} = \tilde{A} \cup \tilde{B} \leftrightarrow D(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$.

2.1.3 Definition

A fuzzy point x_r is a fuzzy set such that :

$\mu_{xr}(y) = r > 0$ if $x = y$, $\forall y \in X$ and $\mu_{xr}(y) = 0$ if $x \neq y$, $\forall y \in X$

The family of all fuzzy points of Ã will be denoted by FP(Ã) .

2.1.4 Remark : Let $\tilde{A} \in I^X$ then $P(\tilde{A}) = \{\tilde{B} : \tilde{B} \in I^X, \mu_{\tilde{B}}$

$(x) \leq \mu_A(x)\} \forall x \in X$.

2.1.5 Definition

A collection T̃ of a fuzzy subsets of Ã, such that $\tilde{T} \subseteq P(\tilde{A})$ is said to be fuzzy topology on Ã if it satisfied the following conditions

1. $\tilde{A}, \phi \in \tilde{T}$
2. If $\tilde{B}, \tilde{C} \in \tilde{T}$ then $\tilde{B} \cap \tilde{C} \in \tilde{T}$
3. If $\tilde{B}_j \in \tilde{T}$ then $\cup_j \tilde{B}_j \in \tilde{T}, j \in J$

$(\tilde{A}, \tilde{T})$ is said to be Fuzzy topological space and every member of T̃ is called fuzzy open set in Ã and its complement is a fuzzy closed set .

2.2 On some types of fuzzy open set Definition 2.3.1 [8,11,12,13,14]

A fuzzy set B̃ in a fuzzy topological space (Ã,T̃) is said to be

1) Fuzzy δ-open [resp.Fuzzyδ-closed set] set if $\mu_{Int(Cl(B}$ (x) ≤ μ [$\mu_{\tilde{B}}$ (x) ≤ $\mu_{Cl\ (Int\ \tilde{B}))}$ (x)]The family of all fuzzy δ-open sets [resp. fuzzy δ-closed sets in a fuzzy topological space(Ã,T̃) will be denoted by FδO(Ã) [resp. FδC(Ã)]

2) Fuzzy regular open [Fuzzy regular closed] set if : $\mu_B(x)$= $\mu_{Int\ (Cl\ B\ )}(x)$[$\mu_B(x)$= $\mu_{Cl\ (Int\ \ B}$ (x)] , The family of all fuzzy regular open [fuzzy regular closed]set in Ã will be denoted by FRO(Ã)[FRC(Ã)].

3) Fuzzy Δ-open set if for every point x_r ∈ B̃ there exist a fuzzy regular semi -open set Û in Ã such that μ_{x_r} (x) ≤ μ_U (x) ≤ $\mu_{\tilde{B}}(x)$, B̃ is called [Fuzzy Δ-closed] set if its complement is FuzzΔ-open set the family of all Fuzzy Δ-open [Fuzzy Δ-closed] sets in Ã will be denoted by FΔO(Ã)[FΔC(Ã)].

4) Fuzzy $\gamma - open$ [$- closed$] set if $\mu_B(x)$ ≤max {$\mu_{Int\ (Cl\ \ (B))}(x)$, $\mu_{Cl\ (int\ \ (B)\ )}$ (x) }, [$\mu_B(x)$ ≥ min {$\mu_{Int\ (Cl\ \ (B\ ))}(x)$, $\mu_{Cl\ (int\ \ (\ B)}$ (x) }]

The family of all fuzzy γ – open [fuzzy γ – closed] sets in Ã will be denoted by FγO(Ã) [FγC(Ã)].

5) Fuzzy θ-open [θ-closed] set if μ_B x =$\mu_{\theta Int\ \ B}$ (x), [μ_B x = $\mu_{\theta Cl(B)}$ (x)]

The family of all fuzzy θ-open (fuzzy θ-closed) sets in Ã will be denoted by Fθ(Ã) [FθC(Ã)].

2.2.1 Proposition

Let (Ã, T̃) be a fuzzy topological space then :

1) Every fuzzy δ-open set (resp. fuzzy δ-closed set) is fuzzy Δ-open set (resp.fuzzy Δ-closed set) **[**fuzzy γ-open set (resp.fuzzy γ-closed set) **]**.
2) Every fuzzyθ-open set (resp. fuzzy θ-closed set) is fuzzy γ-open set (resp. fuzzy γ-closed set) **[**fuzzy δ-open set (resp. fuzzy δ-closed set, fuzzy Δ-open set (resp.fuzzy Δ-closed set) **]**
3) Every fuzzy regular open set (fuzzy regular closed set) is fuzzy δ-open set (resp. fuzzy δ-closed set) **[**fuzzy γ-open set(resp. fuzzy γ-closed set), fuzzyΔ-open set(resp.fuzzy Δ-closed set)**]** **Proof :** Obvious .

2.2.2 Remark

The converse of proposition (2.2) is not true in general as following examples shows

2.2.3 Examples

1) Let X = {a, b}and $\tilde{B}, \tilde{C}, \tilde{D}$, are fuzzy subset in $\tilde{A}$ where $\tilde{A}$ = {(a, 0.9), (b, 0.9) }, $\tilde{B}$ = {(a, 0.0), (b, 0.7) }, $\tilde{C}$ = {(a, 0.8), (b, 0.0) },$\tilde{D}$ ={(a, 0.8), (b, 0.7) }, The fuzzy topology defined on A is $\tilde{T}$= {ϕ $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ }

- The fuzzy set D is a fuzzy Δ-open set but not fuzzyδ – open set (fuzzy regular open set, fuzzy θ – open set).
- let X = {a, b, c}and B G D E are fuzzy subset in A where $\tilde{A}$ = {(a, 0.9), (b, 0.9), (c, 0.9) }, $\tilde{B}$ = {(a, 0.3), (b, 0.3), (c, 0.4)} $\tilde{C}$ = {(a,0.4), (b, 0.3), (c, 0.4) }, $\tilde{D}$ = {(a, 0.5), (b, 0.5), (c, 0.4) }$\tilde{E}$ = {(a, 0.6), (b,0.6), (c, 0.7), The fuzzy topology defined on A is $\tilde{T}$= {∅, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$ }
- The fuzzy set B is a fuzzy γ – open set but not fuzzy δ – open set (fuzzy regular open set, fuzzy θ – open set).

2) Let X ={a, b, c}and B̃, C̃, D̃, Ẽ, F be fuzzy subsets of Ã where:

Ã = {(a,0.8),(b,0.8), (c,0.8)}, B̃ = {(a,0.1), (b,0.1), (c,0.2)}, C̃ = {(a,0.2), (b, 0.1), (c ,0.2)}D = {(a,0.3), (b,0.3), (c,0.2)},Ẽ ={(a,0.4), (b,0.4), (c,0.5)},F = {(a,0.3), (b,0.3), (c,0.3)}

The fuzzy topologies defined on Ã are T̃= {ϕ Ã, B̃ , C̃ , D̃ }The fuzzy set Ẽ is a fuzzy δ-open set but not fuzzy regular open set (fuzzy θ-open set).

2.2.4 Remark

Figure - 1 – illustrates the relation between fuzzy δ-open set and some types of fuzzy open sets.

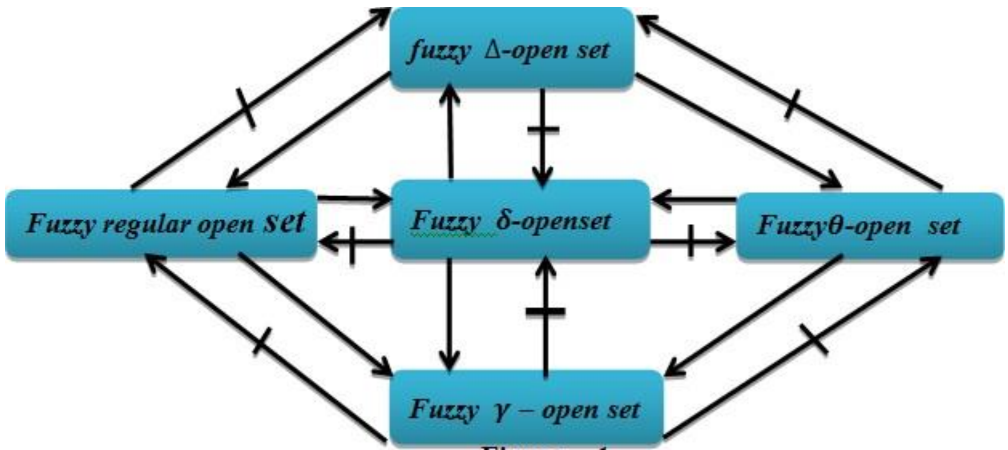

Figure - 1 –

2.3 Some Types Of Fuzzy Separation Axioms

2.3.1 Definition

A fuzzy topological space (Ã, T̃) is said to be **Fuzzy δT_0 − space(FδT_0)** if for every pair of distinct fuzzy points x_r , y_t in Ã there exist B̃ ∈ FδO(Ã) such that either $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_t\, q$B̃ or $\mu_{y_t}(y) < \mu_{\tilde{B}}(y)$, $x_r\, q$B̃ .

2.3.2Theorem

If (Ã, T̃) is a fuzzyδT_0- space then for every pair of distinct fuzzy points x_r $y_t$where $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$,

$\mu_{y_t}(x) < \mu_{\tilde{A}}(x)$ then either δ- cl(x_r) q y_t or δ- cl(y_t) q x_r .

Proof:

Let $x_{r,t}$ be two distinct fuzzy points such that

$\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_t}(x) < \mu_{\tilde{A}}(x)$ then there exist a fuzzy δ- open set B̃ such that either

$\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, B̃

q y_t or $\mu_{y_t}(x) < \mu_B(x)$, B̃ q x_r

If $_x$ $(x) < \mu_B(x)$, $\tilde{B}$ q y_t then B^c q x_r, μ_y $(x) \leq \mu_{B^c}(x)$

Since ^{c} is a fuzzy δ- closed set therefore $\mu_{\delta cl\,(y\,)}(x) \leq \mu_{B^c}(x)$

Hence δ- cl(y_t) q x_r

Similarly if $\mu_{y_t}(x) < \mu_{\tilde{B}}(x)$, B̃ q x_r

2.3.3 Definition :

A fuzzy topological space (Ã, T̃) is said to be **Fuzzy δT_1 − space (FδT_1)** if for every pair of distinct fuzzy points x_r , y_t in Ã there exist two B̃ , C̃ ∈ FδO(Ã) such that$\mu_{x_r}(x) < \mu_B(x)$, $y_t\, q$B̃ and $\mu_{y_t}(y) < \mu_C(y)$,$x_r\, q$ C̃.

2.3.4 Proposition :

Every fuzzy δT_1 – space is a fuzzy δT_0 – space .

Proof : Obvious .

2.3.5 Remark :

The converse of proposition (3.4) is not true in general as shown in **the** following example .

2.3.6 Example :

Let X={ a , b } and B̃, C̃, D̃ are fuzzy subset of Ã where:
Ã = {(a, 0.4), (b, 0.4)}, B̃ = {(a, 0.4), (b, 0.1)}, C̃ = {(a, 0.1), (b, 0.1)}, D̃ = {(a, 0.4), (b, 0.2)}, Ẽ = {(a, 0.3), (b, 0.1)}, T̃ = {∅, Ã, B̃, C̃, D̃ }be a fuzzy topology on A and the FδO(A) = {∅, Ã, B̃, C̃, Ẽ } Then the space (Ã, T̃) is a fuzzy δT_0 - space but not fuzzy δT_1 – space

2.3.7 Theorem :

If (Ã, T̃) is a fuzzy Topological space then the following statements are equivalents :

1) (Ã, Ť) is a fuzzy Ť$_1$ - space.

2) For every maximal fuzzy points x_r , y_t in Ã , there exists a fuzzy open nbhds sets Ũ and Ṽ of

x_r

and y_t respectively in Ã such that $\mu_{x_r}(x)$= min {{$\mu_{\tilde{U}}(x)$,$\mu_{\tilde{U}}(y)$}, {$\mu_{x_r}(x)$, $\mu_{y_t}(y)$}} and $\mu_{y_t}(y)$= min {{

$\mu_{(\tilde{V})}(x)$, $\mu_{(\tilde{V})}(y)$ }, {$\mu_{x_r}(x)$, $\mu_{y_t}(y)$ }}.

3) For every maximal fuzzy points x_r, y_t in Ã, there exists a fuzzy δ-open nbhds sets Ũ and Ṽ of x_r

and y_t respectively in Ã such that $\mu_{x_r}(x)$= min {{$\mu_{\tilde{U}}(x)$,$\mu_{\tilde{U}}(y)$}, {$\mu_{x_r}(x)$, $\mu_{y_t}(y)$}} and $\mu_{y_t}(y)$= min {{

$\mu_{(\tilde{V})}(x)$, $\mu_{(\tilde{V})}(y)$ }, {$\mu_{x_r}(x)$, $\mu_{y_t}(y)$ }}.

Proof:

(1⟹ 2) : Let x_r, y_t ∈ MFP(Ã), ∃ Ũ, Ṽ ∈ FδO(Ã)

such that $\mu_{x_r}(x) < \mu_{\tilde{U}}(x)$,y_t qŨ and

$\mu_{y_t}(y) <$ $_{(V)}(y)$, x_r q Ṽ.then$\mu_{x_r}(x)$=$\mu_{\tilde{U}}(x)$=$\mu_A(x)$,

$\mu_{y_t}(y)$+$\mu_{\tilde{U}}(y) \leq \mu_A(y)$ and

$\mu_{y_t}(y)$=$\mu_{(V)}(y)$=$\mu_A(y)$, $\mu_{x_r}(x)$+ $\mu_{(V)}(x) \leq \mu_A(x)$then $\mu_{\tilde{U}}(y)$= 0 , $\mu_{(V,}$ (x)= 0 , and since Ũ FδO(Ã) then Ũ, Ṽ ∈ T̃

Therefore $\mu_{x_r}(x)$= min {{$\mu_{\tilde{U}}(x)$,$\mu_{\tilde{U}}(y)$}, {$\mu_{x_r}(x)$, $\mu_{y_t}(y)$}}

and$\mu_{y_t}(y)$= min {{$\mu_{(\tilde{V})}(x)$, $\mu_{(\tilde{V})}(y)$ }, {$\mu_{x_r}(x)$, $\mu_{y_t}(y)$ }}.

(2⟹ 3) : Obvious.

(3 ⟹ 1) : Let x_n , y_m ∈ FP(Ã), then every x_r , y_t ∈ MFP(Ã) , there exist Ũ , Ṽ ∈ FδO(Ã) such that

$\mu_{x_r}(x)$= min {{$\mu_{\tilde{U}}(x)$,$\mu_{\tilde{U}}(y)$}, {$\mu_{x_r}(x)$, $\mu_{y_t}(y)$}} and

$\mu_{y_t}(y)$= min {{$\mu_{(\tilde{V})}(x)$, $\mu_{(\tilde{V})}(y)$ },

{$\mu_{x_r}(x)$, $\mu_{y_t}(y)$ }}. thenμ_{x_r}

(x)=$\mu_{\tilde{U}}(x)$=$\mu_A(x)$, $\mu_{\tilde{U}}(y)$= 0 and

$\mu_{y_t}(y)$=$\mu_{(V)}(y)$= $\mu_A(y)$, $\mu_{(V)}(x)$= 0

then$y_t\, q$Ũ and $x_r\, q$ Ṽ**,** Since $\mu_{x_n}(x) < \mu_{x_r}(x)$ and $\mu_{y_m}(y) < \mu_{y_t}$ (y), ∀ n, m ∈ I

then$\mu_{x_n}(x) < \mu_{\tilde{U}}(x)$, y_m qŨ and $\mu_{y_m}(y) < \mu_{(\tilde{V})}(y)$, x_n qṼ Hence the space (Ã, T̃) is a fuzzy δT_1- space.

2.3.8 Definition :

A fuzzy topological space (Ã, T̃) is said to be **Fuzzy $\boldsymbol{\delta T_2}$ − space (F$\boldsymbol{\delta T_2}$)** if for every pair of distinct fuzzy points x_r , y_t in Ã there exist two B̃ , C ∈ FδO(Ã) such that$\mu_{x_r}(x) < \mu_B(x)$,$\mu_{y_t}(y) < \mu_C(y)$ and $\tilde{B}\, q\, C$.

2.3.9 Theorem :

A fuzzy topological space (A $\tilde{T}$) is a fuzzy δ-T_2 –space if and only if min{$\mu_{\delta cl\ U}$ (x) : U is a fuzzy δ-open set ,$x_r(x) < \mu_U(x)$}< $\mu_{y_t}(x)$ any fuzzy point such that $\mu_{y_t}(x) < \mu_A(x)$.

Proof :

(⟹) Let (Ã, T̃) be a fuzzy δ-T_2 –space and x , y_t be a distinct fuzzy points in A such that $\mu_{x_r}(x) < \mu_A(x)$,

$\mu_{y_t}(x) < \mu_A(x)$ and

$\mu_{y_n}(x) = \mu_A(x)$ - $\mu_{y_t}(x)$then$\mu_{y_n}(x) < \mu_A(x)$ and there exists two fuzzy δ-open set U G in A such that

$\mu_{x_r}(x) < \mu_U(x)$, $\mu_{y_n}(x) < \mu_G(x)$, $U \; q \; G$ $\mu_U(x) \leq \mu_{G^c}(x)$ and

$\mu_{\delta cl(U)}(x) \leq \mu_{\delta cl(G^c)}(x) = \mu_{G^c}(x)$

Since $\mu_{y_n}(x) < \mu_G(x)$ and $\mu_{y_n}(x) = \mu_A(x)$ - $\mu_{y_t}(x)$,Then

$\mu_A(x)$ - $\mu_{y_t}(x) < \mu_G(x)$,$_{G^c}(x) < \mu_A(x)$ - $\mu_{y_n}(x)$ and $\mu_{G^c}(x) < \mu_{y_t}(x)$

Since $\mu_{\delta cl(U)}(x) \leq \mu_{G^c}(x) < \mu_{y_t}(x)$ then $\mu_{\delta cl(U)}(x) < \mu_{y_t}(x)$

Hence min{$\mu_{\delta cl\, U_i}$ (x) : i = 1,......, n }< $\mu_{y_t}(x)$

(⟸) Suppose that given condition hold,$_r$, y_t are distinct fuzzy points in A such that $\mu_{x_r}(x) < \mu_A(x)$,

$\mu_{y_t}(x) < \mu_A(x)$ and

Let $\mu_{y_n}(x) = \mu_A(x)$ - $\mu_{y_t}(x)$ then $\mu_{y_n}(x) < \mu_A(x)$

And $\mu_{\delta cl(U)}(x) < \mu_{y_n}(x)$ for every $\mu_{x_r}(x) < \mu_U(x) \leq \mu_{\delta cl(U)}(x)$

, since $\mu_{y_n}(x) = \mu_A(x)$ - $\mu_{y_t}(x)$

Then $\mu_{\delta cl(U)}(x) < \mu_A(x)$ - $\mu_{y_t}(x)$ and $\mu_A(x)$ - $\mu_{y_n}(x) < \mu_{\delta int(U^c)}(x)$hence $\mu_{y_t}(x) < \mu_{\delta int(U^c)}(x)$

let$\mu_G(x) = \mu_{\delta int(U^c)}(x)$ and since $\mu_{\delta int(U^c)}(x) \leq \mu_{(U^c)}(x)$,Then

$\mu_{y_t}(x) < \mu_G(x)$and$\mu_G(x) \leq \mu_{(U^c)}(x)$ we get $U \; q \; G$Hence the space (Ã, T̃) is a fuzzy δ-T_2 – space

2.3.10 Proposition :

Every fuzzy δT_2 – space is a fuzzy δT_1 – space .

Proof : Obvious .

2.3.11 Remark :

The converse of proposition (3.10) is not true in general as shown in the following example .

2.3.12 Example :

Let X={ a , b } and B̃, C̃, D̃ Ẽ are fuzzy subset of A where: = {(a, 0.7), (b, 0.9)},B = {(a, 0.5), (b, 0.0)},C = {(a, 0.0), (b, 0.7)},D = {(a, 0.5), (b, 0.7)},E =

{(a, 0.1), (b, 0.8)}, F = {(a, 0.6), (b, 0.1)}, T = {∅, Ã, B̃, C̃ }be a fuzzy topology on A and the FδO(A) = {∅, Ã, B̃, C̃, Ẽ,F} then the space (A, T) is a fuzzy δT_1 - space but not fuzzyδT_2 – space

2.3.13 Definition :

A fuzzy topological space (Ã, T̃) is said to be Fuzzy $\boldsymbol{\delta}$ 2 1/2 space (F$\boldsymbol{\delta T}$ 2 1/2) if for every pair of distinct fuzzypoints x_r , y_t in Ã there exist two B̃ , C ∈ FδO(Ã) such that $\mu_{x_r}(x) < \mu_B(x)$, $\mu_{y_t}(x) < \mu_C(x)$ and δc(B̃) q $\delta cl(C)$.

2.3.14 Proposition :

Every fuzzy $\delta T_{2\frac{1}{2}}$- space is a fuzzy δT_2- space .

Proof :

Let (Ã, T̃) be a fuzzy $\delta T_{2}1$- space, then every pair of distinct fuzzy points x_r, y_t in Ã there exist two B̃, C

FδO(Ã) such that $\mu_{x_r}(x) < \mu_{B}(x)$,$\mu_{y_t}(x) < \mu_{C}(x)$ and $\delta cl(\tilde{B})\ q\ \delta cl(C)$

Since $\mu_{B}(x) \leq \delta cl(B)$, $\mu_{C}(x) \leq \delta cl(C)$

Then We get $\tilde{B}\ q\ G$ hence (A, T) is a fuzzy δT_2- space

2.3.15 Remark :

The converse of proposition (3.14) is not true in general as shown in the following example

2.3.16 Example :

Let X={ a , b } and, B̃$_1$, B̃$_2$, B̃$_3$, B̃$_4$,B̃$_5$, B̃$_6$, B̃$_7$, B̃$_8$, are fuzzy subset of A where:

A = {(a, 0.9), (b, 0.9)}, B̃$_1$ = {(a, 0.8), (b, 0.1)}, B̃$_2$ = {(a, 0.0), (b, 0.7)},

B̃$_3$ = {(a, 0.8), (b, 0.7)}, B̃$_4$ = {(a, 0.0), (b, 0.1)}, B̃$_5$ = {(a, 0.0), (b, 0.9)},B̃$_6$ = {(a, 0.8), (b, 0.9)}, B̃$_7$ = {(a, 0.0), (b, 0.8)} BB̃$_8$ = {(a, 0.8), (b, 0.0)}, = {∅ A B_1 , B_2 , B_3 , B_4

,B_5 , B_6 }be a fuzzy topology on A and the FδO(A) = {∅, A , B_1 , B_2 , B_4 , B_7 , B_8 }, then the space (A, T) is a fuzzy δT_2- space but not fuzzy $\delta T_{2}1$ –space

2.3.17 Definition :

A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be **Fuzzy δ- regular space (FδR)** if for each fuzzy point x_r in $\tilde{A}$ and each fuzzy closed set F with $x_r \; q \; F$ there exists B C $\in$FδO($\tilde{A}$) such that $\mu_{x_r}(x) \leq \mu_B(x)$, $\mu_F(x) \leq \mu_C(x) \forall$ x ∈ X and $B \; q \; C$

2.3.18 Definition :

A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be **fuzzy δ^*- regular space (Fδ^*R)** if for each fuzzy point x_r in $\tilde{A}$ and each fuzzy δ- closed set F with $x_r \; q \; F$ there exists B , C $\in$FδO($\tilde{A}$) such that $\mu_{x_r}(x) \leq \mu_B(x)$, $\mu_F(x) \leq \mu_C(x) \forall$ x ∈ X and $B \; q \; C$

2.3.19 Proposition :

Every fuzzy δ- regular space is a fuzzy δ^*- regular space.

Proof: Obvious .

2.3.20 Remark :

The converse of proposition (3.19) is not true in general as shown in the following example

2.3.21 Example :

Let X={a, b}and B , C, D, E, F is a fuzzy subset of A where:

A = {(a, 0.7), (b, 0.8)}, B = {(a, 0.0), (b, 0.7)},= {(a, 0.6), (b, 0.0)}, D = {(a, 0.6), (b, 0.7)}, E = {(a, 0.7), (b, 0.0)}, F = {(a, 0.0), (b, 0.8)}

T = {∅, A , B , C , D }be a fuzzy topology on A and the FδO($\tilde{A}$) = {∅, A , B , C, D , E , F } Then the space $(\tilde{A}, \tilde{T})$ is a fuzzyδ^*- regular space but not fuzzy δ- regular space.

2.3.22 Definition :

A fuzzy topological space (Ã, Ť) is said to be Fuzzy $\boldsymbol{\delta T_3}$ − space (F$\boldsymbol{\delta T_3}$) if it is δ- regular space (FδR)as well as fuzzy δT_1 − space (FδT_1) .

2.3.23 Proposition :

Every fuzzy δT_3- space is a fuzzy $\delta T_{2}1$- space .

Proof :

Let $(A\ \ )$ be a fuzzy δT_3 - space,

Then $(A\,T)$ be a fuzzy δ - regular space, for every fuzzy point $x_r \in$ FP(Ã) and $F \in$ FC(Ã) Such that $x_r\ q\ F,\ F = \delta cl(F)$

And since $(A\ \ )$ be a fuzzy δT_1 - space then We get $\{x_r\}$is a fuzzy δ-closed set Let $\{x_r\}= B$ is a fuzzy δ-closed set

Then $cl(B)\ q\ \delta cl(F)$, hence $(A\ T)$ is a fuzzy $\delta T\ \ 1$- space

2.3.24 Remark :

The converse of proposition (3.23) is not true in general as shown in the following example .

2.3.25 Example :

Let X={a, b}and , B_1, B_2, B_3, B_4,B_5, B_6, B_7, B_8, B_9, B_{10} ,B_{11}, are fuzzy subset of A where:A = {(a, 0.8), (b, 0.9)}, B_1 = {(a, 0.8), (b, 0.0)}, B_2 = {(a, 0.0), (b, 0.7)}, B_3 = {(a, 0.8), (b, 0.7)}, B_4 = {(a, 0.1), (b, 0.9)}, B_5 = {(a, 0.6), (b, 0.0)}, B_6 = {(a, 0.1), (b, 0.0)}, B_7 ={(a, 0.6), (b, 0.9)} B_8 = {(a, 0.1), (b, 0.7)}, B_9 = {(a, 0.6), (b, 0.7)}, B_{10} = {(a, 0.0),(b, 0.8)}, B_{11} = {(a, 0.7), (b, 0.0)},T = {∅ , A , B_1 , B_2 , B_3 , B_4,B_5, B_6 , B_7 , B_8 , B_9}be a fuzzy topology on A and the FδO(A) = {∅, A , B_1, B_2, B_4, B_5 , B

$_6$, B_7, B_{10}, B_{11} }, then the space (A, T) is a fuzzy $\delta T_{2}1$ - space but not fuzzyfuzzy δT_3 – space .

2.3.26 Definition :

A fuzzy topological space (Ã, Ť) is said to be fuzzy $\boldsymbol{\delta^* T}$ – space(F$\boldsymbol{\delta^* T_3}$) if it is$\delta^*$- regular space (F$\delta^*$R) as well as fuzzy $\delta\, T_1$ – space (FδT_1)

2.3.27 Proposition :

Every fuzzy δT_3 - space is a fuzzy $\delta^* T_3$ – space.

Proof: Obvious

2.3.28 Proposition :

Every fuzzy $\delta^* T$ - space is a fuzzy δT 1- space .

Proof: Obvious

2.3.29 Remark :

The converse of proposition (3.27)and (2.28) is not true in general

2.3.30 Definition :

A fuzzy topological space (Ã, T̃) is said to be Fuzzy **$\boldsymbol{\delta}$- normal space (F$\boldsymbol{\delta}$N)** if for each two fuzzy closed sets F_1 and F_2in Ã such that $F_1\; q\; F_2$, there exists U_1 , U_2 ∈FδO(Ã) such that $\mu_{F_1}(x) \le \mu_{U_1}(x)$, $\mu_{F_2}(x) \le \mu_{U_2}(x)$ and $U_1\; q\; U_2$.

2.3.31 Definition :

A fuzzy topological space (Ã, T̃) is said to be **Fuzzy δ*- normal space (Fδ*N)** if for each two fuzzy δ-closed sets F_1 and F_2 in Ã such that F_1 q F_2, there exists U_1, U_2 ∈FδO(Ã) such that $\mu_{F_1}(x) \leq \mu_{U_1}(x)$, $\mu_{F_2}(x) \leq \mu_{U_2}(x)$ and U_1 q U_2 .

2.3.32 Proposition :

Every fuzzy δ- normal space is a fuzzy δ*- normal space

Proof: Obvious .

2.3.33 Definition :

A fuzzy topological space (Ã, T̃) is said to be **Fuzzy δ T − space (FδT_4)** if it is

δ-normal space (FδN)as well as fuzzy δ T_1 − space (Fδ T_1) .

2.3.34 Definition :

A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be fuzzy $\boldsymbol{\delta^* T}$ **− space** ($\boldsymbol{F\delta^* T_4}$) if it is δ^*- normal space ($F\delta^*N$) as well as fuzzy $\delta\ T_1$ − space ($F\delta\ T_1$)

2.3.35 Proposition :

Every fuzzy δT_4 – space is a fuzzy $\delta^* T_4$ − space

Proof: Obvious

2.3.36 Proposition :

Every fuzzy δT_4 – space is a fuzzy δT_3 − space

Proof: Obvious

2.3.37 Remark :

The converse of proposition (3.35) and (3.36) is not true in general Definition 3.38:

A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be Fuzzy $\boldsymbol{\delta}$-completely normal if for any two fuzzy δ-separated sets B C in $\tilde{A}$ there exist D $E \in F\delta O(\tilde{A})$ such that $\mu_B(x) \leq \mu_D(x), \mu_C(x) \leq \mu_E x$ and $D\ q\ E$

2.3.39 Definition :

A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be Fuzzy $\boldsymbol{\delta\, T_5}$ **− space** ($\boldsymbol{F\delta T_5}$) if it is δ-completely normal space as well as fuzzy $\delta\ T_1$ − space ($F\delta T_1$) .

2.3.40 Proposition :

Every fuzzy δT_5 – space is a fuzzy δT_4 − space

Proof: Obvious

2.3.41 Remark :

The converse of proposition (3.40) is not true in general

2.3.42 Remark :

Figure (2) illustrate the relations among a certain types of fuzzy δT – space, i = 0, 1, 2, 2^{1},3,4,5.

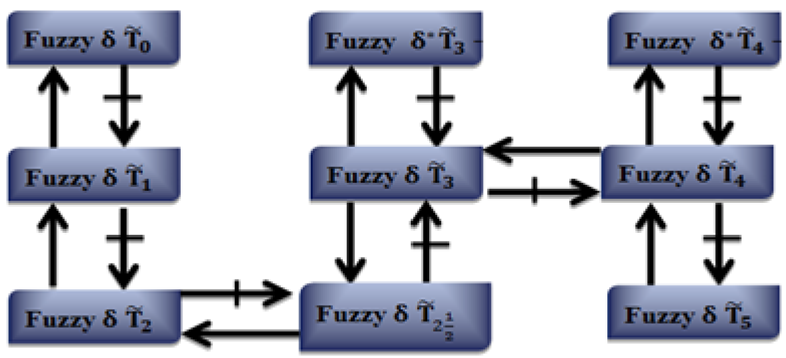

Figure - 2 –

References

[1] A.M.Zahran "fuzzy δ-continuous , fuzzy almost Regularity (normality) on fuzzy topology No fuzzy set" fuzzy mathematics,Vol.3,No.1,1995,pp.89-96

[2] Chakraborty M. K. and T. M. G. Ahsanullah"Fuzzy topology on fuzzy sets and tolerance topology " Fuzzy Sets and Systems, 45103-108(1992).

[3] Chang, C. L. "Fuzzy Topological Spaces", J. Math. Anal.Appl., Vol.24, pp.182-190, (1968).

[4] Ismail Ibedou," A New Setting of fuzzy separation axioms"Department of Mathematics, Faculty of Science, Benha University, Benha, (13518), Egypt

[5] Journal of mathematics,Vol 7, pp 63-70, (May-Jun. 2013).

[6] Kandil1 A., S. Saleh2 and M.M Yakout3 "Fuzzy Topology On Fuzzy Sets: Regularity and Separation Axioms" American Academic & Scholarly Research Journal Vol. 4, No. 2, March (2012).

[7] LuayA.Al.Swidi,AmedS.A.Oon, Fuzzy γ-open sets and fuzzy γ-closed sets " Americal Journal of Scientific research,27(2011),62- 67.

[8] M.E. El-shafei and A. Zakari," θ-generalized closed sets in fuzzy topological spaces " The Arabian Journal for science and Engineering 31(2A) (2006), 197-206.

[9] Mashhour A.S. and Ghanim M.H."Fuzzy closure spaces"J.Math.Anal.And Appl.106,pp.145-170(1985).

[10] N.V.Velicko" H-closed topological space"Amer.math.soc.Transl.(2),78(1968), 103-118.

[11] S.S.Benchalli,R.S.Wali and BasavarajM.Ittanagi " On fuzzy rw-closed sets And fuzzy rw-open sets in fuzzy topological spaces"Int.J.mathematical sciences and Applications, Vol.1,No.2,May 2011.

[12] Shahla H. K. "On fuzzy Δ - open sets in fuzzy topological spaces" M. Sc. Thesis, college of science , Salahddin Univ. (2004). [13]. R.UshaParameswari and K.Bageerathi" On fuzzy γ-semi open sets and fuzzy γ-semi closed sets in fuzzy topological spaces" IOSR

[13] ShymaaAbdAlhassan A " On fuzzy semi-separation Axioms in fuzzy topological space on fuzzy sets"M.Sc Thesis, College of Eduction, Al-Mustansiritah University (2013).

[14] Sinha,S.P." separation axioms in fuzzy topological spaces"suzzy sets and system,45:261-270(1992).

[15] Zadeh L. A."Fuzzy sets", Inform.Control 8, 338-353 (1965).