جامعة القادسية

كليــة التربيـــة

قسم الرياضيات

الزمر التبولوجية النانونية

بحث مقدم من قبل الطالبة

سجى علي مزهر

الى مجلس قسم الرياضيات/كلية التربية/جامعة القادسية كجزء من متطلبات نيل شهادة البكلوريوس في علوم الرياضيات.

بأشراف

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الملخص

تناول هذا البحث موضوع الزمر التبولوجية النانوية وهي الزمر التي يكون الجزء التبولوجي لها هو تبولوجي نانوي ومن هنا يمكن تعريف بقية المفاهيم التبولوجية عليه مثل الاستمرارية النانوية و الفضاءات المرصوصة النانوية و بديهيات الفصل النانوية . وجدنا في هذا البحث العلاقة بين بديهيات الفصل النانوية و الزمر التبولوجية النانوية وكذلك قدمنا تعريف الاتصال النانوي في الزمر التبولوجية النانوية.

<u>الاية</u>

بسم الله الرحمن الرحيم

((أَمَّنْ هو قانتٌ آناءَ اللَّيلِ ساجداً وقائماً يَحذَرُ الآخرةَ ويَرجو رحمةَ ربِّهِ قلْ هل يستوي الَّذين يعلمونَ والَّذين لا يعلمونَ إنَّما يَتَذكَّرُ أولوا الألباب))

الزمر-

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صدق الله العلي العظيم

الاهداء

الى من وهبنا الحياة وديمومتها ....

ربي

الى من علمنا القرآن والثبات على الحق

.................رسولي

إلى العتره الطاهره سيوف الحق وكلمت

الصدق.........أئمتي

إلى بحر الحب والعطاء وروضة الحنان

الطاهرة.............أمي

إلى من أرهقه التعب ومثلي

الأعلى...........................أبي

إلى من وقف بجانبي وسندي...............................اخي

إلى من يشد بهم ازري وتقوى عزيمتي رفقاء دربي

....اصدقائي

إلى كل من يجد القرآن الكريم نورا لدربه .

CHAPTER

ONE

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TWO

REFERENCE

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University of AL-Qadisiyah
Education College
Department of Mathematics

Nano topological group

A paper

Submitted to the council of mathematic dept.-College of education University of Al- Qadisiyah As a Partial Fulfillment of the Requirements for the Degree Of Bachelor of Science in Mathematics

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Abstract:

This research deals with the subject nano-topological groups which are group with topology structure is nano topology , and from this we can define the others concepts topology on its, like the nano continuousm nano compact , and nano-separtion axioms . In this research we find the relation between the nano-separtion axiom and the nano topological group and introduced the definition of nano-connected in nano-topological groups.

Introduction

Topology is an umbrella term that includes several fields of study. These include point set topology, algebraic topology, and differential topology. Because of this it is difficult to credit a single mathematician with introducing topology. The following mathematicians all made key contributions to the subject: Georg Cantor, David Hilbert, Felix Hausdor_, Maurice Frechet, and Henri Poincar.

In general, topology is a special kind of geometry, a geometry that doesn't include a notion of distance. Topology has many roots in graph theory. When Leonhard Euler was working on the famous Konigsberg bridge problem he was developing a type of geometry that did not rely on distance, but rather how different points are connected. This idea is at the heart of topology. A topological group is a set that has both a topological structure and an algebraic structure.

Nano Topology in engineering and medical provides an interdisciplinary forum uniquely focused on conveying advancements in nano science and applications of nano structures and nano materials to the creative conception, design, development, analysis, control and operation of devices and technologies in engineering, medical and life science systems. The notation of nano topology was introduced by Lellis Thivagar which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also depend nano closed sets, nano interior and nano closure. In this paper, we introduced a new class of topological group is called nano topological group and we study some of its properties.

1.1 Topological group

1.1.1 Definition [2]:

A topological group is a set G with two structures :

1) G is a group with respect to * .

2) G is a topological space .

Such that the two structures are compatible, i.e , the multiplication map $M:G\times G\rightarrow G$ and the inversion .

1.1.2 Examples [2]:

Map $U: G\rightarrow G$ are continuous,

I) $(R,+,U_R)$ is a topological group , such that :

1) (R,+) is a group .

2) (R , U_R) is the usual metric topology .

II) $(R,*,T_D)$ is a topological group ,such that :

1) (R,*) is a group .

2) (R, T_D) is a discrete topological space .

1.1.3 Definition[2]:

We say that H is a topological sub group of G , if H is a sub group of G and H is a topological group with respect to the sub space topology,

1.1.3 Example [2]:

Z is the topological sub group of R with a discrete topology on Z .

1.1.4 Definition [2]:

A morphism $f:G\rightarrow H$ of topological groups is continuous group homorphism .

1.1.5 Definition [2]:

1) $l_g:G\rightarrow G$, $l_g(x)=gx$, is called left multiplication by g , This map has inverse l_g^{-1} which is also continuous , so l_g is homeomorphism from G to G .

2) similary, all right multiplication $r_g:G\rightarrow G$,$r_g(x)=xg$ are homeomorphism .

1.1.6 Definition [2]:

A topological space is called a homogeneous space, if for all a,b ϵ G , there is a homeomorphism f: $G\rightarrow G$, such that f(a)=b .

1.1.7 Note [2]:

If (G,*,T) be a topological group then the (G,T) is homogeneous space , because , if a,b ϵ G , then $l_{ba}{}^{-1}(a)=(ba^{-1})(a)=b$, or,$r_{ab}{}^{-1}(a)=a(a^{-1}b)=b$

1.1.8 Definition [2]:

Let A,B $\subseteq$ G and g$\in$G , such that, G is a topological group , then :

i) Ag=A r_g={ag:a$\in$A},is called the right translate of A by g,

ii) $gA=Al_g$

iii) $AB=\cup_{b\in B}Ab=\cup_{a\in A}aB$

iv)$A^{-1} = \{a^{-1}:a\in A\}$

1.1.9 proposition [2]:

let G be a topological group ,A,B $\subseteq$ G,g$\in$G then:

i) G is a homogeneous space .

ii) A open implies Ag and gA open .

iii) A closed implies Ag and gA closed .

iv) A open implies AB and BA open .

v) A closed and B finite implies AB and BA closed .

1.1.10 Definition [2]:

Let G be a topological Group. and let H be a subgroup of G .

let $G/H =\{xH: x\in G\}$ the set of left costs of H.

The map q: $G\rightarrow G/H$ defined by $q(x)=xH$ defines a qoutient topology on G/H . we note that q is open map

If H is a normal sub group of G, then G/H is a topological group .

1.1.11 Definition [2]:

Let X be a topological space :

1) X is T_1−space if for any a,b∈X, a≠b there exists a neighbourhood of a not containing b .

2) X is a T_2−space (or a Hausdorff space) if for any a,b∈X ,a≠b there exists nighbourhoods of a and b which are disjoint .

3) A space X is $T_{\circ}$−space if for any a,b∈X ,a≠b,there exists an open containing exactly one of them.

1.1.12 Note [2]:

X is a T_1−space if and only if every singleton subset of x is closed .

1.1.13 Proposition [2]:

Every of topological group is Hausdorff .

1.1.14 Definition [2]:

Aspace X is regular if every neighbourhood of each point X contains a closed neighbourhood of x,

i.e,for every closed set A⊆ x and every point b∉ A , then exist open sets u,v in x such that A⊆U ,b∈V and U∩V=∅.

1.1.15 Proposition [2]:

A regular T_1−space is Hausdorff space .

Proof:

let X be a regular T_1−space

Let a,b ∈X, a≠b ,since X is T_1−space then there exists an open neighbourhood U such that a∈U and b∉U

→a∉U^c ,U^c closed ,b∈U^c

Now , since X is regular space then there exists open set V,W such that U^c⊆ V ,a∈V and V∩W=∅ → W⊆V^c → a∈V^c , V^c is closed set .

Since U^c⊆V → U^c∩V^c=∅

∴ a∈V^c , b∈U^c , U^c∩V^c =∅ ,U^c,V^c are closed sets.

∴ X is T_2− space .

1.1.16 Proposition [2]:

Every topological group G is regular .

connected if is non−empty and can not be written X=A∪B where A and B are non− empty disjoint open sub sets of X.

1.1.18 Note [2]:

A non_empty space X is connected if and only if the only sub sets of X which are both open and closed are ∅ and X.

1.1.19 Proposition [2]:

A connected topological group has no proper open sub groups .

1.1.20 Proposition [2]:

If x is connected and $f:X\rightarrow Y$ is a continuous function,then f(x) is connected .

1.1.21 Proposition [2]:

Let G be a topological group , and let H be a sub group of G ,then:

i) G connected → G/H connected .

ii) H connected and G/H connected → G connected

1.2 Nano Topological space :

1.2.1 Definition [1]:

Let U be a nano_empty finite set of objects called the universe and R be an equivalence relation on U Ther pair (U,R) is called the approximation space .

Let $X\subseteq U$:

i) The lower approximation of X with respect to R is:

$L_R(X)=\cup_{x\in U}\{R(x):R\subseteq X\}$,where R(x) is the equivalence class of X.

ii)The upper approximation of X with respect to R is :

$U_R(x)=\cup\{R(x):R(x)\cap X\neq\emptyset\}$

iii) The boundary region of X with respect to R is:

$B_R=U_R(x)-L_R(x)$.

1.2.2 Proposition [1]:

let (U,R) is an approximation space and X,Y $\subseteq$U , then:

1) $L_R(X)\subseteq X\subseteq U_R(X)$

2)$L_R(\varnothing)=U_R(\varnothing)$ and $L_R(u)=U_R(u)=U$

3)$U_R(X\cup Y)=U_R(X)\cup U_R(Y)$

4)$U_R(X\cap Y)\subseteq U_R(X)\cap U_R(Y)$

5)$L_R(X\cup Y)\supseteq L_R(X)\cup L_R(Y)$

6)$L_R(X\cap Y)=L_R(X)\cap L_R(Y)$

7)$L_R(X)\subseteq L_R(Y)$ and $U_R(X)\subseteq L_R(Y)$, $X\subseteq Y$

8)$U_R(X^C)=[L_R(X)]^C$ and $L_R(X^C)=[U_R(X)]^C$

9)$U_RU_R(X)=L_RU_R(X) =U_R(X)$

10)$L_RL_R(X)=U_RL_R(X)=L_R(X)$

1.2.3 Definition [3]:

Let (U,R) is an approximation and let $X\subseteq U$

$\tau_R(X)=\{U,\varnothing,L_R(X),U_R(X),B_R(X)\}$

Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X ,We call $(U,\tau_R(X))$ as the nano topological space .

The elements of $\tau_R(X)$ are called as nano open sets.

1.2.4 Definition [3]:

Let $(U,\tau_R(X))$ is a nano topological space with respect to X ,where $X\subseteq U$ and $A\subseteq U$, then :

1) The union of all nano _open subsets of A is called the nano interior of A and it is denoted by $N_{Int}(A)$.

2)The intersection of all nano closed sets containing A is called the nano closure of A and it is denoted by $N_{Cl}(A)$

1.2.5 Examples [5]:

1) Let U={a,b,c,d} with U/R={{a},{c},{b,d}} and X={a,b} .Then the nano topology ,$\tau_R(X)=\{U,\varnothing,\{a\},\{a,b,d\},\{b,d\}\}$.

The nano closed sets are U,∅,{b,c,d},{c} and {a,c}.

2) Let $U=\{p_1,p_2,p_3,p_4\}$ with $U/R=\{\{p_1\},\{p_3\},\{p_2,p_4\}\}$ and $X=\{p_1,p_2\}$,Then $\tau_R(X)=\{\varnothing,\{P_1\},\{P_2,P_4\},\{P_1,P_2,P_4\},U\}$ is a nano topological space .

1.2.6 Definition [4]:

A function $f:(U,\tau_R(x))\rightarrow(V,\tau_R(Y))$ is called a nano_continuous function ,if the inverse images of every nano open(nano closed) set in V is nano open (nano closed) in U .

1.2.7 Example [4]:

Let U={a,b,c,d} ,U/R={{a},{c},{b,d}} and X={a,b}

A nano topology $\tau_R(X)=\{U,\varnothing,\{a\},\{b,d\},\{a,b,d\}\}$

$V=\{x,y,z,w\}, V/R=\{\{x\},\{y,w\},\{z\}\}$, $Y=\{y,w\}$

$\tau_R(Y)=\{V,\varnothing,\{y,w\}\}$

$F:U\rightarrow V$, $f(a)=z$, $f(b)=w$, $f(c)=x$, $f(d)=y$

Then f is nano_continuous .

Chapter Two

2.1 Separation axiom and compactness in nano topological space.

2.1.1 Definition [6]:

Let X be a topological space :

1) X is nano T_1–space if for any a,b∈X, a≠b there exists a nano open set U contains a and not containing b .

2) X is a nano T_2–space (or a nano Hausdorff space) if for any a,b∈X ,a≠b there exists nano nighbourhoods of a and b which are disjoint .

3) A space X is nano $T_\circ$–space if for any a,b∈X ,a≠b,there exists a nano open containing exactly one of them.

2.1.2 Proposition [6]:

X is a nano T_1–space if and only if every singleton subset of x is nano closed .

Proof:

Let X is a nano T_1–space

2.1.3 Proposition [6]:

Every nano topological group is nano Hausdorff .

2.1.4 Definition [5]:

Aspace X is nano regular if every nano neighbourhood of each point X contains a nano closed neighbourhood of x, i.e,for every nano closed set $A\subseteq X$ and every point $b\notin A$, then exist nano open sets u,v in X such that $A\subseteq U$,$b\in V$ and $U\cap V=\emptyset$.

2.1.5 Proposition [5]:

A nano regular nano T_1−space is nano Hausdorff space .

Proof:

let X be a nano regular T_1−space

Let $a,b \in X$, $a\neq b$,since X is nano T_1−space then there exists an nano open neighbourhood U such that $a\in U$ and $b\notin U$

$\rightarrow a\notin U^c$,U^c is nano closed ,$b\in U^c$

Now , since X is nano regular space then there exists nano open set V,W such that $U^c\subseteq V$,$a\in V$ and $V\cap W=\emptyset \rightarrow W\subseteq V^c \rightarrow a\in V^c$, V^c is nano closed set .

Since $U^c\subseteq V \rightarrow U^c\cap V^c=\emptyset$

$\therefore$ $a\in V^c$, $b\in U^c$, $U^c\cap V^c=\emptyset$,U^c,V^c are nano closed sets.

$\therefore$ X is nano T_2− space .

2.1.6 Definition/Nano-compactness [6]:

A collection $\{A_i\}_{i\in I}$ of nano-open sets in a nano topological space (U,R(X)) is called a nano-open cover of a subset B of U if $B \subseteq \bigcup_{i\in I} A_i$ holds.

2.1.7 Definition [6]:

A subset B of U is said to be nano-compact relative to (U,R(X)), if for every collections $\{A_i : i \in I\}$ of nano-open subsets of (U,R(X)) such that $B \subseteq \bigcup_{i\in I} A_i$ if there exists a finite subset I_0 of I such that $B \subseteq \bigcup_{i\in I_0} A_i$

2.1.8 Theorem [6]:

A nano-closed subset of nano-compact space (U,R(X)) is nano-compact relative to (U, R(X)).

Proof.

Let A be a nano-closed subset of a nano compact topological space (U, R(X)). Then A^c is nano-open in (U, R(X)). Let S ={Ai : i∈I} be an nano-open cover of A by nano-open subsets in (U, R(X)). Then $S^* = S \cup A^c$ is a nano-open cover of (U, R(X)).

That is $U = A \cup A^c \subseteq \bigcup_{i\in I} A_i \cup A^c$ [By hypothesis (U,R(X)) is nano-compact] and hence S^* is reducible to a finite subcover of (U,R(X)) say

U = $\bigcup_{i\in I_0} A_i \cup A^c$. Thus a nano open cover S of A contains a finite subocver. Hence A is nano-compact relative to (U,R(X)).

2.1.9 proposition [6]:

The image of a nano-compact space on a nano continuous map is nano-compact.

Proof.

Let f : (U, R(X)) → (V, R1(Y)) be a nano-continuous map from a nano-compact space (U, R(X)) onto a nano topological space (V, R1(Y)). Let {Ai : i ∈ I} be an nano-open cover of (V, R1(Y)). Then {f^{-1}(Ai) : i∈I} is a nano-open cover of (U, R(X)), since f is nano-continuous. As (U, R(X)) is nano-compact, the nano-open cover {f^{-1}(Ai) : i ∈I} of (U, R(X)) has a finite subcover {f^{-1}(Ai) : i = 1, 2, 3, ...n}. Therefore U =$U = \bigcup_{i\in I} f^{-1}(A_i)$. Then f(X) = $\bigcup_{i\in I} A_i$, that is V = $\bigcup_{i\in I} A_i$. Thus {A_1,A_2, ...,A_n} is a finite subcover of {A_i : i ∈I} for (V, R1(Y)). Hence (V, R1(Y)) is nano-compact.

2.1.10 Definition [5]:

A nano topological space (U, R(X)) is countably nano-compact if every countable nano-open cover of (U, R(X)) has a finite subcover.

2.1.11 Theorem [5]:

Let (U, R(X)) be a space and (V,R1(Y)) be a nano-Hausdroff. If f : (U, R(X)) → (V, R1(Y)) is nano-continuous injective, then (U, R(X)) is nano-Hausdroff.

Proof.

Let x and y be any two distinct points of (U, R(X)). Then f(x) and f(y) are distinct points of (V, R1(Y)), because f is injective. Since (V, R1(Y)) is nano-Hausdroff, there are disjoint nano-open sets A and B in (V, R1(Y)) containing f(x) and f(y) respectively. Since f is nano-continuous and $A \cap B = \emptyset$, we have f^{-1}(A) and f^{-1}(B) are disjoint nano-open sets in (U, R(X)) such that x ∈ f^{-1}(A) and y ∈ f^{-1}(B). Hence (U, R(X)) is nano-Hausdroff.

2.1.12 proposition [1]:

If f : (U, R(X)) → (V, R1(Y)) is nano-continuous and bijective and if U is compact and V is nano Hausdroff, then f is a nano homeomorphism.

Proof.

We have to show that the inverse g of f nano-continuous.

For this we show that if A is nano open in (U, R(X)) then the pre-image $g^{-1}(A)$ is nano open in (V, R1(Y)). Since the nano open (or nano closed) sets are just the complements of nano closed (respectively nano open) subsets, and $g^{-1}(V - A) = U - g^{-1}(A)$. We see that the nano continuity of g is equivalent to:

if B is nano closed in (U, R(X)) then the pre-image $g^{-1}(B)$ is nano closed in V. To prove this, let B be a nano closed subset of U. Since g is the inverse of f, we have $g^{-1}(B) = f(B)$, hence we have to show that f(B) is nano closed in V.

implies that B is nano compact. Since V is nano Hausdroff space implies that f(A) is nano closed in (V, R1(Y)).

2.1.13 Definition [6]:

A nano topological space (U, R(X)) is said to be nano-Lindelof space if every nano-open cover of (U, R(X)) has a countable subcover.

2.1.14 proposition [5]:

Every nano-compact space is a nano-Lindelof space.

Proof:

Let (U, R(X)) be nano-compact. Let $\{A_i : i \in I\}$ be an nano-open cover of (U, R(X)). Then $\{A_i : i \in I\}$ has a finite subcover $\{A_i : i = 1, 2, 3, ...n\}$, since (U, R(X)) is nano-compact. Since every finite subcover is always a countable subcover and therefore, $\{A_i : i = 1, 2, ...n\}$ is countable subcover of $\{A_i : i \in I\}$ for (U, R(X)). Hence (U, R(X)) is nano-Lindelof space.

2.1.15 Theorem [6]:

The image of a nano-Lindelof space under a nano-continuous map is nano-Lindelof .

Proof.

f : (U, R(X)) $\rightarrow$ (V, R1(Y)) be a nano-continuous map from a nano-Lindelof space (U, R(X)) onto a nano topological space (V,R1(Y)). Let $\{A_i : i \in I\}$ be an nano-open cover of (V, R1(Y)), then $\{f^{-1}(A_i) : i \in I\}$ is an nano-open cover of (U, R(X)), since f is nano-continuous. As (U, R(X)) is nano-Lindelof, the nano-open cover $\{f^{-1}(A_i) : i \in I\}$ of (U, R(X)) has a countable subcover $\{f^{-1}(A_i) : i = 1, 2... \}$. Therefore $X = \bigcup_{i \in I} f^{-1}(A_i)$which implies f(U) = V = $\bigcup_{i \in I}(A_i)$, that is {A1,A2..... } is a countable subfamily of $\{Ai : i \in I\}$ for (V, R1(Y)). Hence (V, R1(Y)) is nano-Lindelof space.

2.2 Nano topological group.

2.2.1 Definition:

1) G is a group with respect to * .

2) G is a nano topological space .

Such that the two structures are compatible, i.e , the multiplication map $\mu:G\times G\rightarrow G$ and the inversion $\nu:G\rightarrow G$ are nano continuous.

2.2.2 Definition:

We say that H is a nano topological sub group of G is a sub group of G and H is a nano topological group with respect to the sub nano space topology,

2.2.3 Definition:

A nano morphism $f:G\rightarrow H$ of nano topological groups is nano continuous group homorphism .

2.2.4 Definition:

1) $l_g:G\rightarrow G$, $l_g(x)=gx$, is called nano left multiplication by g , This map has inverse l_g^{-1} which is also nano continuous , so l_g is nano homeomorphism from G to G .

2) similary, all nano right multiplication $r_g:G\rightarrow G$,$r_g(x)=xg$ are nano homeomorphism .

2.2.5 proposition :

let G be a nano topological group ,A,B ⊆ G,g∈G then:

i) A nano open implies Ag and nano open .

ii) A nano closed implies Ag and gA nano closed .

iii) A nano open implies AB and BA nano open .

iv) A nano closed and B finite implies AB and BA nano closed .

Proof:

i) We note that a nano right multiplication r_g:G→G ,$r_g(x)$=xg is nano homeomorphism for some g, thus $r_g(A)$=Ag is nano-open .

Similarly, we can prove (ii),(iii),and (iv).

2.2.6 Definition :

Let G be a nano topological Group. and let H be a subgroup of G .

let G/H ={xH: x∈G} the set of left costs of H.

The map q: G→G/H defined by q(x)=xH defines a qoutient nano topology on G/H . we note that q is nano open map

If H is a normal sub group of G, then G/H is a nano

topological group .

2.2.7 Proposition :

Every nano topological group G is nano regular .

Proof:

Consider the map $f:G\times G\rightarrow G$ defined by :

$f(a,b)=ab^{-1}$. This map is always nano-continuous in a nano-topological group. Now take $x\in U$, where U is an nano-open set in G. Then $f^{-1}(U)$ contains (x,e), so we have $(x,e)\in V\times W\subseteq f^{-1}(U)$ for some nano-open subsets V and W such that $x\in V$ and $e\in W$. Hence, $x\in V$. **Furthermore,** $V\cap(X-U)W=\emptyset V\cap(X-U)W=\emptyset$**, since any element in the intersection corresponds to** $a\in V$, $b\in W$**, such that** $ab^{-1}\notin U$**, which is a contradiction.**

Since (X−U)W is an nano-open set containing X−U, X is nano-regular.

2.2.8 Definition:

A nano topological space X is called nano connected if is non−empty and can not be written $X=A\cup B$ where A and B are non− empty disjoint nano open sub sets of X.

2.2.9 Remark:

1- A non-empty space X is nano connected if and only if the only sub sets of X which are both nano open and nano closed are ∅ and X.

2- A nano connected topological group has no proper nano open sub groups .

2.2.10 Proposition:

If X is nano connected and $f:X \to Y$ is a nano continuous function,then f(X) is nano connected .

Proof:

Suppose that X is nano connected and $f:X \to Y$ is a nano continuous function,and f(X) is not nano connected then by Definition of nano-connected space we have:

There are two non− empty disjoint nano-open sets A,B such that: $f(X)=A \cup B$

$X \subseteq f^{-1}(f(X))= f^{-1}(A) \cup f^{-1}(B)$ but $f^{-1}(A) \cup f^{-1}(B) \subseteq X$

Thus $X= f^{-1}(A) \cup f^{-1}(B)$, but $f^{-1}(A)$, $f^{-1}(B)$ are disjoint nano-open set since f is nano-continuous, this contradiction , thus we have f(X) is nano connected.

2.2.11 Definition [1]:

Let $(U,\tau_R(X))$ be a nano topological space and $A \subseteq U$, Then A is said to be: i) nano semi_open if $A \subseteq N\ cl\ (N\ int\ (A))$

ii) nano pre_open if $A \subseteq N\ int\ (N\ cl\ (A))$

iii) nano α_ open if $A \subseteq N\ int\ (N\ cl\ (A))$

NSO (U,X) , NPO(U,X) and $\tau^{\alpha}_R(x)$ respectively denote the families of all nano semi_open , nano pre_open and nano α_open subsets of U .

2.2.12 Definition [1]:

Let $(U,\tau_R(X))$ be a nano topological space and $A \subseteq U$. A is said to be nano α_closed (respectively,nano semi _closed , nano pre _closed),if its complement is nano α_open (nano semi _open , nano pre_open).

2.2.13 Example :

Let U={a,b,c,d} with U/R={{a},{c},{b,d}} and X={a,b}. Then the nano topology $\tau_R(X)$ ={U,∅,{a},{a,b,d} , {b,d}}. The nano closed sets are U ,∅,{a,b,d},{c} and {a,c} .Then ,NSO (U,X)={U,∅,{a},{a,c},{a,b,d},{b,c,d}},

NPO (U,X)={U,∅,{a{,{b},{d},{a,b},{a,d},{b,d},{a,b,c},{a,b,d},{a,c,d}} and $\tau^{\alpha}{}_R(X)$={U,∅,{a},{b,d},{a,b,d}}.We note that ,NPO(U,X) does not form a topology on U, since {a,c} and {b,c,d}∈NSO(U,X) but {a,c} ∩{b,c,d}={c}∉NSO(U,X). Similarly, NPO(U,X) is not atopology onU,since {a,b,c}∩{a,c,d}={a,c}∉NPO(U,X), even though {a,b,c} and {a,c,d}∈NPO(U,X). But the sets of $\tau^{\alpha}{}_R(X)$ form a topology on U. Also, we note that {a,c} ∈NSO(U,X) but is not in NPO(U,X) and {a,b}∈NPO(U,X) but does not belong to NSO(U,X). That is , NSO(U,X) and NPO(U,X) are independent.

2.2.14 Theorem [1]:

$\tau_R{}^{\alpha}(X) \subseteq NSO(U,X)$ in a nano topological spce $(U,\tau_R(X))$.

Proof: If $A \in \tau_R{}^{\alpha}(X), A \subseteq N\ Int\ (N\ cl\ (N\ int\ (A))) \subseteq (N\ cl(N\ int(A))$, and hence $A \in NSO(U,X)$.

2.2.15 Remark :

The converse of the above theorem is not true. In example (2.2.13), {a,c} and {b,c,d} and nano semi-open but are not nano α-open inU .

2.2.16 Theorem [5]:

$\tau_R^{\alpha}(X) \subseteq$N PO(U,X) in a nano topological space (U,τ(X)).

Proof: If $A \in \tau^{\alpha}_R(X)$,$A \subseteq$Nint(N cl (N int (A))) . SinceN int (A) $\subseteq$A ,N int (N cl (N int(A)))$\subseteq$(N int (N cl (A)) . That is,A$\subseteq$ N int (N cl (A)) . Therefore,A$\in$NPO(U,X) . That is,$\tau^{\alpha}_R(X) \subseteq$NPO(U,X) .

2.2.17 Remark:

The converse of the above theorem is not true. In example *, the set {b} is nano pre-open but is not nano α-open in U

2.2.18 Theorem [5] :

If, in a nano topological space $(U,\tau_R(X))$, , then $L_R(X)=U_R(X)=X$,then U,∅,$L_R(X) =U_R(X))$, and any set $A \supset L_R(X)$ are the only nano-α-open sets in U.

Proof:

Since $L_R(X)=U_R(X)=X$, the nano topology, $\tau_R(X)=\{U,\varnothing,L_R(X)\}$. Since any nano-open set is nano-α-open U, ∅ and $L_R(X)$ are nano α -open in U. If $A \subset L_R(X)$, then N Int(A)=∅ , since ∅ is the only nano-open subset of A. Therefore N cl (N Int (A))=∅ and hence A is not nano α-open. If $A\supset L_R(X)$, $L_R(X)$, is the largest nano-open subset of A and hence,N Int(N Cl(N Int(A))) =N Int (N Cl ($L_R(X)$))=N Int ($B_R(X)^c$)=N int (U) , since $B_R(X)=\varnothing$. Therefore N Int (N Cl (N int (A)))=U, and hance, $A \subseteq$ N Int (N Cl (N Int (A))) . Therefore, A is nano α-open. Thus ,U,∅,$L_R(X)$ and any set $A\supset L_R(X)$ are the only nano α -open sets in U, if $L_R(X)=U_R(X)$.

2.2.19 Theorem[5]:

U,∅,$U_R(X)$ and any set $A\supset U_R(X)$ are the only nano α_open sets in a nano _topological space (U, $\tau_R(X)$) if $L_R(X)=\varnothing$

Proof:

since $L_R(X)=\varnothing$, $B_R(X)=U_R(X)$. Therefore , $\tau_R(X)=\{U ,\varnothing ,U_R(X)\}$and the members of $\tau_R(X)$ are nano_open in U. Let $A \subset U_R(X)$.Then N Int (A)=∅ and hence largest nanom_open subset of A. Therefore , N Int (N Cl (N Int (A)))=N Int (N Cl ($U_R(X)$)=N Int (U) and hence $A \subseteq$ N Int (N Cl (N Int (A))). Thus any set $A \supset U_R(X)$ is nano α_open in U . Hence , U ∅ ,$U_R(X)$ and any superset of $U_R(X)$ are the only nano α_open sets in U .

2.2.20 Theorem [5] :

If U R(X)=U and $L_R(X)\neq\varnothing$, in a nano topological space (U , $\tau_R(X)$) , then U,∅,- $L_R(X)$ and $B_R(X)$ are the only nano α -open sets in U.

Proof:

Since $U_R(X)=U$ and $L_R(X)\neq\varnothing$, the nano-open sets in U are U ,∅ ,$L_R(X)$ and b $B_R(X)$ and hence they are nano α -open also. If A=∅ ,then A is nano α -open. Therefore, let $A\neq\varnothing$. When $A\subset L_R(X)$,N Int(A)=∅ , since the largest open subset of A is ∅ and hence $A\not\subset$ N Int (N Cl(N Int(A))) , unless A is ∅ . That is, A is not nano α-open in U . When $L_R(X)\subset A$, N Int (A)= $L_R(X)$ and therefore,N Int(N Cl ($L_R(X)$)) =N Int($B_R(X)^c$) =N Int ($L_R(X)$) =$L_R(X) \subset A$. That is,$A\not\subset$N Int (N Cl (N Int (A))) . Therefore, A is not nano α-open in U . Similarly, it can be shown that any set $A\subset B_R(X)$ and $A \supset B_R(X)$ are not nanoα -open in U . If A has atleast one element each of $L_R(X)$ and $B_R(X)$, then N Int (A)=∅ and hence A is not open sets in U when U_R(X=U and $L_R(X)\neq\varnothing$.

2.2.21 Corollary :

$\tau_R(X)= \tau_R^{\alpha}(X)$, if $U_R(X)=U$.

2.2.22 Theorem [5]:

Let $L_R(X) \neq U_R(X)$ where $L_R(X) \neq U$ and in a nano topological space $(U, \tau_R(X))$. Then $U, \varnothing, L_R(X)$, $B_R(X), U_R(X)$ and any set $A \supset U_R(X)$ are the only nanoα -open sets in U .

Proof:

The nano topology on U is given by $\tau_R(X)=\{U,\varnothing,L_R(X),B_R(X),U_R(X)\}$, and hence U , $\varnothing, L_R(X)$, $B_R(X)$ and $U_R(X)$ are nano α-open in U. Let $A \subseteq U$ such that A $\supset U_R(X)$.ThenN Int (A)= $U_R(X)$ and therefore, N Int (N Cl ($U_R(X)$)) = U . Hence , A $\subseteq$N Int (N Cl(N Int(A)). Therefore , any $A \supset U_R(X)$ is nano α_open in U . when $A \subset L_R(X)$,N Int (A)=$\varnothing$ and hence N Int (N Cl (N Int (A)))= and hence A is not nano α_open in U . when $A \subset U_R(X)$ such that A is neither a subset of $L_R(X)$ nor a subset of $B_R(X)$ N Int (A)=$\varnothing$ and hence A is not nano α_open in U . Thus , U ,$\varnothing, L_R(X)$, $B_R(X), U_R(X)$ and any set $A \supset U_R(X)$ are the only nano α _open sets in U .

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