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Applying Adomian Decomposition Method to Solve The Burger's Equation

A research submitted to the Council of the Department of Mathematics College of Education / University of Qadisiya, as part of a bachelor's degree in Mathematics requirement

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Dedication

To the fountain of patience and optimism and hope To each of the following in the presence of God and His Messenger

..... my mother dear

To those who have demonstrated to me what is the most beautiful

...... of my brother's life

To the big heart

......my dear father

To the people who paved our way of science and knowledge ***....... All our teachers Distinguished***

To those who defended us and without them we couldn't do this work

......All soldiers in Iraqi army and all

Abstract.

Adomian decomposition method is presented as a method for the solution of the Burger’s equation, a popular PDE model in the fluid mechanics. The method is computationally simple in application.

Burger’s equation is a nonlinear partial differential equation. Burger’s equation is the simplest equation combining both nonlinear propagation effects (uu_x) and diffusive effects (u_{xx}). We interest to derive, find the solution and apply this equation in ground water. The Adomian decomposition method used to solve the Burger's equation.

1. Introduction.

Burgers (1948) first developed this equation primarily to throw light on turbulence described by the interaction of two opposite effects of convection and diffusion. The term uu_x will have a shocking up effect that will cause waves to break and the term νu_{xx} is a diffusion term like the one occurring in the heat equation. Burgers’ equation is obtained as a result of combining nonlinear wave motion with linear diffusion and is the simplest model for analyzing combined effect of nonlinear advection and diffusion. This equation is balance between time evolution, nonlinearity, and diffusion. This is the simplest nonlinear model equation for diffusive waves in fluid dynamics.

Burger’s equation is a fundamental partial differential equation in fluid mechanics. It is also a very important model encountered in several areas of applied mathematics such as heat conduction, acoustic waves, gas dynamics and traffic flow [17] , model of traffic, turbulence, shock waves and fluid flow[6].

Analytical solutions of the partial differential equations modeling physical phenomena exist only in few of the cases. Therefore the need for the construction of efficient numerical methods for the approximate solution of these models always exists. Many of the analytical solutions to the Burger’s equation involve Fourier series. According to [3], the convergence of such Fourier-series based solutions is very slow. Several researchers have proposed various numerical methods for the solution of the Burger's equation. [21] and [7] used the finite element method for the solution of the Burger's equation. [20] and [4] used the finite difference method. [19] used the direct variational method while [9] used the projection method by B-spline. A decomposition method which provides convergent solutions to nonlinear stochastic operator equations was developed in [2]. Many authors Bateman H[5], Burger J.M[6], Cole, J.D[8], Mittal R.C and Singhal P[18], Dogan A[10], Aksan, E.N., A. and T. [10], Caldwell, J., P. Wanless and A.E. Cook [12] have discussed the numerical solution of Burger's equation using Finite Difference Methods and Finite Element Methods.

2. Mathematical Formulation

As mentioned previously, the equation of Burger has appeared in many applications, including groundwater, so we will study in this section how to derive this equation in this area.

In isotropic homogeneous medium the motion of water is given by Darcy's law as

$$V = -K\,\nabla\emptyset \qquad (1)$$

where V is volume flux of moisture content, K is coefficients of aqueous conductivity and $\nabla\emptyset$ is gradient of the whole (total) moisture potential.

Through unsaturated porous media, the equation that governed the motion of water flow is the continuity equation

$$\frac{\partial(\rho s\theta)}{\partial t} = -\nabla . M \qquad (2)$$

where ρs is the bulk density of medium on dry weight basis, θ is the moisture content at any depth Z on a dry weight basis and M is a mass of flux of moisture at any time $t \geq 0$.

Using incompressibility of the water from the equation (1) and (2)

$$\frac{\partial(\rho s\theta)}{\partial t} = -\nabla.M = -\nabla.(\rho.V) = \nabla.(\rho.K.\nabla\phi) \qquad (3)$$

where ρ is the flux density of the medium.

Since, in the present problem, flow takes place only in the vertical direction, therefore (3)

reduces to

$$\rho s\,\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(\rho\,K\,\frac{\partial\psi}{\partial z}\right) - \frac{\partial}{\partial z}\rho\,K\,g \qquad (4)$$

where ψ is the pressure (capillary) potential, g is the gravitation constant and φ = ψ−z g the positive direction of z−axis is the same as that of gravity.

Considering ψ and φ to be connected by a single valued function, equation (4) may be written as [13],

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(D\,\frac{\partial\theta}{\partial z}\right) - \frac{\rho}{\rho S}\,g\,\frac{\partial\,K}{\partial z} \qquad (5)$$

Where $D = \frac{\rho}{\rho S}\,K\,\frac{\partial\psi}{\partial z} = \frac{\rho}{\rho S}$ is called the diffusivity coefficients [1].

Replacing D by its average value D_a over the whole range of the moisture content [16] and $K \propto \theta^2$ [15].

i.e. $K = K_0\,\theta^2$, where K_0 is constant. Hence equation (5) becomes

$$\frac{\partial\theta}{\partial t} + \frac{\rho}{\rho S}\,2gK_0\theta\,\frac{\partial\theta}{\partial z} = D_a\,\frac{\partial^2\theta}{\partial z^2} \qquad (6)$$

Substituting $\frac{\rho}{\rho S}\,2gK_0 = K_1$, equation (6) becomes:

$$\frac{\partial\theta}{\partial t} + K_1\theta\frac{\partial\theta}{\partial z} = D_a\frac{\partial^2\theta}{\partial z^2} \qquad (7)$$

For the sake of simplicity of the problem, the value of the constant $K_1 = 1$ is considered.

We choose new variable as,

$$Z = \frac{Z}{L} \quad , \quad T = t\,\frac{D_a}{L^2} \quad , \qquad 0 \le Z \le 1 \ , \qquad 0 \le \ T \le 1 \qquad (8)$$

Hence the equation (7) can be written as

$$\frac{\partial\theta}{\partial T} + \theta\frac{\partial\theta}{\partial Z} = \frac{\partial^2\theta}{\partial Z^2} \qquad 0 \le Z \le 1 \qquad (9)$$

The equation (9) is the governing non-linear partial differential equation known as Burger's

equation for the moisture content distribution phenomenon.

3. The Adomian decomposition method (ADM)

George Adomian introduced the ADM method firstly in 1981. Then this method used to solve the differential equations . And up to now a large number of research papers have been published to show the feasibility of the decomposition method.

There are many advantages to this method, most importantly is that it can be applied directly to all types of differential and integral equations, linear or non-linear, homogeneous or inhomogeneous, with constant or variable coefficients. Another important advantage is that, the method is capable of greatly reducing the size of computational work while still maintaining high accuracy of the numerical solution. This method decomposes a solution into an infinite series which converges rapidly to the exact solution. The convergence of the ADM has been disscused by a number of authors . [1] gave proof of convergence of Adomian decomposition method when applied to differential equations. The non-linear problems are solved easily and elegantly without linearising the problem by using ADM. It also avoids linearisation, perturbation and discretization unlike other classical techniques.

The differential equation will be represented as follows

$$L(u) + R(u) + N(u) = g \qquad (10)$$

where g is a function of independing variables and u is the depinding variable, and L is a linear operator to be inverted, which usually is just the highest order differential operator, R is the linear remainder operator, and N is the nonlinear operator, which is assumed to be analytic.

Rewrite (10) to be:

$$L(u) = g - R(u) - N(u) \qquad (11)$$

Generally, if we choose $\frac{d^p}{x^p}(.)$ for pth-order differential equations and thus its inverse L^{-1} follows as the p-fold definite integration operator from 0 to x. We have $L^{-1}L\,(u) = u - \varphi$, where φ incorporates the initial values.

Applying the inverse linear operator L^{-1} to both sides of (11):

$$L^{-1}L(u) = L^{-1}g - L^{-1}[R(u) + N(u)\,] \qquad (12)$$

And this gives:

$$u = \gamma - L^{-1}[R(u) + N(u)] \qquad (13)$$

Where $\gamma = \varphi + L^{-1}g.$

The ADM decomposes the solution of (10) into an infinite series:

$$u = \sum_{n=0}^{\infty} u_n \qquad (14)$$

And the nonlinear term *Nu* can be decomposed into an infinite series as:

$$Nu = \sum_{n=0}^{\infty} A_n \qquad (15)$$

where the *An*, depending on $u_0, u_1, \ldots, u_n$ are called the Adomian polynomials, and are obtained for the nonlinearity *Nu*=*f*(*u*) by the definitional formula [11]

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial \lambda^n}\left[f\left(\sum_{k=0}^{\infty} u_k\, \lambda^k\right)\right]_{\lambda=0} \quad , n = 0,1,2,\ldots \qquad (16)$$

where λ is a grouping parameter of convenience.

Upon substitution (14) and (15) into (13), we have

$$\sum_{n=0}^{\infty} u_n = \gamma - L^{-1}\left[R\left(\sum_{n=0}^{\infty} u_n\right) + \sum_{n=0}^{\infty} A_n\right] \qquad (17)$$

The recursive relationship is found to be:

$$\left.\begin{array}{l} u_0 = \gamma \\ u_{n+1} = -L^{-1}[R(u_n) + A_n], \qquad n \geq 0 \end{array}\right\} \qquad (18)$$

The *n*-term approximation of the solution is

$$\Psi_n = \sum_{k=0}^{n-1} u_k \qquad (19)$$

3.1 Application of ADM to Differential Equations

Now we will apply the ADM method to some examples to illustrate it.

Example 3.1 Consider the equation

$$u_t + \frac{3}{2}\mathrm{u}_x - u = 0 \qquad (20)$$

With initial condition;

$$u(x,0) = 2x \qquad (21)$$

And the exact solution is;

$$u(x,t) = (2x - 3t)\, e^{t}$$

From (20) and (21) we have:

$$\mathrm{L}=\frac{\partial}{\partial \mathrm{t}} \; ; \; \mathrm{R}(u) = \frac{3}{2}\mathrm{u}_x - u \; , \;\; N(u) = 0 \quad ; \; g(\mathrm{x,t}) = 0 \;\; ; \quad f = 2x$$

by (18) we get:

$$u_o = f = 2x$$

$$u_1 = -L^{-1}(\mathrm{R}\,(u_0)) = -\int_0^t (3 - 2x)dt = -3t + 2xt$$

$$u_2 = -L^{-1}(\mathrm{R}\,(u_1)) = -\int_0^t (6\,t - 2xt)\ \mathrm{d} = \ -3t^2 + xt^2$$

$$u_3 = -L^{-1}(\mathrm{R}\,(u_2)) = -\int_0^t \left(\ \frac{3}{2}\ t^2 \quad +3t^2 - xt^2\right) dt = \frac{-3}{2}t^3 + \ x\frac{t^3}{3}$$

And so on

$$u(x,t) = u_0 + u_1 + \ \ u_2 + u_3 + \cdots$$

$$= 2x - 3t + 2xt - 3t^2 + xt^2 - \ \ \frac{3}{2}\ t^2 + x\frac{t^3}{3} + \cdots$$

$$= (2x - 3t) + (\,2x - 3t)t \ + (\,2x - 3t)\frac{t^2}{2} + \cdots$$

$$= (2x - 3t)\left(\,1 + t \ + \frac{t^2}{2} + \frac{t^3}{3!} \ + \cdots\right) = (2x - 3t)\, e^{t}$$

$$\therefore\, u(x,t) = (2x - 3t)\, e^{t}$$

Example 3.2 Consider the equation

$$u_t - \frac{1}{4}u_x = 0 \tag{22}$$

With initial condition;

$$u(x,0) = 2e^{4x} \tag{23}$$

And the exact solution is;

$$u(x,t) = 2e^{4x+t}$$

From (22) and (23) we have:

$$\mathrm{L}=\frac{\partial}{\partial \mathrm{t}}\ ;\ \mathrm{R}(u) = -\frac{1}{4}u_x\ \ ;\ \ \mathrm{N}(u) = 0\ \ ; g(x,t) = 0\ \ ;\ \ \ f = 2e^{4x}$$

by (18) we get:

$$u_o = f = 2e^{4x}$$

$$u_1 = -L^{-1}\big(\mathrm{R}\ (u_0)\big) = \int_0^t \frac{1}{4} 8\ e^{4x}\mathrm{dt} = 2te^{4x}$$

$$u_2 = -L^{-1}\big(\mathrm{R}\ (u_1)\big) = \int_0^t 2\ t\ e^{4x}dt = 2\frac{t^2}{2}e^{4x}$$

$$u_3 = -\mathrm{R}\ (u_2) = -\int_0^t 2te^{4x}\ dt = 2\frac{t^3}{3!}e^{4x}$$

And so on.

$$u(x,t) = u_0 + u_1 + \ u_2 + u_3 + \cdots = 2e^{4x}\left(\ 1 + t\ + \frac{t^2}{2} + \frac{t^3}{3!}\ + \cdots\right)$$

$$= 2e^{4x+t}$$

$$\therefore u(x,t) = 2\, e^{4x+t}$$

Example 3.3 Consider the equation

$$u_t - \mathrm{u}u_x = 0 \qquad (24)$$

With initial condition;

$$u(x,0) = \frac{x}{10} \qquad (25)$$

And the exact solution is;

$$u(x,t) = -\frac{x}{t-10}$$

From (24) and (25) we have:

$$\mathrm{L}=\frac{\partial}{\partial \mathrm{t}}\ ;\ \mathrm{R(u)} = 0\ ;\quad \mathrm{N\ (u)} = -\mathrm{u}u_x \quad ;\quad g(\mathrm{\ x,t}) = 0\ ;\quad f = \frac{x}{10}$$

by (18) we get:

$$u_0 = \frac{x}{10}$$

$$A_0 = -u_0 u_{0x} = -\frac{X}{10} \cdot \frac{1}{10} = -\frac{x}{100}$$

$$u_1 = -L^{-1}A_0 = -\int_0^t A_0 dt = -\int_0^t \left(-\frac{x}{100}\right) dt = \frac{x}{100} t$$

$$A_1 = -\left(u_0 u_{1X} + u_1 u_{0X}\right) = -\frac{x}{100} \cdot \frac{t}{100} - \frac{x}{100} + \frac{1}{10} = -\frac{xt}{500}$$

$$u_2 = -L^{-1}A_1 = -\int_0^t \left(-\frac{Xt}{500}\right) dt = \frac{xt^2}{1000}$$

$$A_2 = -(u_0 u_{2X} + u_1 u_{1X} + u_2 u_{0X}) = -\left(3\frac{xt^2}{10000}\right)$$

$$u_3 = -L^{-1}A_2 = -\int_0^t \left(-3\frac{xt^2}{10000}\right) dt = \frac{xt^3}{10000}$$

$$A_3 = -(u_0 u_{3X} + u_1 u_{2X} + u_2 u_{2X} + u_3 u_{0X}) = \frac{xt^3}{25000}$$

$$u_4 = -L^{-1}A_3 = -\int_0^t A_3\,dt = \frac{xt^4}{100000}$$

And so on.

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots = \frac{x}{10} + \frac{x}{100}t + \frac{xt^2}{1000} + \frac{xt^3}{10000} + \cdots$$

$$= \frac{x}{10}\left[1 + \frac{t}{10} + \left(\frac{t}{10}\right)^2 + \left(\frac{t}{10}\right)^3 + \cdots\right] = \frac{x}{10} \cdot \frac{1}{\frac{t}{10} - 1}$$

$$= -\frac{x}{t-10}$$

$$\therefore u(x,t) = -\frac{x}{t-10}$$

Example 3.4 Consider the system

$$\frac{dy}{dx} - y = 0 \qquad (26)$$

With initial condition;

$$y(0) = 1 \qquad (27)$$

And the exact solution is;

$$y(x) = e^x$$

From (24) and (25) we have:

$$L = \frac{d}{dx};\ R(y) = -y,\ N(y) = 0,\ g = 0,\ f = 1$$

by (18) we get:

$y_0 = f = 1$

$$y_1 = -L^{-1}(R(y_0)) = \int_0^x dx = x$$

$$y_2 = -L^{-1}(R(y_1)) = \int_0^x x\,dx = \frac{x^2}{2}$$

$$y_3 = -L^{-1}(R(y_2)) = \int_0^x \frac{x^2}{2}\,dx = \frac{x^3}{6}$$

$$y_4 = -L^{-1}(R(y_3)) = \int_0^x \frac{x^3}{6}\,dx = \frac{x^4}{24}$$

And so on.

$$y(x) = y_0 + y_1 + y_2 + y_3 + \cdots = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = e^x$$

$$\therefore\ y(x) = e^x$$

Considering the first five components, the solution can be approximated as

$$y(x) \approx y_0 + y_1 + y_2 + y_3 + y_4 = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

For application purposes only few terms of the series will be computed . Table 1 compares the ADM result with exact solution.

x	Exact	Adomian
0	1	1
0:1	1:1052	1:1052
0:2	1:2214	1:2214
0:3	1:3499	1:3498
0:4	1:4918	1:4917
0:5	1:6487	1:6484
0:6	1:8221	1:8214
0:7	2:0138	2:0122
0:8	2:2255	2:2224
0:9	2:4596	2:4538
1	2.7183	2.7083

Table 1 Exact versus Adomian

4. Using ADM to Solve Burger's Equation

we will use the symbol *t* instead of *T* and *z* instead of *Z*, then (9) becomes:

$$\frac{\partial\theta}{\partial t}+\theta\frac{\partial\theta}{\partial z}-\frac{\partial^2\theta}{\partial z^2}=0 \qquad 0\le z\le 1 \tag{28}$$

With initial condition;

$$\theta(z,0)=\frac{1+0.2e^{0.4z}}{1+e^{0.4z}} \tag{29}$$

From (28) and (29) we have:

$$\mathrm{L}=\frac{\partial}{\partial \mathrm{t}}\ ;\ \mathrm{R}(\theta)=-\,\theta_{zz}\ ;\ \mathrm{N}(\theta)=\theta\theta_z\quad ;\quad g(z,t)=0$$

$$;\quad f=\frac{1+0.2\,e^{0.4z}}{1+e^{0.4z}}$$

by (18) and using matlab software we get:

$$\theta_0=f=\frac{1+0.2\,e^{0.4z}}{1+e^{0.4z}}$$

$$A_0=\theta\theta_z=-\frac{8}{125}\frac{e^{0.4z}(5+e^{0.4z})}{(1+e^{0.4z})^3}$$

$$\theta_1=-L^{-1}[R(\theta_0)+A_0]=-\int_0^t[R(\theta_0)+A_0]dt=\frac{24}{125}\frac{t\,e^{0.4z}}{(1+e^{0.4z})^2}$$

$$A_1=-(\theta_0\theta_{1z}+\theta_1\theta_{0z})=-\frac{48}{3125}\frac{t\,e^{0.4z}(-5+8\,e^{0.4z}+e^{0.8\,z})}{(1+e^{0.4z})^4}$$

$$\theta_2=-L^{-1}[R(\theta_1)+A_1]=-\int_0^t[R(\theta_1)+A_1]dt=\frac{72}{3125}\frac{t^2\,e^{0.4z}\,(-1+e^{0.4z})}{(1+e^{0.4z})^3}$$

$$A_2=-(\theta_0\theta_{2z}+\theta_1\theta_{1z}+\theta_2\theta_{0z})$$
$$=-\frac{144}{78125}\frac{t^2e^{0.4z}(5-31\,e^{0.4z}+13\,e^{0.8\,z}+e^{1.2\,z})}{(1+e^{0.4z})^5}$$

$$\theta_3=-L^{-1}[R(\theta_2)+A_2]=-\int_0^t[R(\theta_2)+A_2]dt=\frac{144}{78125}\frac{t^3\,e^{0.4z}\,(1-4\,e^{0.4z}+e^{0.8z})}{(1+e^{0.4z})^4}$$

$$A_3=-(\theta_0\theta_{3z}+\theta_1\theta_{2z}+\theta_2\theta_{1z}+\theta_3\theta_{0z})$$
$$=-\frac{288}{1953125}\frac{t^3e^{0.4z}(-5+82\,e^{0.4z}-132\,e^{0.8\,z}+22\,e^{1.2\,z}+e^{1.6\,z})}{(1+e^{0.4z})^6}$$

$$\theta_4 = -L^{-1}[R(\theta_3) + A_3] = -\int_0^t [R(\theta_3) + A_3]dt$$

$$= \frac{216}{1953125} \frac{t^4\, e^{0.4z}\,(-1 + 11\, e^{0.4z} - 11\, e^{0.8z} + e^{1.2\,z})}{(1 + e^{0.4z})^5}$$

$$A_4 = -(\theta_0\theta_{4z} + \theta_1\theta_{3z} + \theta_2\theta_{2z} + \theta_3\theta_{1z} + \theta_4\theta_{0z})$$

$$= -\frac{432}{48828125} \frac{t^4 e^{0.4z}(5 - 189\, e^{0.4z} + 724\, e^{0.8\,z} - 484\, e^{1.2\,z} + 39\, e^{1.6\,z} + e^{2\,z})}{(1 + e^{0.4z})^7}$$

$$\theta_5 = -L^{-1}[R(\theta_4) + A_4] = -\int_0^t [R(\theta_4) + A_4]dt$$

$$= \frac{1296}{244140625} \frac{t^5\, e^{0.4z}\,(1 - 26\, e^{0.4z} + 66\, e^{0.8z} - 26\, e^{1.2\,z} + e^{1.6\,z})}{(1 + e^{0.4z})^6}$$

$$A_5 = -(\theta_0\theta_{5z} + \theta_1\theta_{4z} + \theta_2\theta_{3z} + \theta_3\theta_{2z} + \theta_4\theta_{1z} + \theta_5\theta_{0z})$$

$$= \frac{2592}{6103515625} \frac{t^5 e^{0.4z}(5 - 408\, e^{0.4z} + 3117\, e^{0.8\,z} - 4832\, e^{1.2\,z} + 1647\, e^{1.6\,z} - 72\, e^{2\,z} - e^{2.4z})}{(1 + e^{0.4z})^8}$$

$$\theta_6 = -L^{-1}[R(\theta_5) + A_5] = -\int_0^t [R(\theta_5) + A_5]dt$$

$$= \frac{1296}{6103515625} \frac{t^6\, e^{0.4z}\,(-1 + 57\, e^{0.4z} - 302\, e^{0.8z} + 302\, e^{1.2\,z} - 57\, e^{1.6\,z} + e^{2\,z})}{(1 + e^{0.4z})^7}$$

By [14] the exact solution for the system (28) and (29) is:

$$\theta(z,t) = \frac{1 + 0.2\, e^{(0.4z - 2.4t)}}{1 + e^{(0.4z - 2.4t)}} \qquad (30)$$

If we approximate the exact solution by the obtained reslts i.e.

$$\theta(z,t) \approx \theta_0 + \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \qquad (31)$$

We can use Table 2 to compare the ADM result (31) with theexact solution (30).

z	t=0.1		t=0.2		t=0.3	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0	0.60479977	0.60479977	0.609598157	0.609598157	0.614393782	0.614393782
0.1	0.596800068	0.596800068	0.601599991	0.601599991	0.606399454	0.606399454
0.2	0.588802926	0.588802926	0.593600546	0.593600546	0.598400009	0.598400009
0.3	0.580814732	0.580814732	0.585606218	0.585606218	0.590401843	0.590401843
0.4	0.572841847	0.572841847	0.577623386	0.577623386	0.582411349	0.582411349
0.5	0.564890582	0.564890582	0.569658395	0.569658395	0.574434895	0.574434895
0.6	0.556967182	0.556967182	0.561717532	0.561717532	0.566478805	0.566478805
0.7	0.5490778	0.5490778	0.553807005	0.553807005	0.558549336	0.558549336
0.8	0.541228485	0.541228485	0.545932931	0.545932931	0.550652663	0.550652663
0.9	0.533425161	0.533425161	0.538101309	0.538101309	0.542794856	0.542794856
1	0.525673608	0.525673608	0.530318008	0.530318008	0.534981867	0.534981867

Table 2. Exact versus ADM

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