**Subclass for Higher-Order Derivatives of Multivalent Analytic Functions on Complex Hilbert Space with Some Applications in Fractional Calculus**

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**Abstract.**In the present investigation, we introduce and study a certain subclass for higher-order derivatives of multivalent analytic functions defined on complex Hilbert space. We determine some properties of this class, like, coefficient estimates, radii of starlikeness and convexity and convex combination. Also, we give an applications of the fractional calculus techniques.

**Mathematics subject classification:** 30C45,30C50.

**Keywords:** Multivalent functions, Higher-order derivatives, Hilbert space, Radii of starlikeness and convexity, Fractional calculus.

**1. Introduction**

Let indicate the family of all functions of the form:

which are analytic and multivalent in the open unit disk

Let denote the subclass of consisting of functions of the form:

Upon differentiating both sides of (1.2) -times with respect to , we obtain (see [2])

where

Several researchers have investigated higher-order derivatives of multivalent functions, see, for example [1,9,14,15,17].

A function is said to be multivalent starlike of order if it satisfies the condition:

and is said to be multivalent convex of order if it satisfies the condition:

Denote by the classes of multivalent starlike and multivalent convex functions of order , respectively, which were introduced by Owa [12]. It is known that (see [8] and [12])

The classes = were studied by Own [11].

Let be a complex Hilbert space and be a bounded linear operator on . For a complex analytic function on the unit disk , we denoted , the operator on defined by the usual Riesz-Dunford integral [4]

where is the identity operator on , is a positively oriented simple closed rectifiable contour lying in and containing the spectrum of in its interior domain [5]. Also can be defined by the following series:

which converges in the norm topology [6].

**Definition 1.1 [13] .** The fractional integral operator of order is defined by

where is analytic function in a simple connected region of - plane containing the origin.

**Definition 1.2[13].** The fractional derivative for operator of order is defined by

where is analytic in a simply connected region of the – plane containing the origin .

For , from Definitions 1.1 and 1.2 by applying a simple calculation, we get

and

**Definition 1.3.** A function is said to be in the class if satisfies the inequality:

where and for all operator with , ( denote the zero operator on ).

The operators on Hilbert space were considered recently by Xiaopei [18] ,Joshi [10], Chrauim et al . [3], Ghanim and Darus [7], selvaraj et al .[13] and Wanas and Jebur [16].

**2 . Coefficient Estimates**

In this section, we derive coefficient estimates for the function to be in the class

**Theorem 2.1.** Let be defined by (1.2). Then for all if and only if

where

The result is sharp for the function given by

**Proof.** Assume that the inequality (2.1) holds. Then, we get

Therefore

To show the converse, let . Then

Simple calculations gives

Taking in the above inequality, we have

 Upon clearing denominator in (2.3) and letting we obtain

or

This completes the proof of the theorem.

**Corollary 2.1.** If then

**3. Radii of Starlikeness and Convexity**

**Theorem 3.1.** If , then will be valently starlike of order in the disk , where

The result is sharp for the function given by (2.2).

**Proof.** It is enough to show that

We have

Hence (3.1) will be satisfied if

In view of Theorem 2.1, if , then

By making use of (3.3), we observe that (3.2) holds true if

or equivalently

This gives the desired result.

**Theorem 3.2.** If , then will be valently convex of order in the disk , where

The result is sharp for the function given by (2.2).

**Proof.** It is enough to show that

The result follows by application of arguments similar to the proof of Theorem 3.1.

**4 . Convex Combination**

**Theorem 4.1.** The class is closed under convex combinations.

**Proof.**Forlet *,* whereis given by

Then by (2.1), we obtain

Forthe convex combination of may be written as

It follows from (4.1) that

Thus,

**Corollary 4.1.**The class is a convex set.

**5. Applications of the Fractional Calculus**

**Theorem 5.1.** If then

and

The result is sharp for the function given by

**Proof.** Let By (1.3), we deduce that

Putting

Then, we obtain

Since for is a decreasing function of , then we get

Now, by the application of Theorem 2.1 and using (5.4), we find that

which gives (5.1), we also have

which gives (5.2).

 By taking in Theorem 5.1 , we conclude the following Corollary:

**Corollary 5.1.** If , then

and

**Proof.** By Definition 1.1 and Theorem 5.1 for , we have the result is true.

**Theorem 5.2.** If then

and

The result is sharp for the function given by (5.3).

**Proof .** Let By (1.4), we deduce

where

Since for is a decreasing function of , thus we have

Also, by using Theorem 2.1, we obtain

Thus

Then

and by the same way, we conclude that

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