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**Topological Aspects of Asymptotic Behaviors of Random Dynamical Systems**

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بسم الله الرحمن الرحيم

**وَعَلَّمَكَ مَا لَم تَكُن تَعلَمُ وَكَانَ فَضَلُ اللَّهِ عَلَيّكَ عَظِيماً**

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**List of Symbols**

|  |  |
| --- | --- |
| Symbols | Meaning |
|  | Family of allsubset of |
|  | the set. |
|  | the set of all measurable functions from to . |
|  | Set of real number |
|  | Real n-dimesional enclidean |
|  | the set. |
|  | Sequence { converges to |
|  | If ,then |
| RDS | Random dynamical systems |
| URDS | Uniform random dynamical system |

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Abstract

The aim of this thesis is to study the asymptotic behaviors of random dynamical systems in two different topological structures ( in a metric space and uniform space), where we study in the first part of this work the Omega Limit Set and introduced the concepts of the First Prolongation and the Prolongational Limit Set for Random Dynamical Systems., where some new properties are proved such as the relation among the orbit closure, orbit and omega limit set in RDS; also we prove that the First Prolongation of a closed random set containing this set, the First Prolongation is closed and invariant, also it is connected whenever it is compact provided that the phase space of the RDS is locally compact. Then we study the Prolongational Limit Set for RDS and proved some essential properties of this set. Where we prove that the Prolongational Limit Set for RDS is closed and invariant. Also the relation among the the First Prolongation, the Prolongational Limit Set and the positive trajectory of a random set is given and proved. Also if the phase space of RDS is locally compact then the following statements are true : if the Prolongational Limit Set for RDS is nonempty and compact, then the omega-limit set is non-empty; the Prolongational Limit Set for RDS is nonempty and compact if and only if the the First Prolongation is compact. Finally the Prolongational Limit Set for RDS is connected. The second part of this work the stability theory of RDS is studied when the phase space of the RDS is any metric space. This study orphanage through the study of the recursiveness , minimal random sets, and the concepts of region of weak attraction, attraction and uniform attraction. The third final part of our work is to study the the asymptotic behaviors of random dynamical systems in uniform space, i.e., when the phase space of RDS is uniform space. Throught the concepts of uniform RDS and the uniform random set are introduced. Also we study the concepts of Uniform Transitive, Uniform Sensitivity , Sequence of Maps in a Uniform space**,** Uniformly Equicontinuous RDSand Proximal and Distal .

**Chapter One**

**Random Dynamical Systems**

Random dynamical systems arise in the modeling of many phenomena in physics, biology, economics, climatology, etc, and the random effects often reflect intrinsic properties of these phenomena rather than just to compensate for the defects in deterministic models. The history of study of random dynamical systems goes back to Ulam and von Neumann in 1945 [33] and it has flourished since the 1980s due to the discovery that the solutions of stochastic ordinary differential equations yield a cocycle over a metric dynamical system which models randomness, i.e. a random dynamical system. This chapter consists of three sections. In section 1 we state the definition of MDS in terms of any locally compact group and give some examples . In section 2 we state the definition of RDS and give some basic properties of RDS .In section 3 we study random set.

**1.1 Metric Dynamical Systems ( Definitions and Examples):**

In this section the definition of MDS given in [3,11] is stated in terms of locally compact (semi) group with Haar measure and some examples are given.

**Definition 1.1.1:**

The 5-tuple is called a ***metric dynamical system*** ( Shortly MDS) if is a probability space and

1. is measurable,
2. ,
3. and
4. , for every and every .

Note that we write either in the form ( as a function of two variable or in the form . If is locally compact semigroup, then we say that is called a ***semi metric dynamical system*** ( Shortly SMDS).

From an applied point of sight the use of metric dynamical systems to model external perturbations adopts implicitly that the external influence is fixed in some sense (as we shown in the following examples). That is we do not consider possible transient (random) process in the environment, i.e. we assume that all these processes are finished before we start to observe the dynamics of our system. This is also the cause why we consider MDS with two-sided time. We note that any one-sided MDS ( with time ) possesses a natural two-sided extension. Now, we state several important examples of MDS's.

**Example 1.1.2 [11]:**

Let be a probability space with . is the Borel algebra generated by the compact-open topology of , is the Wiener measure on . Define by , . Then is an MDS.

**Example 1.1.3[11] (Ordinary Differential Equations):**

MDS can be also generated by ordinary differential equations (ODE). Let us consider a system of ODEs in :

, . (1.1.1)

Suppose that the Cauchy problem for this system is well-posed. We define by , where is the solution of (1.1.1) with . Suppose that a nonnegative smooth function satisfies the stationary Liouville equation

(1.1.2)

and possesses the property . Then is a density of a

probability measure on . By Liouville’s theorem

for any bounded continuous function on and so in this site an MDS rises with , and . Here is the Borel algebra of sets in . Some times it is also possible to construct an MDS related with the system (1.1.1), when the solution to (1.1.2) is not integrable but the problem (1.1.1) possesses a first integral (e.g., if (1.1.1) is a Hamiltonian system) with suitable properties.

**Example 1.1.4[11] (Stationary Random Process):**

Let be a stationary random process on a probability space , where is the algebra generated by . Suppose that in the continuous case () the process possesses the c`adl`ag property: all paths are right-continuous and have limits from the left. Then the shifts create an MDS. See Arnold [3] and the references therein for details.

**Example 1.1.5[11] (Wiener Process):**

Let be a Wiener process with values in and two-sided time . Let be the corresponding canonical Wiener space. More specifically, let be the space of continuous functions from into such that endowed with the compact-open topology, i.e. with the topology induced by the metric

, .

Let be the conforming Borel algebra of , and let P be the Wiener measure on . We suppose is the subset in consisting of the functions that have a growth rate less than linear for and is the restriction of to . In this realization , where , i.e. the elements of are recognized with the trajectories of the Wiener process. We define an MDS by . These transformations preserve the Wiener measure and are ergodic. Thus we have an ergodic MDS. The flow is called the Wiener shift. We note that the algebra is not complete with respect to and we cannot use its completion to construct MDS because is not a measurable mapping from into . This is one of the causes why the completeness of is not assumed in the basic definitions. See Arnold [3].

**1.2 Random Dynamical Systems:**

In this section we state the definition of random dynamical system [7] and give some basic properties of such system.

**Definition 1.2.1[3] (Random Dynamical System):**

A measurable random dynamical system on the measurable space over (or covering, or extending) an MDS with time is a mapping , with the following properties:

1. Measurability, is measurable.
2. Cocycle property: The mappings form a cocycle over , i. e. they satisfy

for all (if ), (1.2.1)

for all . (1.2.2)

If there is no ambiguity the RDS is denoted by rather than .

Note that axiom (1.2.1) of Definition 1.2.1 is not redundant. However, if the mappings are known to be invertible, (1.2.2) implies (1.2.1). It is very useful to imagine an RDS move on the (trivial) bundle , as Fig. 2.1 depicts: While is shifted by the dynamical system in time to the point on the base space , the cocycle moves the point in the fiber over to the point in the fiber over . The cocycle property is also clearly "visible" on this bundle.



Figure 2.1 A random Dynamical Systems as an action on bundle

**Definition 1.2.3 [3]** (**continuous (RDS)**.

A continuous or topological RDS on the topological space over the MDS is a measurable RDS which satisfies in addition the following property: For each the function , , is continuous.

**Definition 1.2.4[3] (Smooth RDS):**

A smooth RDS of class , or a RDS, where , on a dimensional () manifold is a topological RDS which in addition satisfies the following property: For each the mapping

, ,

is (i. e. times differentiable with respect to x, and the derivatives are continuous with respect to ).

**Definition 1. 2.5[3] (Linear RDS):**

A continuous RDS on a (for simplicity) finite-dimensional vector space is called a ***linear*** RDS, if for each , , where is the space of linear operators of .

If we endow the vector space with its natural manifold structure, then . Hence a linear RDS is automatically .

**Notations 1.2.6[3]**:

1. We often omit specifically mentioning the underlying metric DS (or abbreviate it as ) and speak of an "RDS " (over ), thus identifying an RDS with its cocycle part. Whenever we speak of a RDS we assume .
2. We denote by or the semigroup or group of continuous mappings or homeomorphisms of a topological space endowed with its compact-open topology. If is a locally compact Hausdorff space, this is a Hausdorff topological semigroup or group, and the evaluation mapping: is continuous.
3. Finally, we denote by or the semigroup or group of mappings or diffeomorphisms of a manifold , respectively, endowed with its compact-open topology. This is a Polish topological semigroup or group, and the evaluation mapping is with respect to . In the manifold case also is a Polish group.

**Remark 1.2.7 [3]**

1. If is discrete, measurability of is equivalent to measurability of for each fixed , continuity of for each is equivalent to continuity of for each fixed , and the smoothness of is just with respect to for each fixed .
2. A measurable/continuous/ RDS with continuous time is also a

measurable/continuous/ RDS if restricted to discrete time .

1. A measurable/continuous/ RDS with two-sided time is also a

measurable/continuous/ RDS if restricted to one-sided time .

1. We stress that we never allow our exceptional sets in the definition

of a cocycle to depend on . In fact, it is one of the basic problems of a theory of RDS in an infinite-dimensional space that

often holds only outside a set of measure zero which depends on and on .

**Example1.2.8:**

Deterministic DS and DS in the sense of ergodic theory are particular cases of RDS. Indeed, if is independent of then the RDS decouples in to a metric DS and a deterministic measurable/continuous/ DS on .In case time is a group, the underlying metric DS is invertible with . Equations (1.2.1) and (1.2.2) then force the coeyele to be invertible too. More precisely, we have the following far-reaching consequences of the cocycle property.

**Theorem 1.2.9[3] (Basic Properties of RDS with Two-Sided Time).**

Suppose is a group (i. e. or ).

1. Let be a measurable RDS on a measurable space over . Then for all , is a bimeasurable bijection of and

for all , (1.2.3)

or, equivalently,

for all . (1.2.4)

Moreover, the mapping is measurable.

1. Let be a continuous RDS on a topological space . Then for all

we have . If

1. ,or
2. and is a topological manifold, or
3. and is a compact Hausdorff space

then is continuous for all .

1. Let be a RDS on a manifold . Then for all , . Moreover, is with respect to for all .

**Remark 1.2.10[3]:**

1. It is somewhat surprising that under the assumptions of part (ii) of the above theorem the function

is continuous in although was assumed to be only measurable in .

1. Let be a continuous RDS with time . If is not locally Euclidean or compact Hausdorff we can in general not conclude that is continuous. This is due to the appearance of the shift operator in formula (1.2.3) for the inverse. This , is continuous. We still could conclude that is a homeomorphism. In fact, this weaker assumption suffices for most things we do with continuous RDS. The reason we stay with the stronger version of a topological RDS as given in Definition 1.2.2 is that we automatically obtain such continuous RDS when solving random or stochastic differential equations.

**Note 1.2.11**

In this thesis we shall discuss the perfection problem for the following setting

1. The group is replaced with a locally compact Hausdorff topological group . Our proof relies on the fact that such group has a Haar measure.
2. , , is an action of ( on the left ) on the set .
3. The cocycle as a family of self-mappings of some space is replaced with a group-valued cocycle over .

**Definition 1.2.12:**

Let and be two RDSs. The order triple

is said to be isomorphism between the two RDS and if

1. is bijective and measurable,
2. it topological group isomorphism,
3. for every .
4. is homeomorphism map and
5. .

If there exists such order triple we say that and are ***equivalent*** via and write .

**Definition 1.2.13[11]: (Equivalence of RDS)**

Let) and be two RDS over the same MDS with phase spaces and resp. These RDS and are said to be (topologically) equivalent (or conjugate) if there exists a mapping

: × → with the properties:

(i) the mapping → (, ) is a homeomorphism from onto for

every ∈ ;

(ii) the mappings → (, ) and → (, ) are measurable for every ∈ and ∈ ;

(iii) the cocycles and are cohomologous, i.e.

(, , (, )) = (, (, , )) for any ∈ .

**Theorem 1.2.14:**

For each , is a homeomorphism.

**Proof.** For any the mapping is continuous by definition of RDS. To see that is one –to-one and onto observe that if , then

.

Again, if . Set . Then

.

Then is an onto map.

By definition of RDS the mapping is continuous. The inverse of is defined as follows such that

, for all .

Since is continuous for all , then so is . This means that is homeomorphism.

**Theorem 1.2.15:**

If and are RDS's, then so is , where defined by

for all .

**Proof**:

It is sufficient to show that is cocycle over . Since and are continuous, then so is . Thus is continuous. Since is defined by

form a vector valued function, then it is measurable [7,8]. Also the function is continuous since

and

are continuous functions and . Now,

.

.

Thus is RDS.■

**Exampl 1.2.16 RDS form Random Differential Equations[ 5 ]:**

Let and be a metric DS. We will now establish absically one-to one correspondence between (local) continuous/ RDS over which are absolutely continuous with respect to and random differential equations driven by .The correspondence is given by

, (1.2.5)

Which is valid in the local case for all an open interval of containing 0, and in the giobal case for all .If (1.2.5) holds ,we say (sometimes called solution in the sense of Caratheodory),or that the random differential equation generates . Not hat (1.2.5) implies for all and .

If the solution is differentiable with respect to and satisfies for

, (1.2.6)

It is called a classical solution of . Non –classical solutions are important since they allow us to consider discontinuous noisc .

**Exampl 1.2.17 Random Differential Equations from RDS[3]:**

We now with the inveres problem of when for a given RDS on over with time there exists a random differential equation which generates .We will also determine the only possible from in which is coupled to , namely

**Example 1.2.18:**

Let be any MDS and be any locally compact group. Let be a map with the property that is continuous homomorphism for every , is measurable for every , and . Then the mapping defined by is a cocycle over and hence is a URDS.

**1.3 Random Sets in Random Dynamical System:**

In this section one of important tools in the study of RDS's which is the random is studied and some new properties of such sets are proved. The origin of the modern concept of a random set goes as far back as the seminal book by A.N. Kolmogorov [22](first published in 1933) where he laid out the foundations of probability theory. In this section assumed a Polish space. The set

is called the section of *.* Let *,* be a function whose values are subsets of *.* Such a function is uniquely determined by its graph . Conversely, every subset defines such a function via *.*

**Definition 1.3.1[3]:**

Let be a measurable space and be a metric space which is considered a measurable space with Borel algebra . The set-valued function  *,* is said to be ***random set*** if for each the function is measurable. If is connected closed (compact) for all , it is called a ***random*** ***connected closed(compact)******set***.

The algebra of universally measurable sets associated with the measurable space is defined as ,where the intersection is taken over all probability measures on and denotes the completion of with respect to .

**Proposition 1.3.2[3]:**

Let the set-valued function take values in the subspace of closed subsets of a Polish space . Then:

1. A is a random closed set if and only if for all open sets the set is measurable.
2. If is a random closed set then .
3. Conversely, if contains the algebra of universally measurable sets (in particular', if is complete), then implies that is a random closed set.

The property of being a random closed set is thus slightly stronger than being measurable and being closed.

**Example 1.3.3[27](Random Sets Defined from Random Points):**

1. The singleton is a random closed set.
2. A ball with and radius is a random closed set if is a random vector and is a non-negative random variable. If the joint distribution of depends on a certain parameter, we obtain a parametric family of distributions for random balls.
3. A random triangle obtained as the convex hull of is a random closed set. Similarly, it is possible to consider random polytopes that appear as convex hulls of any (fixed or random) number of points in the Euclidean space.

**Example 1.3.4[27](Random Sets Related to Deterministic and Random Functions):**

1. Let be a deterministic function, and let be a random variable. If is continuous, then is a random set. If is upper semicontinuous, i.e. for all , then is closed and so also defines a random closed set. Its distribution is determined by the distribution of and the choice of . In these both case can be obtained as the inverse image of a random set, e.g. as or .
2. Let , be a real-valued stochastic process. If has continuous sample paths, then is a random closed set. If has almost surely upper semicontinuous sample paths, then the excursion set and the hypograph are random closed sets.

**Example 1.3.5[11] (Random Ball):**

Let . Suppose that is a random variable and is a random vector from . Then the set-valued function

is a random compact set . Here is the Euclidean distance in . This fact follows from the formula

which implies that . It is also clear that is a random (open) set.

**Proposition 1.3.6[27]:**

1. If is a random closed set, then so is , the closure of .
2. If is a random open set, then is a random closed set.
3. If is a random closed set, then , the interior of , is a random open set.
4. If and are random compact sets, then so is .

**Definition 1.3.7 [3]:**

Let be a measurable RDS and a set.

1. is called forward invariant if for

a.s.

equivalently

a.s..

1. is called invariant if for all

a.s.,

for two-sided time equivalent to

a.s.

In the following the definition of invariant set that given in [3] is sated in terms of locally compact group.

**Definition 1.3.8[3]:**

Let be a measurable RDS. A random set is said to be invariant set if there exists a full measure subset such that

, for every .

**Proposition 1.3.9 [10]:**

1. Let be a measurable RDS. Then
2. arbitrary unions and intersections of invariant sets is invariant;
3. is invariant if and only if is invariant;
4. Let be a continuous RDS. Then
5. If is invariant, then so is .
6. If is an abelian group, and if is invariant, then so is , , and .

**Definition 1.3.10[10]:**

Let be a family of random closed sets which is closed with respect to inclusions (i.e. if and a random closed set possesses the property for all , then ). Sometimes the collection D is called a ***universe***of sets.

**Definition 1.3.11 (Asymptotically Compact RDS)**[11]:

An RDS is said to be ***asymptotically compact*** in the universe , if there exists an attracting random compact set , i.e., for any and for any we have

, (1.3.1)

where .

**Chapter Two**

**Prolongation Limit Random Sets in Random Dynamical**

**Systems**

In this chapter we recall same basic definition and facts about random dynamical system . This chapter consists four sections. In sections 1 prove some measurable properties of the trajectories. In sections 2 we study random fixed points and periodic random points for random dynamical system are introduced and proved. Here the time space considered any locally compact space and the phase space is any metric space. Also some new concepts are introduced here such as Topological metric dynamical system, uniform converge and closed set. In sections 3 we study Omega-Limit set. In sections 4 we generalize the definition of prolongations and prolongational limit sets RDS given in N.P. Bhatia, G.P. Szegö [10]. and prove some new properties of the studying of prolongations and prolongational limit sets .

**2.1Trajectory :**

In this section we describe some measurable properties of the trajectories of RDS.

**Defdinition 2.1.1[11]:**

Let be a multifunction. We call the multifunction

the tail (from the moment ) of the pull back trajectories emanating from . If is a single valued function, then is said to be the ***(pull back) trajectory*** ( or orbit) emanating from .

In the deterministic case  is a one-point set and is a semigroup of continuous mappings. Therefore in this case the tail has the form **.**

**Definition 2.1.2:**

Let and and be the mappings form in to defined as follows

(1)

(2)

(3)

For every , the sets , , and are respectively called the ***trajectory, the forward semi-trajectory and backward semi-trajectory.***

**Definition 2.1.3:**

Let and and be the mappings form in to defined as follows

(1)

(2)

(3)

For every, the sets , , and are respectively called the trajectory, the forward semi-trajectory and backward semi-trajectory of .

**Proposition 2.1.4:**

For and , the sets , , and are invariant random sets.

**Proof.** Let . To show that is an invariant. Let and . Then there exists such that . Now

,where

, .

.

Thus for every and , we have

.

This means that the set is an invariant. In a similar way we can show that , and are invariant random sets.

**2.2 Random Fixed Points and Random Periodic Points :**

In general one cannot expect that one point is fixed by (almost) all mappings . However, there is an appropriate generalization of the notion of a fixed point. In this section some new properties of random fixed point and random periodic point for RDS's are studied. In the following simple modification on the definition of random fixed point given in [3,11] is made.

**Definition 2.2.1[19]:**

A measurable function is said to be a ***random fixed point*** (some time called ***random invariant point*** or ***stationary solution***) for the RDS if for all .

**Remark 2.2.2:**

If consists of a single point, then an RDS over is just a dynamical system generated by the single homeomorphism . In this case the notation of a random fixed point coincide with that of a deterministic fixed point.

**Example 2.2.3[19]:**

Let , be the algebra of Lebesgue measurable sets and be the Lebesgue measurer on . Then the triple form a probability space. Let and . Define by and . Also define by and . Clearly that is RDS and the random variable defined by is a fixed point of .■

Hence

.

.

Hence

**Example 2.2.4[19]:**

Let be any non-trivial MDS and let be any injective random variable. Define a cocycle over by

. Then is RDS. This RDS has no random fixed point.■

**Lemma 2.2.5:**

If and for some , then for all integer .

**Proof.** If , for some . Then

Then . Thus we need to prove the result for positive integers only. This follows from induction. If , then by hypothesis we have

for some .

Now, suppose that the statement is true for . i.e.,

, for some .

To show that this statement true for . Let

.

Let .

,

Thus

.

**Theorem 2.2.6 ;**

Let **.** Then the following are equivalent:

1. is random fixed point,

2. ,

3. ,

4.,

**Proof**.(1) (2): Suppose(1) holds, then

,

where ,

. Conversely, suppose (2) holds, then

. But , then

, then

That is and ,

. Thus

. Set .Then for every

and. Consequently is an R.F.P.

(2)(3). Suppose (2) holds. Since , we conclude that . Conversely, suppose (3) holds. Then . That is, . Then , where . Thus

(2) (4). As in (2) (3). This end the proof. ■

**Theorem 2.2.7:**

Let be an RDS with considered as a topological MDS. is continuous, then the following are equivalent:

1. is random fixed point,

2. There is a sequence , , with .

**Proof** . To prove (1)(2). Assume (1). Since is random fixed point, then

for all .

Thus we can say that there exists a sequence , , with .

Conversely, assume (2), let . If for some integers and , then by Lemma (2.2.5) . If , then for every , there exists such that and moreover an integer with

.

Thus clearly the so constructed sequence has the property that . Now since is continuous for every , then , for every .

Since for every then

, for every .

Again, since and is continuous for every , then , and since is continuous, then for every . Thus for every . This means that for all .

**Note.** The implication (1)(2) is true when is any MDS. ■

**Definition 2.2.8** **:**

The set is said to be ***distinguishable*** if for every , there exist two random open sets and in and such that , and for every we have , and .

The set is said to be ***distinguish set.***

**Lemma 2.2.9:**

Let is distinguishable with distinguish set . If is not random fixed point, then there exist two random open sets and such that for every with and we have and .

**Proof.** Note that if is random open set in , then for every , is random open set in , since is homeomorphism. Since is distinguishable with distinguish set , then there exist two random open sets and in such that for every we have and and for every . Since for every , then for every . Set

.

Then , for every . Set

.

Then , for every . Clearly that and for every . But for every , this implies that , for every . ■

**Theorem 2.2.10:**

Let is distinguishable with distinguish set . Then is random fixed point if and only if every random neighborhood of , contains the set , .

**Proof** Suppose that is random fixed point, then so that contained in every random neighborhood of . Conversely, suppose that every random neighborhood of contains semi-trajectory. Assume contrary that is not random fixed point, then there exists , for every with ,

, for some .

By Lemma (2.2.9) there exist two random open sets and such that for every with and we have and . Since for each , we have , , then , . But this is a contradiction.

**Notation** the set of all random fixed point

**Theorem 2.2.11:**

Let is distinguishable with distinguish set .The set of all random fixed point is closed.

**Proof.** Let the set of all random fixed point. If , then the prove is end. If . Assume contrary that is not closed. If the set of random fixed points is not closed, then there is a sequence in with is uniform converge to and . Thus there is with . By Lemma( 2.2.9) there exist two random open balls and such that for every with and we have and . Since is uniform converge to , then and we have for all sufficiently large . Then for the above and in particular . But 's are random fixed points and therefore and . This contradiction proves the theorem.

**Theorem 2.2.12:**

Let is distinguishable with distinguish set .If and , for every as ( or ). Then is random fixed point.

**Proof.** Let be a random neighborhood of . Since , for every as , there exists such that , for every , for all . Hence for all we have . Then for all , . Or for all , . Or for all , , where and . That is contains semi-trajectory , consequently, by Theorem 2.2.10 is R.F.P. ■

**Proposition** **2.2.13:**

Let be a random fixed point. If with , then .

**Proof.** Suppose that is a random fixed point. Let with . Assume contrary that

.

Then .

Since is bijective, then .

So .Contradiction. ■

**Definition 2.2.14:**

A random variable is said to be random ***periodic point*** of a RDS if there exists such that

.

**Remark 2.2.15**

In any RDS every random fixed point is random periodic point.

In the following assertion gives another description of random periodic point.

**Proposition 2.2.16:**

A random variable is random periodic point if and only if there exists such that

, for every

**Proof.** Suppose that is random periodic point. Then there exists such that

.

If and only if

,

If and only if

, .

This complete the proof**. ■**

**Theorem 2.2.17;**

Let be an RDS with be a stable TMDS and let be a random periodic point and continuous but not R.F.P. Then there exists such that is the smallest positive period of .Further, if is any other positive period of , then for some integer .

**Proof.** Consider the set . If period of , then .Let , then . Since, then . Now, set

. To show that

, :

. Hence . Since either or is positive, then the set is nonempty. Now, set . We calem that . Indeed , and if , then there exists a sequence in with . Since for each , then by Theorem( 2.2.7) is random fixed point which contradicts our hypothesis. Thus . Since , then there is a sequence in with . Since is continuous for every , then for every , .So for every , .

Since is continous for every , then for every . Again, since is continuous, then for every . But , i.e., there exists a full measure subset of such that for every . Hence , .On the other hand, , for every . Since is metric space, then from the uniqenss of the limit we have . That is, . It follows that . By definition of it is also the smallest positive period of . Finally, let be a positive periodic. If , for any integer, then there is an integer with . Then by Lemma( 2.2.9) we have

.

Since the TMDS is satble, then .

Therefore by Lemma( 2.2.9 )we have . So.Thus

where . Then satisfy (2.2.9). Since , we get a contradiction to the fact that was the smallest positive period of period of . This complete the prove. ■

In the following we need to show that the set of random periodic point for random dynamical system (under certain conditions) is closed. To this end the following notations are introduced.

**Definition 2.2.18:**

Let be a probability space with considered as a topological space and be any metric space. A sequence in is said to be uniform converge in if there exist and such that converge uniformly (shortly u.c.) to for every . That is for every , there is a positive integer such that , for every and for every .

**Definition 2.2.19:**

Let be a probability space with considered as a topological space and be any topological space. A subset of is said to be closed if any sequence in is uniform converge in .

**Lemma 2.2.20:**

Let be an RDS with is be a TMDS. If be a sequence of continous random periodic point in with positive periodic , and uniform converge in , then is random fixed point.

**Proof.** For a given , there are integers such that . Since , we have . Since is uniform converge to , then . Let ,

then But is continuous for every , then for every , .

Since is continuous for every , then for every . Therefore for every .

Since , for every , then for every . Since is random periodic point for every , then

Set . Then . So , for every . , for every . Then,

, for every .Since for every , it follwos that , for every . Since ,then is random fixed point.

**Theorem 2.2.21:**

Let be an RDS with is be a TMDS .Given any , the set of all such that is (continuous) random periodic point with positive period is closed.

**Proof.** Let be a set of all random periodic point with positive period . Suppose that be uniform converge sequence in . Then for every , is random periodic point with periods and then . Set ,

then . Since , then , where .Also for every , .

Since , either in which case is random fixed point by Lemma 2.2.20 and hence random periodic, or there is a subsequence , , then by the continuity axiom for every and also for every , Since is continuous, then for every . Also we have is continuous for every , then , for every .Therefore . for every . Consequently , for every

. Since .,then is random periodic point. ■

**2.3 Omega-Limit set :**

In this section we study concept in RDS called Omega-Limit set and give some properties of such systems.

**Definition 2.3.1[11] :**

Let be multifunction. We call the multifunctions

is said to be ***omega limit*** of the trajectories emanating from .Also the

are said to be the omega (alpha) -limit set of the trajectories emanating from respectively.

If , the we have

.

The following assertion gives another description of omega-limit sets.

**Theorem 2.3.2[11]:**

Let be the omega-limit set of the trajectories emanating from . Then .

**Proof.** Suppose that, the for any there exists inandin such that . Hence . Thus

.Therefore , for all . Thus .

To prove the converse inclusion, let , then for all . In particular,

for all .

Therefore there exists a sequence in such that . Thus and , . It follows that there exists and such that . That is . Consequently, .

In the following we show omega-limit set is random closed set .

**Theorem 2.3.3:**

is random closed set.

**Indirect Proof.** By above theorem we have

.

Since is closed an invariant, then so is .

**Direct proof.** Let . Then there exists in be a sequence in such that . We wish to show that . Indeed for each positive integer , there is a sequence in and in with and . We assume without loss of generality that

and for . Consider now the sequence in with and a sequence in with . Then and we claim that .To see this observe that

.

Since and tend to zero we conclude that

Consequently and . Thus , i.e., is closed.

**Theorem 2.3.4:**

Let be any metric space and . Then

**Proof.** To prove .Let .Then

,where . Thus we have . Then

Then .Now letThen there issuch that.Then By continuity , ,

.Thus we have .Then .Then

**Theorem 2.3.5:**

If is continuous, then .

**Proof.** First, note that .By Theorem (2.3.2) we have . Therefore . To prove the converse inclusion, let . then there exists a sequence in such that . Now , then there exists a sequence with for every and in such that . We have two cases:

**Case I**: The sequence has the property that , in which case .

**Case II:** There is a subsequence in such that (as is closed). But then (since is continuous). Since , then from the uniqueness of the limit we have .

From Case I and Case II, we have . Hence

.

Therefore .

**Corollary 2.3.6:**

For any and .

**Proof.** By the definition we have.To show that ,let .Then there is a sequence { in such that . Since in .Then for a in .Either the sequence { has the property that ,in which case or there is a subsequence (as is closed).But then , and since also we have .Thus .Thus .

**2.4The First Prolongation and the Prolongational Limit Set:**

The concepts of first prolongations and prolongational limit sets are played an essential role. In the deterministic dynamical system the formal definition of prolongation is due to T. Ura [34] and the concept of prolongational limit set is due to N.P. Bhatia, G.P. Szegö [10]. In this section we study The First Prolongation and the Prolongational Limit Set of RDS .We press the notions of The First Prolongation and the Prolongational Limit Set of RDS and ready their base properties.

**Definition2.4.1:**

Let be multifunction.The multifunction , where

, is said to be to be ***first positive prolongation of*** . If the set replaced by in above we get the notation of first negative prolongation of and shall denoted by .

If , the we have

}.

The following result show that the The First Prolongation is closed and invariant .

**Theorem 2.4.2:**

is closed.

**Proof.** To show that is closed. Let , then there exists sequence in such that . Since for every . Then by definition of there exists sequences and such that and . We assume by taking subsequences if necessarily that , and for . Now consider the sequences , . Clearly and . Note that

.

Since and tend to zero, then , then . This means and so is closed.

**Theorem 2.4.3:**

If is invariant, then so is .

**Proof.** We need to show that .

Let , then there exists such that

.

To show that .Since , there exist sequences in and in with and

. Since is continuous, then

, then

,where a sequences in and in with

Then we have . then . To prove the converse inclusion, let by definition, there exist sequences in and in with and so for all . Since is an invariant , then , then there exists such that , then . Now, .

Then for all .

Then there exists such that for all . Since , i.e.

, or .

Hence , with . (2.4.1)

From (1.3.1) we have that as . Since is compact, there exist and such that as . Moreover by Def. . From (2.4.1) we obtain that . Therefore for all and . Thus is invariant.

We now discuss about the connectedness of the First Prolongation set .

**Theorem 2.4.4:**

Let be locally compact. Then is connected whenever it is compact.

**Proof** Let be compact but disconnected. Then there are two compact non- empty sets and such that and .Since and are compact ,.Thus there is such that ,are compact and disjoint .Now or .Let .Then there is a sequence in and a sequence in such that and .We may assume and .Then the trajectory segments intersect ,and therefor is a sequence , such that .Since is compact we may assume that .Then ,but as .Thus contradiction shows that is connected.

**Definition 2.4.5:**

Let be multifunction.The multifunction , where

, is said to be to be ***first positive prolongational limit set of .*** If the set replaced by in above we get the notation of first negative prolongational limit set of and shall denoted by .

If , then the definition of becomes

The following theorem showing the prolongational limit set is closed and invariant .

**Theorem2.4.6:**

is closed.

**Proof.** To show that is closed. Let , then there exists sequence in such that . Since for every . Then by definition of there exists sequences and such that , and . We assume by taking subsequences if necessarily that , and for . Now consider the sequences , . Clearly and . Note that

.

Since and tend to zero, then , then . This means and so is closed.

**Theorem 2.4.7:**

If is invariant, then so is .

**Proof.** We need to show that .

Let , then there exists such that .

To show that .Since , there exist sequences in and in with , and . Since is continuous, then

.

By the cocycle property, we have

,where and in with . Then by Definition

, then . To prove the converse inclusion, let

. by Def. there exist sequences in and in with , and . By the cocycle property we have

, with . (2.4.2)

From (1.3.1) we have that as . Since is compact, there exist and such that as . Moreover by definition . From (2.4.2) we obtain that . Therefore for all and . Thus is invariant.

**Theorem 2.4.8:**

.

**Proof.** . To prove the converse inclusion. Let by Def. there exist sequences in and in with and . We may assume that either or , if necessarily by taking subsequences. In the first case (since is continuous for every ). By uniqueness of the limit we have . In the second case by Def. of . Thus . Hence .

**Corollary 2.4.9:**

.

**Proof.** By definitions . To prove the converse inclusion. Let by definition there exist a sequences in and a sequences with such that . We may assume that either or , if necessarily by taking subsequences. In the first case (since is continuous for every ). By uniqueness of the limit we have . In the second case byDefinition of . Thus . Hence

**Theorem 2.4.10:**

Let

. Then if and only if .

**Proof.** Suppose that . Then there exist sequences in and in with , and . Set and . Then is a sequence in with and is a sequence in and .

Finlay we need to show that .

,

Then we have .Thus .

Similarly we can prove the converse.

**Theorem 2.4.11:**

**=**

**Proof.** To prove .Let . Then there is a sequence in with and a sequence in with such that ,

,where

Thus we have . Then

then  . To prove the converse inclusion

Let . Then there is with and a sequence in with and a sequence in with such that . By the continuity of ,

, where ,.

Thus  **,**we have **.**

Then **=**

The following result show that the omega-limit set is non-empty .

**Theorem 2.4.12:**

If is locally compact.Then whenever is non-empty and compact. .

**Proof** .If possible let .Then we claim that is closed and disjoint with.That is closed follows from = as ,That follows from the fact that if ,then by invariance of , Since is compact ,we will have and compact(remember that any sequence{}in a compact set Q has a convergent subsequence ). This again contradicts the assumption = .Thus is closed and .Since is non-empty and compact we have . Thus there is a such that is compact and disjoint with . Now choose any of . There is a sequence { in and a sequence { in such that and and, .We may assume that , for all n .Then the trajectory segments with 0 , intersect and therefor there is a sequence { , 0 ,such that .Since is compacte we may assume that .By taking subsequences we may assume that either or . if , then by the continuity axiom ,i.e, which contradicts .If ,then ,but this contradicts as = .

**Theorem 2.4.13:**

Let be locally compact. Then is non –empty

and compact if and only if is compact.

**Proof** . Let be non –empty and compact .Then is non empty and compact .But then is compact( is closed with be locally compact).Hence = is compact .Now if is compact . Since .Then is compact.

**Theorem 2. 4.14:**

If is locally compact. Then is connected.

**Proof** Let be compact . If = there is nothing to prove. So let .If is disconnected ,then there are non-empty compact sets such that and .Since is non- empty and compact ,hence connected ,we have or .Let .Since = as and is compact. Then is compact. Now let ,then .But must be invariant .Thus will show that ,aconradiction .Then is compact and disjoint from ,= .since and are disjoint compact sets we have is disconnected. Thus is a contradiction . Then is connected.

**Chapter Three**

**Stability of Random Dynamical Systems**

Throughout this chapter we study three concepts .In section 1 we defin Recursive concepts poisson stable , non-wandering point and reucurrent .In section 2 we give definition of minimal random set and prove some properties of minimal RDS’s . In section 3 we study the weakly attracted, attracted, uniformly attracted and stability of RDS .

**3.1 Recursive concepts:**

In this section we study those concepts wich are conncted with the concept of recursiveness defined a down . these are concepts of poisson stable , non-wandering point ,and reucurrent .

**Definition (Recursiveness) 3.1 .1:**

A random set is said to be ***positively recursive*** with respect to a random set if for each there is a and an such that .***Negative recursiveness*** may be defined by using the inequality . We will say that a set is self positively recursive whenever it is positively recursive with respect itself.

**Example 3.1.2:**

The set , where is a random periodic point for is self recursive.

**Solution.** Since is a random periodic point for , then there exists is a random periodic point for such that

Now

This means that is self recursive.

**Definition 3.1.3:**

A point is said to be ***positively Poisson stable*** if every random neighborhood of is positively recursive with respect to .

**Theorem 3.1.4:**

Let . Then the following are equivalent.

(1) is positively Poisson stable

(2) given a random neighborhood of and ,

for some ,

(3) for every random variable there is such that

.

**Proof.(1) iff (2):**

**(1) implies (2)**:Suppose that is positively Poisson stable. Let be a random neighborhood of and . By hypothesis is positively recursive with respect to , i.e., there is such that

.

**(2) implies (1)**: Let be a random neighborhood of . Let . If , then by hypothesis there exists such that

.

If . Then , and by hypothesis

for some .

**(2) iff (3):** Suppose (2). Let be a random variable. Then be a random open ball centered with . By (2) for every we have for some .

Suppose (3). Let be a random neighborhood of and . Then there exists a random variable such that .By hypothesis we have for some .

Then , for some .

**Theorem3.1.5**:If . Then for every random variable there is such that

.

**Proof**. Suppose that . Let be a random variable.

The there is a sequence in with and for all . Then eventually in every neighborhood of . Thus for every random variable there exist such that

, for all .

**Theorem 3.1.6:**

Let be a complete metric space. Let be a positively Poisson stable. Then the set is dense in .

**Proof.** Since is positively Poisson, we have . To see that . Let there is a monotone increasing sequence , , such that . Choose such that . then (otherwise will be random periodic point). Hence . Set

.

Then

and .

Having defined and , choose such that

(possible because of positive Poisson of ). Then define

where . note that as the motion is not periodic. Clearly

,

and .

The sequence has the property that

for . is, therefore, a Cauchy sequence which converges to a point as the space is complete. Since and , we have . Further , so that . Notice further that . For, otherwise, if , we will have . But there is an such that , so that . However,

and by construction , i.e., . This contradiction proves that and the theorem is proved.

**Definition 3.1.7:**

A point is said to be ***non-wandering*** if every neighborhood of is self positively recursive.

**Theorem** **3.1.8:**

For any , the following are equivalent.

(1) is non-wandering,

(2) ,

(3) every neighborhood of is self negatively recursive,

(4)

**Proof.** Assume (1). Consider a null sequence of random variables, , , and a sequence in with . Since each is self positively recursive, we have an and a with . Since we have and and since we conclude . Thus (2) holds. Now suppose (2). Then there exists a sequence in and a sequence in with and such that . Now for any random neighborhood of and there is such that for and for every and for and for every . Thus is self positively recursive. Consequently is non-wandering and (1) holds.

(3) iff (4). Is proved in the same way.

(2) iff (4): by theorem (2.4.10)**.**

**Definition 3.1.9:**

The MDS is said to be ***stable*** if for each sequence with the property that , we have .

The following result show that the point which reside in the limit sets are non-wandering .

**Theorem 3.1.10:**

Let . Every is non-wandering.

**Proof.** We have to show that if , for some , then . indeed there is a sequence with . Since , we may assume, if necessary by taking a subsequence for each , then . Setting we have . Now

Thus . By Theorem ( 3.1.8 ) is non-wandering.

**Theorem 3.1.11:**

Let be a random set in be such that every is either positively or negatively Poisson stable. Then every is non-wandering.

**Proof.** Let . Then there exists such that . We must prove that . Indeed for each we have either or . Thus by taking a subsequence, we may assume that for all or for all . Assume for all . For each there is with . Then

.

This shows that and consequently . In the second case similar consideration show that . Thus Theorem(3.1.8 ) is non-wandering.

**3.2 Minimal Random Sets:**

Minimal systems are natural generalizations of periodic orbits, and they are analogues of ergodic measures in topological dynamics. They were defined by G. D. Birkhof in 1912. The basic fact discovered by G. D. Birkhof is that in any compact system there are minimal sets. This follows immediately from the Zorn's lemma.

**Definition 3.2.1:**

Let be a random dynamical system. A random subset is said to be minimal set if

1. is nonempty random closed set.
2. is invariant;
3. does not contains any nonempty proper random subset invariant under . That is, if satisfy (1) and (2), then .

**Theorem 3.2.2:**

Let be a non-empty random set in . Thenthe following statement are equivalent:

(i)  is minimal in ;

(ii) for every .

(iii) is closed an invariant and for every non-empty random open subset of , either

Or .

**Proof. (i)(ii)**.Suppose (i). Then for each ,then and as is closed and invariant. Since is closed and invariant set we must have , for otherwise will be a non-empty proper subset of , a contradiction to minimality of .

**(ii)(iii).** Suppose (ii). Then is closed and invariant. Let be a non-empty random open subset of such that . If , by (ii) Since is homeomorphism for every and for every , then is open for every and for every . Since , then there exists and ,for every and . Hence there exists sequence in such that and , for every and . Therefore there exists such that , for every , and . Then , for every , and . This means that for every and . Then there exists and for every and . Thus there exists such that and . Then

,

.

Then . Consequently,

.

(iii) (i).Suppose (iii). Let be closed and invariant subset of such that . Then is nonempty open random set (since if , then ). By (iii)

or .

If , then

.

If , then . Then, then .Consequently, is minimal.

**Theorem 3.2.3:**

is minimal if and only if .

**Proof.** Suppose that is minimal. Let . If , then by Theorem (3.2.2) ( , so . Conversely, suppose that . To show that is minimal. We have is non-empty closed invariant random set. Suppose that be a nonempty closed invariant subset of . If , then and by hypothesis . Since is closed and invariant, then , so .Then . By a similar argument we have but . Thus we have . This means that is minimal.

**Theorem 3.2.4:**

Let be a compact minimal random set. Then for every non-empty random open subset of , either or for some finite subset of .

**Proof.** Then is closed and invariant. Let be a non-empty random open subset of such that . Then . So .Since is invariant, then or . Since is minimal then by Theorem(3.2.2) we have .Since and is homeomorphism for every and for every , then is open for every and for every . Then . But is compact, then . Set , then

.

**Theorem 3.2.5:**

Let be a non-empty and compact random set. Then the following are equivalent. ( )

(1) is minimal,

(2) for every ,

(3) for every ,

**Proof.** (1) (2). Theorem ( 3.2.2 )

(1) (3): Suppose (1). Let . Let , then there exists a sequence in with such that Set . Since , then . So that ( because is an invariant.) Then be a sequence in with . But is closed. Thus . Then . Since is minimal and is a non-empty invariant closed subset of . Thus we must have .

**Theorem** **3.2.6::**

If is minimal and , then is open.

**Proof.** To prove that is open we must show that every point of is an interior point of . Let . Since is minimal, then by Theorem (3.2.2) . Since , then there exists such that .Then . That is is a limit point of . Thus . Then there exists . Then and . So there exists such that . Then or . Since is open subset of , then is an interior point of . Therefore is open.

The following theorem shows that the minimal random sets in RDS's are disjoint.

**Theorem 3.2.7:**

Any two minimal random sets must have empty intersection.

**Proof:**

Suppose that and are two distinct minimal sets, and . Then is closed random set. By hypothesis we have and for all . Then

.

Thus is invariant. But then is proper subset of both and which is closed invariant nonempty random set, contradicting the fact that and are minimal.■

**Theorem 3.2.8:**

Let with is isometric. If is minimal in , then is minimal in .

**Proof.** Suppose that is minimal in . To show that is minimal in . By hypothesis is a nonempty random closed set and invariant under . Since is an isometric and homeomorphism, then is nonempty random closed set and invariant under . Now, let be a non-empty invariant closed random subset of . Then is non-empty random closed and invariant under . By hypothesis either or . Then either or . ■

**3.3Stability of random dynamical system:**

This chapter is devoted to the study of stability and attraction. will denote a non- empty compact subset of

**Definition 3.3.1:**

With the given we associate the sets

1. ,

2. ,

3..

The sets , , and are respectively called region of ***weak attraction, attraction, and uniform attraction*** of the set .

Moreover, any point in , , or may respectively be said to be weakly attracted, attracted, or uniformly attracted.

**Definition 3.3.2:**

With the given we associate the sets

1. ,

2. ,

3..

The sets , , and are respectively called region of weak attraction, attraction, and uniform attraction of the set .

Moreover, any random set in , , or may respectively be said to be weakly attracted, attracted, or uniformly attracted.

**Proposition** **3.3.3:**

Given , a point is

1.weakly attracted to if and only if there is a sequence in with and ,

2.attracted to if and only if as ,

3. uniformly attracted to if and only if for every neighborhood of there is a neighborhood of and a with , .

**Proof.** Let . Then and . Then There is ,, then there is in with such that ,, or ,. Now , since

,

,.

That is , .

Conversely, Suppose that there is a sequence in with and . Then there is such that . But by definition of we have . Then we have . Consequently .

2. Let . Then and . Then there is . Then there is in with such that , or ,. Since , then

3- First assume that for an , . Then ().Assume now that there is a neighborhood of , such that for every neighborhood of and there is with . By (2) there is a such that for .This show that there exist sequences and in , , and a sequence in , such that ,but . Since may be assume compact (the space is locally compact) we conclude that there is sequence ,,and a sequence , with .Then but .This is a contradiction. Now assume the converse requirement that for every neighborhood of there is a neighborhood of and a with for . Then indeed . This shows indeed that for and since we may take to be compact, we have consequently . Clearly , for every neighborhood of and every .Hence for every neighborhood of . Thus

.

This proves the proposition.

**Theorem 3.3.4:**

For any random set

**Proof** Let Then . Then by theorem(2.4.13) .Thus .Then Then .Then .

**Definition 3.3.5:**

A random set is said to be

(1) a ***weak attractor*** if is neighborhood of ,

(2) a ***attractor*** if is neighborhood of .

(3) a ***uniform attractor*** if is neighborhood of .

(4) if every for any , there is a random open set of the form with the following properties:

(ii) ,

(iii) for every , we have

(5) asymptotically stable if it is stable and is an attractor ,

(6) unstable ,if it not stable.

The following theorem provide as the characterization of the stable**.**

**Theorem 3.3.6:**

A random set is stable if and only if

**Proof** Let and suppose if possible is not stable. Then there is ,a sequence } and sequence }with , and .We may assume without loos of generality that has been chosen so small that and hence is compact. Further we may assume that .We can now choose a sequence } ,such that Since is compact, we may assume that .Then clearly ,but .This contradiction show that is stable. Now assume that is stable. Then if every for any , there is a random open set of the form with the following properties:

(ii) ,

(iii) for every , we have

Since for any , for any neighborhood of . For, let , then there exist sequence , and sequence with such that . Since then there is such that for every . Then for every . Then . But, , so . Then . Hence . Thus for any neighborhood of . Hence

as is compact. Now to show that .Let , then for every sequence in and every in with such that . Since is stable, then there is a sequence in . But is compact, so .

The following theorem shows the relation between the uniform attractor

and stable .

**Theorem 3.3.7:**

If and is a uniform attractor ,then is stable .

**Proof** .Since uniform attractor. Then is neighborhood .Thus .Thus in particular ,. Since .Since as . Then and by Theorem(3.3.6) is satble.

**Corollary 3.3.8:**

If and is a uniform attractor, then is asymptotically stable.

**Proof.** The proof follows immediately from above theorem and Def.

**Theorem 3.3.9:**

A set is stable if and only if every component of is stable.

**Proof .** Not that if is compact ,then every component of is compact. further if is positively invariant ,so is every one of its component. Now let where is an index set, and are components of . Let each be stable ,i.e., .Then ,i.e., is stable . Let is stable .Then .since is compact. Then is compact and connected set and Since is component of we have ,then as .Thus is stable.

**Theorem** **3.3.10:**

Let be minimal and asymptotically stable. Then for every is asymptotically stable with .

**Proof** .Since is minimal and asymptotically stable. Then by theorem(3.2.2) , and is of .Thus

.

Then is stable. Now since is of for .Then is of .Thus is of for .Thus is asymptotically stable .

**Chapter four**

**Asymptotic Behaviour of the Uniform Random Dynamical System**

In this chapter we state the definition of uniform random set in a uniform random dynamical system and give some properties of such sets. this chapter consists of six sections . In section 1we definition of uniform random dynamical system and definition of uniform random set and give some properties of such that. In section 2 we study transitive of URDS . In section 3 we study sensitive of URDS .In section 4 we study sequence of maps in a uniform space . In section 5 we study uniformly equicontinuous RDS . In section 6 we prove some properties of proximal and distal of URDS.

**4.1 Uniform Random Set:**

Concepts and results including random sets performed in probabilistic and statistical literature long time ago. The origin of the modern concept of a random set goes as far back as the seminal book by A.N. Kolmogorov [22] where he laid out the foundations of probability theory.

**Definition 4.1.1:**

A uniform random dynamical system URDS on a uniform space over the MDS is a measurable RDS which satisfies in addition the following properties:

(i) For each *,* is uniformly continuous with respect

to the uniformity on *.* (4.1.1)

(ii) For every *,* there exists so that implies  for all; i.e.,

implies (4.1.2)

The cocycle is called uniform cocycle.

**Definition 4.1.2[ 20]:**

If and are cocycles where is a locally compact group *,* then a map of cocycles, or cocycle *map,* is a map satisfies

(i) is measurable for all ;

(ii) is uniform continuous for all ;

(iii) for all .

**Definition 4.1.3:**

Let be a uniform space and be a measurable space.

The collection of set-valued maps is said to be random uniformity if it is satisfy the following conditions:

[RU1] for every ;

[RU2]If , then ,

[RU3] For every , there exists such that ,

[RU4] For every , there exists such that ,

[RU5] If and , then ,

[RU6] If , then .

The members of are called uniform random sets. We say that is closed (open, compact) uniform random set if it is closed (resp. open, compact) in the product (with the product of the uniform topology).

**Proposition 4.1.4:**

[RU3] and [RU4] is equivalent to the following statements

For every , there exists such that .

**Proof.** Suppose that [RU3] and [RU4] aresatisfied. Let . By [RU3], there exists such that .By [RU4], there exists such that . Let , then . Then the condition above holds.

Conversely, suppose that the condition above holds. Let , there exists such that . Then easily, and if , then and . Thus [RU3] and [RU4] hold.

**Definition 4.1.5[20]:**

For and , we define

Thus if is URS, then we define .

**Remarks 4.1.6:**

**(1)** If , then .

**(2)** The set-valued map defined by is uniform random set. In fact it is closed URS.

(3) The empty set and are URS.

(4) The closure of a URS is (closed) URS.

(5) If , then . For, if , then by hypothesis . If , then . Thus . If

**Theorem 4.1.7:**

Let where are compact spaces and . If the set-valued map is uniform random set then so is the set-valued map defined by

for every .

**Proof.** First, we need to show that . Since is uniform isomorphism on a compact space , then it is homeomorphism and consequently it is bimeasurable. Thus is bimeasurable. Therefore . Now, to show

for every .

Let . Since is uniform isomorphism, then there exists such that . Now

.

Thus . Since is surjective, then there exists such that . Since is uniform random set, then

.

Since is bimeasurable, then

.

Thus . Therefore is uniform random set and this complete the proof.

**Theorem 4.1.8:**

If be a sequence of uniform random closed set, then so are and .

**Proof** Let be a uniform space and be a measurable space. If , then

.

This means that is uniform random closed set. To show that is uniform random closed set observe that

,

for every (in fact for any set ) so that

.

**Corollary 4.1.9:**

If and are two uniform random closed sets, then so are and .

**Theorem 4.1.10[ 20]:**

Let be a measurable spaces and , be uniform spaces. If is uniform random set in , and is uniform random set in , then is uniform random set in .

**Theorem 4.1.11[20]**:

Let be a uniform space be a continuous function and be a random variable. If the set is nonempty for all , then it is uniform random closed set.

**4.2 Uniform Transitive :**

In this section we universal the concept of transitive uniform random dynamical system and give some things of such systems .

**Definition 4.2.1:**

A URDS is said to be ***uniform transitive*** at and the point is said to be uniform transitive point under if for every URS , there exists such that

.

**Definition 4.2.2:**

A URDS is said to be uniform transitive if for every URS and , there exists such that

.

**Notation** For we define a subset of by

.

**Remark 4.2.3:**

The URDS is UT if and only if , for every URS's and .

**Definition 4.2.4:**

A uniform dynamical system is said to be ***uniform mixing*** (UM) if for every URS's , there exists such that

.

**Notation 4.2.5:**

For we define a subset of by

.

**Remark 4.2.6:**

The URDS is UM if and only if , URS's and .

**Proposition 4.2.7:**

The following statements are equivalents for every .

(i) is transitive at .

(ii) If is URS's, then .

(iii) If is URS's, then , for some .

**Proof.** (i)(ii): Suppose (i). Let be a URS's, there exists such that . Thus

(ii)(iii): Suppose (ii). Let be a URS's. By hypothesis .

Then there exists such that . Hence

.

i.e. (iii) holds.

(iii)(i): Suppose (iii). By hypothesis there exists such that

.

Then

, i.e., (i) holds.

**Proposition 4.2.8:**

The following statements are equivalent:

(i) is transitive.

(ii) For every URS's, .

**Proof** (i)(ii): Suppose (i). Let and be URS's. By hypothesis there exists such that . Hence

.

Therefore

.

(ii)(i): Suppose (ii). Let and be URS's. Then

.

Then there exists and . Hence there exist and such that . But . Therefore there exists such that and . That is for some ,

, i.e. (i) holds.

**Theorem 4.2.9:**

If the set of all transitive point is non-empty, then is transitive.

**Proof.** Suppose that be the set of all transitive point and . Let . By hypothesis there exists and such that

, and .

Then

and

Therefore

Then ,

where and

and

That is, . Or,

This implies that is transitive.

**Theorem 4.2.10:**

Let . If is transitive then so is .

**Proof**. Suppose that with is transitive. To show that is transitive. Let and be two URS's in . Since is uniform continuous , then by Theorem be two URS's in . By hypothesis there exists such that

.

That is,

Then

Therefore

.

Consequently is uniform transitive.

**Proposition 4.2.11:**

Let be a URDS and . Then the following statements are satisfying

(1) .

(2) If , then and .

**Proof.** (1) If , then there is nothing to be proved. Assume that , then and . So . This implies that .

(2) First, note that since , then and . Therefore and . To show that . Let , then . But

.Then . This means that and consequently . To show that . Let , then

.

Thus

.

This means that and consequently .■

**Corollary 4.2.12**

If a URDS is mixing, then it is transitive.

**Proof .** Let be URS's. Since is mixing .Then there exist such that  .Thus .Then is transitive.

**Definition 4.2.13:**

The URDS is said to be ***attracting*** if

for every URS's and every .

**Proposition 4.2.14:**

Let be a URDS and be URS's . If denotes translation by on *,* then for all we have

**(1)** .

(2) .

**Proof.** Let , then there exists such that . Thus .

.

This means that

.

(2) Let ), then there exists such that . Thus .

.

This means that

.

**Corollary 4.2.15:**

Let be a URDS and be URS's. If is an attracting and denotes translation by on *,* then for all we have

**(1)** .

(2) .

**Proposition 4.2.16:**

Let and be two equivalent URDS via . If and , then

1. .
2. .

**Proof:** First, note that by and.

1. Let

.

This means that .

1. Let

(

.

This means that.■

**Definition 4.2.17:**

Let be a URDS. A uniform closed subset of is called a uniform transitive if for any choice of uniform subset of and uniform random open subset of we have .

**Theorem 4.2.18:**

Let be a URDS be a uniform closed subset of . Then the following are equivalent.

(i) is uniform transitive.

(ii) If is a uniform open subset of and is a uniform random open subset of we have for some .

**Proof** (i) implies (ii): Let be a uniform transitive. Then for any uniform random open subset of and any open subset of , we have . Thus there exists such that

.

Hence ,

.

, where .

(ii) implies (i): Let be any uniform random open subset of and be a uniform open subset of , . By (ii) there exists such that , which implies .

. Consequently ; i.e., is uniform transitive.

**Theorem 4.2.19:**

Let . If is closed and transitive subset of , then is transitive subset of .

**Proof.** Suppose that is transitive subset of . Since is homeomorphism, then is uniform random closed set. Let be a uniform subset of and be a uniform open subset of with . Then is a uniform open subset of and is a uniform open subset of with . By hypothesis ; i.e, there exists such that

Hence . That is

. Then . Thus is transitive subset of .■

**4.3 Sensitivity of URDS:**

In this section we introduce the concept of Sensitive of URDS and study some new properties of such system that analogues to that in deterministic dynamical systems.

**Definition 4.3.1:**

The URDS is said to be ***uniform sensitive*** on , if for all and every neighborhood (with respect to the uniform topology generated by ) of , there exist ,a point , and a URS's with such that

.

Such a uniform random set is then called a ***sensitive random set*** for the uniform dynamical system .

**Examples 4.3.2:**

**(i)** Let be a uniform dynamical system with , then is not uniform sensitive. Note that generates the indiscrete topology on .

**(ii)** Let be a uniform dynamical system with , then is not uniform sensitive. Note that generates the discrete topology on .

**Theorem 4.3.3:**

Let and be two equivariant uniform dynamical systems via . If is sensitive on , then so is .

**Proof.** Suppose that is sensitive on . To show that is sensitive. Let and neighborhood of .Since is uniform conjugacy then is an open set in containing . By hypothesis there exist , and a URS's with such that

.

)

)

where

=) .Since , then .

**Theorem 4.3.4:**

Let *,*  be two uniform random dynamical systemsare sensitive, then so is their product.

**Proof.** Let and let neighborhood of . Then and neighborhood of in and in such that . Since is sensitive then there exists , and a URS with of such that. Similarly, since is sensitive then there exists , and a URS with of such that. Now, let , and we have

Since and then .Thus .

This means that the product of and is sensitive.

**Theorem 4.3.5:**

Let and be two URDS and , be equivariant topologically conjugate via. (, is uniform sensitive, if and only if is uniform sensitive.

**Proof.** Let is uniform sensitive. Let and neighborhood of .Since is bijective. Then there exists such that and this implies that . Put is neighborhood of .Since is uniform sensitive .Then there exist ,a point , and a URS with such that

**(** .

Since .Then , .Since .Then . ,where

**(** .

This implies that

**(** .

**(** .

This mean that is uniform sensitive. The converse also follows analogously.

**4.4 (Sequence of Maps in a Uniform space):**

Letbe a uniform space. For any sequence  of uniform random operator , define map for any by

,

for all and for any.

**Definition 4.4.1:**

Let be a uniform random dynamical system and ,, be a sequence of maps then is said to have uniform sensitive dependence on initial conditions in the iterative way if there exists an URS set such that for any and any neighborhood of , there exists a point and a positive integer such that

,

**Definition 4.4.2:**

Let be a uniform dynamical system and ,, be a sequence of maps then is said to have uniform sensitive dependence on initial conditions in the successive way if there exists an URS such that for any and any neighborhood of , there exists a point and a positive integer ,n such that

**Remark 4.4.3:**

Under trivial action of on , Definition and Definition are coincide.

**Theorem** **4.4.4:**

Let *,*  be two uniform random dynamical systems and *,*  be two sequences of maps on respectively. If there exists an equivariant uniform isomorphism such that and are conjugate, then is uniform sensitive on in the successive) way if and only if is uniform sensitive on .

**Proof.** Suppose has uniform sensitive on in the iterative way. There exists a uniform random set such that for any and any neighborhood of , there exists a point , a positive integer syndetic subset of and uniform set such that

,

Since is uniform continuous therefore there exists a uniform random set such that for any with

, ,

where . Hence for any , we have

. (4.4.1)

Observe that for any and any neighborhood of , is a neighborhood of . We therefore have with and a positive integer such that

,

Now we use (4.4.1) and observe that for all

Since , and is equivariant, we have

and hence

for all and . This establishes has uniform sensitive dependence on initial conditions in the iterative way. The converse statement can be proved similarly. The case of uniform sensitive dependence on initial conditions in the successive way also follows analogusly

**4.5 Uniformly Equicontinuous RDS:**

In this section we study concept in URDS called Equicontinuous URDS

**Notation 4.5.1:**

If  and uniform random set, then we define

**Definition 4.5.2:**

A URDS is said to be ***equicontinuos*** if for every and every URS and there exist an nhd of , andsyndeticsubset of such that

, for every .

**Definition 4.5.3:**

A URDS is said to be ***uniformly equicontinuos*** if for every URS and there exists URS , andsyndeticsubset of such that

, for positive integer for every .

**Theorem 4.5.4:**

A URDS is uniformly equicontinuos if and only if for every URS ,there exist a URS , andsyndeticsubset of such that

implies ,

for every positive integer .

**Proof.** Suppose that is uniformly equicontinuos. Let be URS. By hypothesis there exists URS , andsyndeticsubset of such that

.

Let . Then .

.

.

Let . Since , for every , then , for every so , for every . Therefore

**.**

Conversely, Let be URS. By hypothesis there exists URS , andsyndeticsubset of such that implies

, for every positive integer .

Let , then there exists with such that . By hypothesis

,

or equivalently

. So .

Therefore

.

This complete the proof.

The following theorem provide as the characterization of the uniformly equicontinuos .

**Theorem 4.5.5:**

A URDS is uniformly equicontinuos if and only if for every URS there exists a URS such that

**Proof.** Suppose that is uniformly equicontinuous. Let be a URS. By hypothesis there exists a URS such that

implies **.**

Let

Thus for some . But by assumption

6 or .

This means that

Conversely, let be a URS, then by hypotheses there exists a URS such that .Let . Since , and , then . Thus is uniformly equicontinuous.

**Definition 4.5.6:**

If the collection of all random set generate the uniform topology, then the uniform topology is called uniform random topology**.**

**Theorem 4.5.7:**

If is equicontinuous, then

.

**Proof.** Suppose that is equicontinuous. Let and , we want to show that . So consider a URS and let be an nhd of such that , for every integer .

Since , then is a limit point of . Thus . Then there is so and . Then there is such that . so , then and . Then , where .

**Definition 4.5.8:**

A URDS is said to be ***uniformly almost periodic*** if

: is syndetic and

.

**Notation 4.5.9:**

For , put

.

**Lemma 4.5.10:**

If is any URS in a URDS , then .

**Proof.** Let , then there exists such that . Now if and only if such that

,

if and only if if and only if

if and only if

if and only if

if and only if

,

if and only if

if and only if . Therefore .

**Proposition 4.5.11:**

If is any URS in a URDS and , then if and only if

**Proof**. if and only if if and only if if and only if (by above lemma).

**Proposition 4.5.12:**

If is any URS in a URDS and , then

(1) if and only if such that

.

(2) if and only if such that

**Proof.**(1) Suppose that . Let , then . There exist and such that .Now if and only if and . So

, then

.

Conversely, suppose that such that

.

We have . To prove the converse inclusion. Let , by hypothesis there exists such that

.

if and only if

, where

if and only if

if and only if

if and only if

.

This means that . Thus or . Therefore , so .

(2)Suppose that . Let , then . There exist and such that .by (1) we have

.

.

Conversely, suppose that such that

.

We have . To prove the converse inclusion. Let , by hypothesis there exists such that

.

if and only if

, where

if and only if

if and only if

if and only if

.

This means that . Thus or . Therefore , so .

**Theorem 4.5.13:**

A URDS is uniformly almost periodic if and only if is syndetic subset of T for every .

**Proof.** Suppose that is uniformly almost periodic, then : is syndetic and

.

Then

Then is syndetic. Conversely, suppose that is syndetic, then is syndetic. Now

.

Set

.

Then is syndetic, so . This means that is uniformly almost periodic.

**Definition 4.5.14:**

AURDS is said to be ***totally bounded*** if for every there is a finite subset of such that

**Hypothesis 4.5.15:**

Let, and be topological spaces, let be a continuous mapping and let and be compact subsets of and , respectively. If the topology of generated by a uniformity . Then

,

for all .

We shall prove the following theorem with above hypothesis

**Theorem 4.5.16:**

Let is be totally bounded with be compact Hasudorff space. Then is uniformly equicontinuous if and only if it is uniform almost periodic.

**Proof.** Suppose that is uniformly equicontinuous. To show that it is sufficient to show that for every for every . By hypothesis

, then

, then

, then , but , so . Set , then is compact set. Hence . So is syndetic. By above theorem is uniform almost periodic. Conversely, let . There exists with and by uniform almost periodicity, there is a compact subset such that . Since is continuous, for every , there exists a nbd such that for all :

, for all . (4.5.1)

Now for arbitrary write with and . Then so by (4.5.1) we have

It follows that

for .

**Theorem 4.5.18:**

Let be two uniform random dynamical systems and , are two uniform random operators equiv-ariant topologically conjugate via . If is equicontinuous, then so is .

**Proof.** Suppose that is equicontinuous. Let be a uniform random subset of and . Since is uniform isomorphism, then there exists a uniform random subset of such that

implies . (4.5.2)

Since is equicontinuous, then there exist a uniform set , andsyndeticsubset of such that

implies (4.5.3)

for all . Since is uniform continuous, there exists a uniform subset of such that

implies . (4.5.4)

By (4.5.3) we have

By (4.5.2) we have .

Since , be equivariant topologically conjugate via , then

,.

Thus

This means that is uniformly equicontinuous.

**Definition 4.5.19:[20]**

The URDS be a uniform random dynamical system is said to be ***expansive*** if

, there exists a uniform random set such that

Such is called an expansivity characteristic.

**Theorem 4.5.20:**

Let be a uniform space and . If is uniformly equicontinuous map, then its uniformly expansive.

**Proof.** Suppose that is uniformly equicontinuous. Let with . Let be a non-symmetric uniform random set. By hypothesis there exists a uniform random set asyndeticsubset of such that for every integer we have

implies , for all .

Since is non- symmetric and uniform random set, then

, for all .

This means that is expansive.

**4.6 Proximality and Distality of URDS:**

In this section we shall introduce the notation of proximality and distality in URDS case .

**Definition 4.6.1:**

A pair of points is said to be ***proximal*** (and the points and in are said to be proximal to each other) whenever for every URS there exists such that

.

The cocycle (and also the URDS) is called proximal whenever all pairs of points are proximal.

**Definition 4.6.2:**

A pair of points is said to be distal (and the points and in are said to be distal to each other) whenever or is not proximal pair. So if then is distal pair if there exists URS such that for all

.

(here usually depends on ). The cocycle (and also the URDS) is called distal whenever all pairs of points are distal.

**Notation 4.6.3:**

Let be a URDS and let (. Set.

**Proposition 4.6.4:**

A pair of points is proximal if and only if for all URS such that

**Proof** Let be a proximal and be a URS Then there exists such that

. If and only if

**Proposition 4.6.5:**

A pair of points is proximal if and only if

for all URS .

**Proof** Let and be a URS. Then

, hence

.

, for some

, this means that

. Consequently

.Then is a proximal. Conversely, suppose that is a proximal and URS .Then

Since iff

, iff there exists such that

iff

Then

**Theorem 4.6.6:**

Let be a compact and let ( Then

1. ( is a proximal pair then .
2. If such that every URS containing , then ( is a proximal pair.

**Proof** 1.Suppose that ( is a proximal pair. Assume contrary that Then . Since is closed set in for every , then is open set in for every . Since , for every , then , for every and every . Thus for every we have , this means that is an open uniform random set containing . Then . Hence ( is not proximal, but this is a contradiction.

2. Suppose that such that every URS containing . To show that is a proximal. There exists . Then every URS is a neighborhood of and . So , i.e. is a proximal.

**Theorem 4.6.7:**

Let be a compact and let ( Then ( is a distal pair iff or .

**Proof.** Let ( is a distal pair. Then (is not a proximal or ,by (1) or .Now let ( and or .by (1) (is not a proximal or .Hence ( is a distal pair.

**Theorem 4.6.8:**

Let be a compact. (is a proximal iff there are a net in and a point such that for .

**Proof .**Let (is a proximal. Then .Thus ( Then there is net such that .Since .Then , , Then for . Conversely let ( and there are a net in and a point such that for .Thus .Then .Then Hence ( is a proximal.

**Theorem 4.6.9:**

A pair point (is a proximal if and only if : .

**Proof** . Let (is a proximal .Then therer exists .Then there a seqaunes ,) in such that ,),there is such that

for . Then for every nbhd of y ,such that for .Then .Hence , . Conversely let ( and : .Thus there is for .Then and thus .Then for is nbhd of .Thus .Hence (is a proximal.

The following Proposition show the relative between the distal and uniformly equicontinuous.

**Proposition 4.6.10:**

Every uniformly equicontinuous is distal.

**Proof .** Let is uniformly equicontinuous and (.There exists URS U with ( ,and by uniformly equicontinuous ,there exists URS V such that . So ( , impels .

, where .. Hence ( is not a proximal pair. Then ( is distal.

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**المستخلص**

الهدف من هذه الرسالة هو دراسة السلوك المحاذي للنظم الديناميكية العشوائية في بنيتين تبولوجيتين مختلفتين هما الفضاءات التبولوجية و الفضاءات المتسقة, حيث درسنا في الجزء الاول من هذه الرسالة مجموعة الغاية من النمط اوميكا و قدمنا مفهوم الاستطالة الاولى ومجموعات الغاية المستطيلة للنظم الديناميكية العشوائية حيث تم برهان بعض الخواص الجديدة مثل العلاقة بين اغلاق المسار و المسار و مجموعة الغاية من النمط اوميكا بالنسبة للنظم الديناميكية العشوائية, وكذلك برهنا بان الاستطالة الاولى لمجموعة عشوائية مغلقة تحتوي تلك المجموعة وان الاستطالة الاولى لمجموعة تكون مغلقة وغير متغايرة بشرط ان تلك المجموعة غير متغايرة, وكذلك تكون مجموعة مترابطة عندما تكون متراصة بشرط ان فضاء الطور للنظام الديناميكي العشوائي متراص محليا. ومن ثم درسنا مجموعة الغاية المستطيلة للنظم الديناميكية العشوائية و برهنا بعض الخواص الاساسية, حيث برهنا ان مجموعة الغاية العشوائية المستطيلة تكون مغلقة و غير متغايرة. كذلك برهنا العلاقة بين الاستطالة الاولى و مجموعة الغاية المستطيلة و المسار الموجب لمجموعة عشوائية. وكذلك عندما يكون فضاء الطور لنظام ديناميكي عشوائي متراص محليا فتكون العبارات الاتية صحيحة اذا كانت مجموعة الغاية المستطيلة لنظام ديناميكي عشوائي غير خالية و متراصة, فان مجموعة الغاية من النمط اوميكا تكون غير خالية, ان مجموعة الغاية المستطيلة لنظام ديناميكي عشوائي تكون غير خالية و متراصة اذا و فقط اذا كانت الاستطالة الاولى متراصة. اخيرا مجموعة الغاية المستطيلة تكون مترابطة. خصص الجزء الثاني من هذا العمل لدراسة نظرية الاستقرارية للنظم الديناميكية العشوائية عندما يكون فضاء الطور لنظام ديناميكي هو اي فضاء متري. تمت هذه الدراسة من خلال دراسة, التكرار, المجموعات العشوائية الصغرى, مفهوم منطقة الجذب الضعيف, الجذب, والجذب المتسق. خصص الجزء الثالث و الاخير من هذا العمل لدراسة السلوك المحاذي للنظم الديناميكية العشوائية المتسقة, هذا يعني ان فضاء الطور للنظام الديناميكي العشوائي يكون فضاء متسق. خلال هذا الجزء تم تقديم و دراسة مفهوم النظام الديناميكي العشوائية المتسق, المجموعات العشوائية المتسقة. وكذلك قدمنا مفاهيم التعدي المتسق, الحساسية المتسقة, النظام الديتاميكي العشوائي متساوية الاستمرارية بشكل متسق, و النظم الديناميكية العشوائية المتدانية و المتقاصية.

 **جمهورية العراق**

**وزارة التعليم العالي والبحث العلمي  
 جامعة القادسية**

**كلية علوم الحاسبات و تكنولوجيا المعلومات  
 قسم الرياضيات**

واجهات تبولوجيه للسلوكيات المحا ذية

للنظم الديناميكية العشوائية

رسالة مقدمة الى مجلس كلية علوم الحاسبات و تكنولوجيا المعلومات في جامعة القادسية كجزء من متطلبات نيل درجة ماجستير علوم في الرياضيات  
  
من قبل  
**سندس طالب محسن**

بأشراف  
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