The Usage of Disturbance Estimator in the Buck DC-DC Converter

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Abstract- This paper discusses the design of a discrete-time controller for self-tuning that takes into consideration random disturbances that effect on the Buck DC-DC, this controller implements the Minimum Variance Control (MV) with disturbance estimator. The MV method is based on the input /output measurements and holds the output voltage as stable as possible in spite of the parameters variations, while the disturbance estimator increases the accuracy.
Finally, we verify the control method by digital simulation to show the performance and the effectiveness of the controller added to the Buck DC-DC converter.
***Keywords**:* Minimum Variance Control(MVC), Nonlinear systems, State- space, Buck converter, Estimator.

1.Introduction

The consumer demands of having a stable, economic power supply is increasing every day due to the evolution of the electronic devices. These devices implements the buck DC-DC converters as power supplies, which ensure an output voltage that is less than the source(input) voltage. The buck DC-DC converter basic circuit consist of several circuits shown in Fig.1:

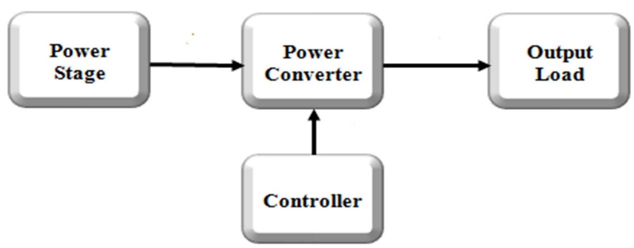

Figure 1. Block diagram of a characteristic power supply .

The most important circuit is the controller circuit which its function is to overcome the changes in the input voltage and load current providing requested overall dynamic response using different methods and strategies. The sliding mode control is one of these methods which simplify design procedures, enhance robustness, reduce the number of components, and prevent chattering. The disadvantage of this method is that the system operates at an infinite switching frequency which leads to the significant losses in coils, as well as in transformer cores, producing electro-magnetic interference. That is why sliding mode control is combined with pulse-width-modulation control techniques, which operates at fixed frequency. Sliding mode control belongs to the class of nonlinear control known as variable structure control that its system dynamics is equivalent to the system dynamics with PWM controller, so that the SM equivalent control u_{eq} is equal to the duty cycle control signal d [1].
Due to the presence of various non-idealities, such as dead zones, hysteresis, time delays, neglected small time constants, a real sliding mode in continuous-time SMC does not differ much from quasi-sliding mode (QSM) in digital sliding mode (DSMC) systems. However, the ease and flexibility in the realization of DSMC using microprocessor technology enable the increasingly frequent use of DSMC in control systems.
The minimum variance (MV) method is one of these methods which can be used to design such control systems because of the advantages such as its implementation in both cases when the system parameters are known or unknown , it can follow the changes of the system parameters, taking into account the disturbances in practical cases, straightforward method and is not time-consuming , it remarkably decreases the deviation of the controlled signal around its desired value . The Minimum Variance control was first introduced by Astrom assuming the linear plant was a minimum phase and later derived the MV controller for processes that could be non-minimum phase [2], [3]. It has been proven in [4] that minimum variance (MV) control corresponds to DSM equivalent control, when the controller design is based on input/output plant models.
In this paper, we present a digital control designed by using the input-output model of buck converter in the form of a discrete-time transfer function, which eliminates the need for additional current sensor. This means

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that the measuring of inductor current is not mandatory. The proposed control is the combination of MV control algorithm and one-step-delayed disturbance estimator [4], [5]. The remainder of this paper is organized as follows: Section 2 gives an introduction to the theory of Buck DC-DC converters. The implementation of the digital controller based on minimum variance with disturbance estimator is summarized in Section 3. Section 4 presents the results of the developed simulation environment, and Section 5 provides a comprehensive summary of the main conclusions.

2. Digital Controller Design

The DC-DC converters are nonlinear systems, which is the consequence of the use of semiconductor devices (switches), parasitic capacitance and inductance, another source of this nonlinearity comes from the control circuits comprise of comparators, PWM, internal and external disturbances [6], Therefore the system cannot be easily solved analytically using Laplace transform see Fig. 2 .

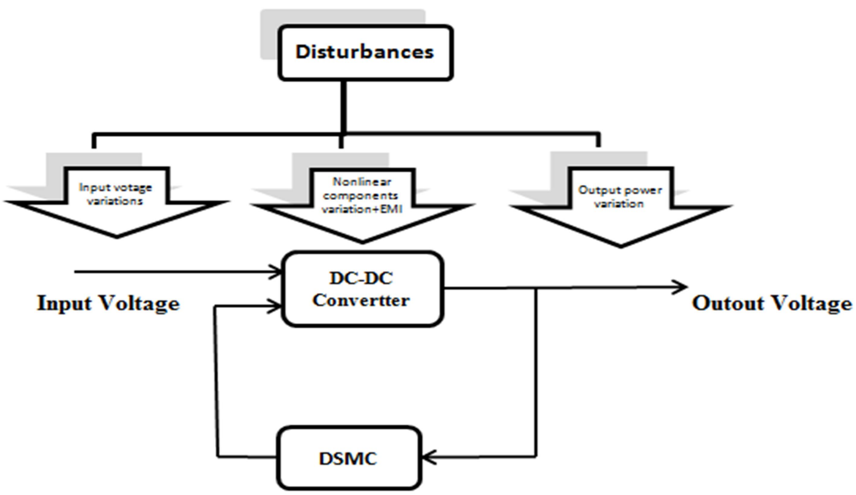

Figure 2. Different types of disturbances which effect on DC-DC converter.

This article consider a digital controller based on minimum variance control with the disturbance estimator. The first step in designing a digital controller is to derive the DC-DC buck converter model as the discrete-time transfer function, which can be obtained from the state-space model given in [7], Using Fig. 3, assuming that the output voltage and its time-derivative as the state coordinates ($x_1=v_o$, $x_2=\mathrm{d}v_o/\mathrm{d}t$), we can conclude the linearized small signal state-space model of buck converter in continuous – time domain :

$$\dot{x}(t)=\begin{bmatrix} 0 & 1 \\ -\dfrac{1}{LC_k} & -\dfrac{1}{R_L C} \end{bmatrix} x(t)+\begin{bmatrix} 0 \\ \dfrac{\beta V_i}{LC} \end{bmatrix}(t) \quad (1)$$

$$y(t)=\begin{bmatrix} \beta & 0 \end{bmatrix} x(t) \quad (2)$$

And its transfer function in continuous-time domain is :

$$W_{buck}(s)=\frac{Y(s)}{U(s)}=\frac{\dfrac{\beta V_i}{LC}}{s^2+\dfrac{1}{R_L C_k}s+\dfrac{1}{LC_k}} \quad (3)$$

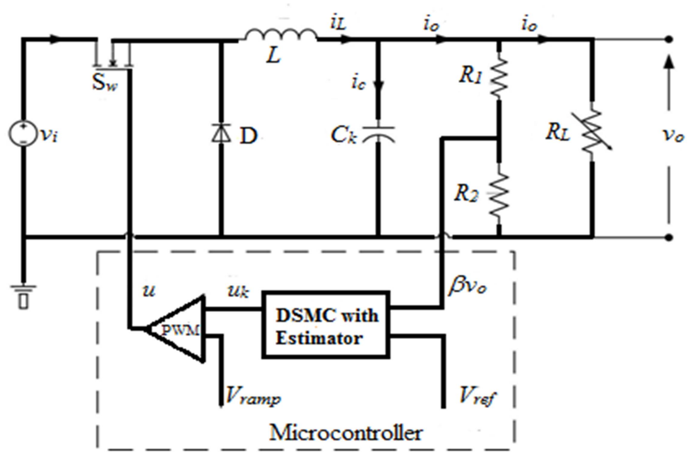

Figure 3. Buck converter with digital controller.

Where : L, C_k and R_L denote an inductance, a capacitance and a load resistance of the converter, i_L, i_c, i_o are inductor, capacitor, and output (load) currents, V_{ref}, v_{in}, v_{out} are reference, input and output voltages respectively, β is the sensor gain, and u represents the signal driving the power switch *Sw*.

Using the advantages of the control systems that possess digital control algorithms enables us in constructing robustness system against the external disturbances and parameters variation, and holding the system error in the steady-state in the desired limits, while the adaptive mechanisms and linear techniques for eliminating disturbances dose not ensure the knowledge about the system dynamics.

In this paper the input-output model is used , where the output of the plant is measured only, taken in concern that the reference signal can be generated in the microcontroller . This method of digital quasi-sliding control is called the digital quasi-sliding control which use multiple selecting periods of the signal.

The linear dynamic system with SISO can be represented as :

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{d}f$$
$$y = \mathbf{C}\mathbf{x} \tag{4}$$

Where : $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_n]^T \in R^n$ which represents state-vector, $u \in R$ the input of the system under control, $f \in R$ external disturbances, $y \in R$ the output of the scheme, $n-$ is the order of the system, and $\mathbf{x}(t) \in R^n$ is a state-space vector, $\overline{u}(t) \in R$ is input, $y(t)$ is output.

Now we write the discrete-time representation of the model (1) in the following form:

$$\mathbf{x}_{k+1} = \mathbf{\Phi}\mathbf{x}_k + \mathbf{\Gamma}u_k + \mathbf{h}_k$$
$$y_k = \mathbf{C}x_k \tag{5}$$

Here :

$$\mathbf{\Phi} = e^{\mathbf{A}T}\ ,\ \mathbf{\Gamma} = \int_0^T e^{\mathbf{A}\lambda}\mathbf{B}d\lambda\ ,$$
$$\mathbf{h}_k = \int_0^T e^{\mathbf{A}\lambda} df(k+1)T - \lambda)d\lambda \tag{6}$$

The mathematical model of the converter system in discrete time domain [4]:

$$y_k = \frac{z^{-1}\mathbf{B}(z^{-1})}{\mathbf{A}(z^{-1})}u_k + \frac{z^{-1}\mathbf{D}(z^{-1})}{\mathbf{A}(z^{-1})}\mathbf{h}_k \tag{7}$$

Where :

$$\mathbf{A}(z^{-1}) = z^{-n}\det(z\mathbf{I} - \mathbf{\Phi}), \tag{8}$$
$$\mathbf{B}(z^{-1}) = z^{-n+1}\mathbf{C}\ \mathrm{adj}(z\mathbf{I} - \mathbf{\Phi})\mathbf{\Gamma}, \tag{9}$$
$$\mathbf{D}(z^{-1}) = z^{-n+1}\mathbf{C}\ \mathrm{adj}(z\mathbf{I} - \mathbf{\Phi}). \tag{10}$$

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where z^{-1} - is the unit delay, i.e., $z^{-1} = e^{-sT}$, s denotes a complex variable, $\bullet(k) = \bullet(kT)$ and :

$$A(z^{-1}) = A_n(z^{-1}) + \Delta A(z^{-1}),$$
$$B(z^{-1}) = B_n(z^{-1}) + \Delta B(z^{-1}). \quad (11)$$

$A_n(z^{-1})$, $B_n(z^{-1})$, $\Delta A(z^{-1})$, $\Delta B(z^{-1})$ represent the polynomials with nominal and perturbed values of Buck converter parameters, respectively.

1. The Minimum Variance Control: The goal of using such method is to establish a control law u_k :

$$u_k = -\frac{1}{E(z^{-1})B(z^{-1})}\left[F(z^{-1})y_k - C(z^{-1})r_{k+1}\right] \quad (12)$$

Where : $E(z^{-1})$ and $F(z^{-1})$ are the polynomials obtained as the solutions of the Diophantine equation:

$$E(z^{-1})A(z^{-1}) + z^{-1}F(z^{-1}) = C(z^{-1}) \quad (13)$$

Here: $C(z^{-1})$ is a polynomial with all zeros inside the unit disk. This control law will minimize the variance of the variable :

$$S_{k+1} = C(z^{-1})(y_{k+1} - r_{k+1}) + Q(z^{-1})u_k \quad (14)$$

r_{k+1} is a known reference input . $Q(z^{-1})$ is chosen as :

$$Q(z^{-1}) = Q_1(1 - z^{-1}) \quad (15)$$

By substituting (11) in (7), taking into account (12), we get S_{k+1} in the form :

$$S_{k+1} = E(z^{-1})\mathbf{D}(z^{-1})\mathbf{h}_k = O(T) \quad (16)$$

In the steady –state (when $k \to \infty$ and $z = 1$), the system output is defined by :

$$y_\infty = r_\infty + \frac{S_\infty}{C(1)} \quad (17)$$

As S_{k+1} is $O(T)$, in accordance with (16), the steady-state accuracy will be within $O(T)/C(1)$ boundaries.

2. The Minimum Variance with Disturbance Estimator : In this case, the control law now is:

$$u_k = -\frac{1}{E(z^{-1})B(z^{-1})}\left[F(z^{-1})y_k - C(z^{-1})r_{k+1} + E(z^{-1})\mathbf{D}(z^{-1})\mathbf{h}_{k-1}\right] \quad (18)$$

To avoid the need to know the system state coordinates, we use the disturbance estimator obtained from (6) in the form of :

$$\mathbf{D}(z^{-1})\mathbf{h}_{k-1} = A(z^{-1})y_k - B(z^{-1})u_{k-1} \quad (19)$$

Substituting (17) in (6), and taking in account (12), will yields :

$$S_{k+1} = E(z^{-1})\mathbf{D}(z^{-1})(\mathbf{h}_k - \mathbf{h}_{k-1}) \ ,$$
$$\mathbf{h}_k - \mathbf{h}_{k-1} = \int_0^T e^{A\lambda} d \int_{kT-\lambda}^{(k+1)T-\lambda} \dot{f}(\sigma) d\sigma d\lambda = O(T^2) \quad (20)$$

Here the precision is $O(T^2)$ if $\dot{f}(t)$ is limited. Therefore, the system accuracy in the steady -state will be within the $O(T^2)/C(1)$ boundaries.

The block diagram of the minimum variance control with disturbance estimator is given in Figure 4.

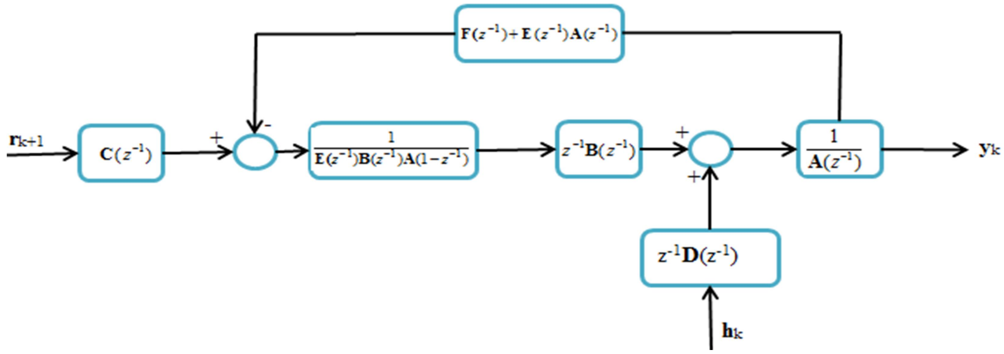

Figure 4. The block diagram of the MVC with disturbance estimator.

In other words , using MV with disturbance estimator gives better accuracy than when applied the same controller without estimators, which means that it will provide zero error tracking the reference input signal in the case of the operation of step disturbances , while in case when the disturbances in the form of tilting signal this error to be $O(T^2)$ limits[8]. The implementation of the disturbance estimator with MV has the same effect such as adding additional integral member in the front of this controller. Knowing that :

$$\mathbf{D}(z^{-1})\mathbf{h}_{\mathbf{k-1}} = A(z^{-1})y_k - B(z^{-1})u_{k-1} \tag{21}$$

Which represents the equation of the disturbance estimator delayed for one sampling time, substituting (**20**) in (17) gives :

$$u_k = -\frac{\left[F(z^{-1})y_k - C(z^{-1})r_{k+1} + E(z^{-1})A(z^{-1})y_k - E(z^{-1})B(z^{-1})u_{k-1}\right]}{E(z^{-1})B(z^{-1})} \tag{22}$$

Multiplying (21) with $E(z^{-1})B(z^{-1})$, and putting the member $E(z^{-1})B(z^{-1})u_{k-1}$ on the right side of the equation, taking in account (14), we have :

$$u_k = -\frac{\left[(F(z^{-1}) + E(z^{-1})A(z^{-1}))y_k - C(z^{-1})r_{k+1}\right]}{(E(z^{-1})B(z^{-1})} \tag{23}$$

This control law doesn't differ from (11) only by the integral member $1/(1-z^{-1})$ and $E(z^{-1})A(z^{-1})y_k$, using an algorithm (22) on the system (6), will give the same results on the system accuracy.

3. Simulation Results

A digital simulation that shows the use of disturbance estimator together with the DSMC based on MVC is tested in order to improve the steady-state accuracy, which means providing zero error tracking of the reference input signal, both in the case of the input voltage and the load resistance variations. The implementation of the MV with disturbance estimator has the same effect as adding additional integral term in control algorithm [7].
The scenario is to change the load resistance of the buck DC-DC converter from R_L = 33 Ω, R_L =16.5 Ω, and R_L =11 Ω, in case of input voltage values : minimum (Vi = 20 V), nominal (Vi = 24 V) and maximum (Vi = 27 V), with and without disturbance estimator . The results are given in the form of output voltage v_o as in Fig. 5 to Fig. 13. the output voltages without disturbance estimator are in the blue color, while the estimator ones are in black color. The dependence of output voltage verses load resistance for the different values of the input voltage can be summarized in Fig. 14, depending on this conclusion one can calculate the load and line regulation properties of the proposed DC-DC buck converter with MV control incorporating disturbance estimator .
The load regulation is :

$$\frac{\Delta V_{out}}{V_{out}(\text{nominal} = 11.9v)} \text{x}100\% \tag{24}$$

where : $\Delta V_{out} = V_{out(33\Omega)} - V_{out(11\Omega)}$

And the line regulation is :

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$$\frac{\Delta V_{out}}{V_{out}(\text{nominal} = 11.9v)} \text{x}100\% \tag{25}$$

where : $\Delta V_{out} = V_{out(27v)} - V_{out(21v)}$

Also the digital sliding mode parameters are calculated as :

$f_0 = 0.4379, f_1 = -0.7$, $A(z^{-1}) = 1 - 1.9802z^{-1} + 0.9802z^{-2}$, $B(z^{-1}) = 1.3515 - 1.3425z^{-1}$, $C(z^{-1}) = 1 - 1.067z^{-1} + 0.2846z^{-2}$

The converter parameter values are given in Table 1.

Table 1. Buck converter parameter values

Description	Parameter	Value
Input voltage	V_i	24 V
Desired output voltage	V_o	12 V
Capacitance.	C_k	1500 µF
Capacitance resistance	r_C	69 mΩ
Inductance.	L	1000 µH
Inductance resistance	r_L	0.12 Ω
PWM frequency	f_{pwm}	7.874 kHz
Sampling period	T	0.5 ms
Minimum load resistance	$R_{L\ min}$	11 Ω
Maximum load resistance	$R_{L\ max}$	33 Ω

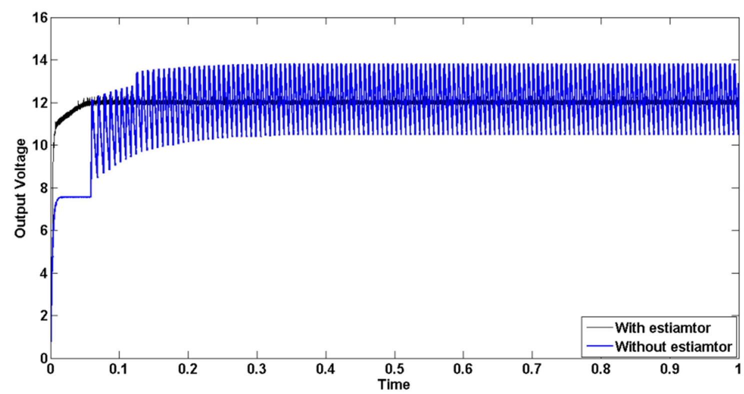

Figure 5. The buck DC-DC converter at 20v , 11Ω

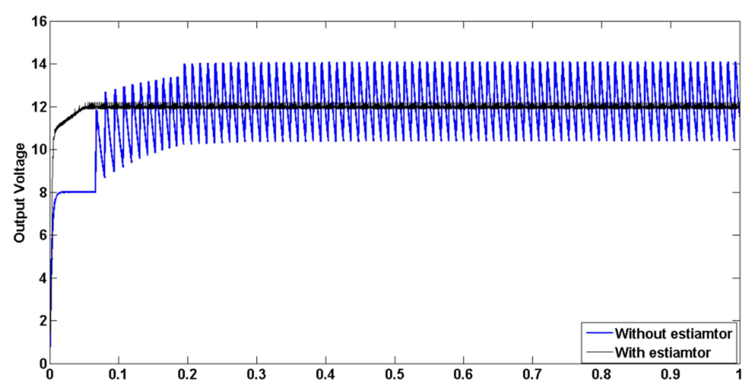

Figure 6. The buck DC-DC converter at 20v , 16.5Ω

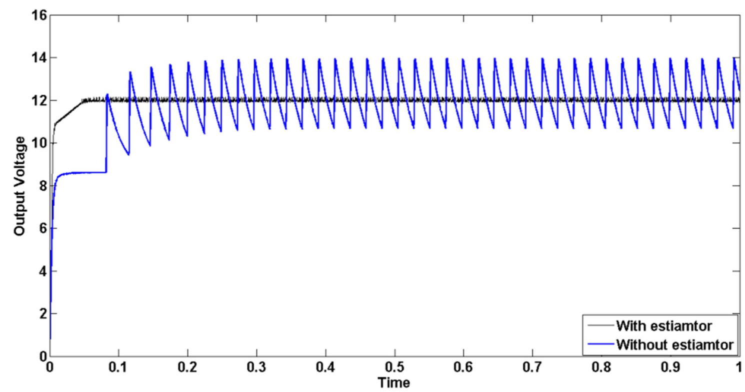

Figure 7. The buck DC-DC converter at 20v , 33Ω

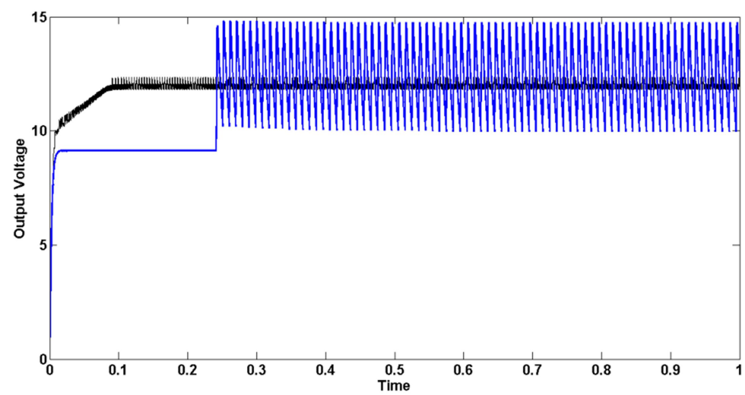

Figure 8. The buck DC-DC converter at 24v , 11Ω

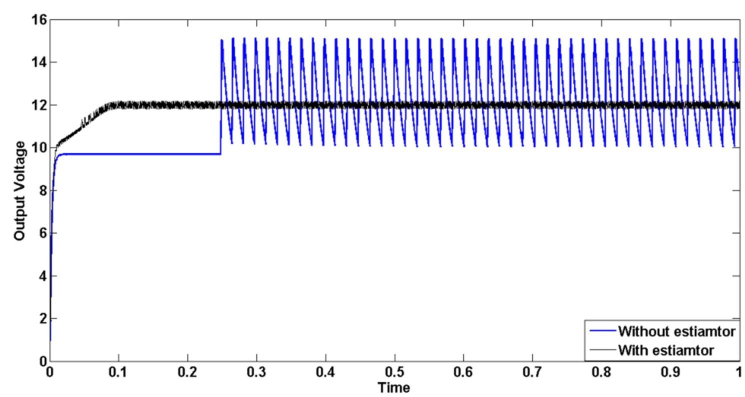

Figure 9. The buck DC-DC converter at 24v , 16.5Ω

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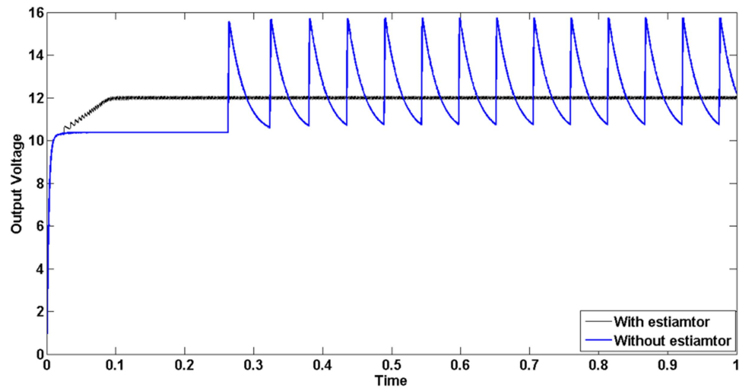


Figure 10. The buck DC-DC converter at 24v , 33Ω

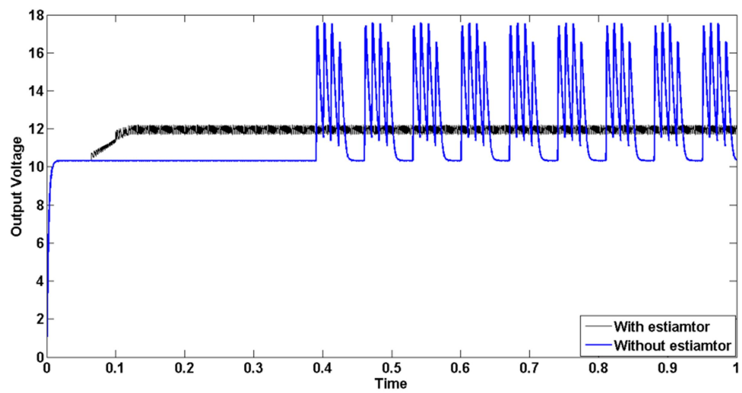


Figure 11. The buck DC-DC converter at 27v , 11Ω

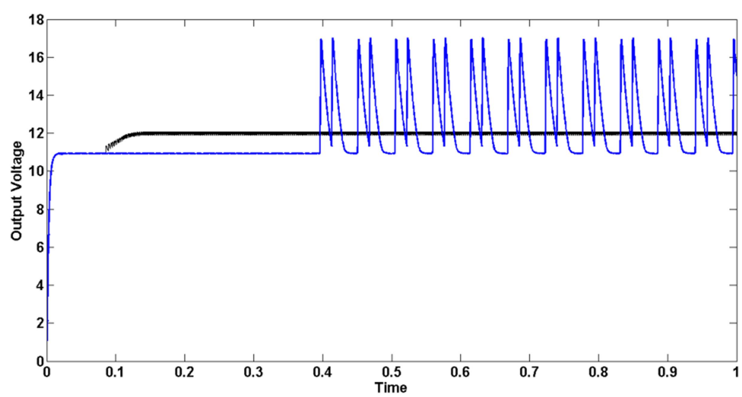


Figure 12. The buck DC-DC converter at 27v , 16.5Ω

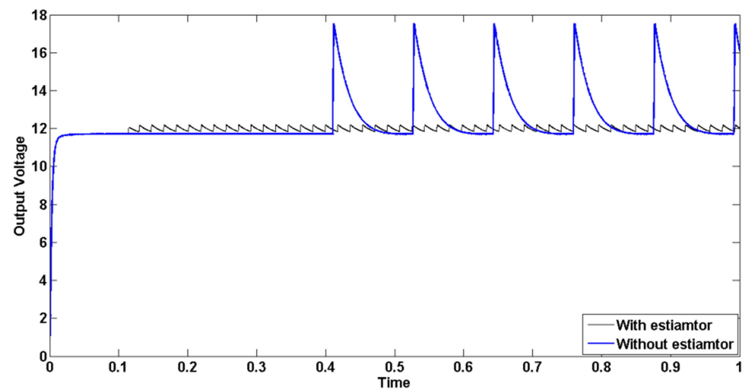
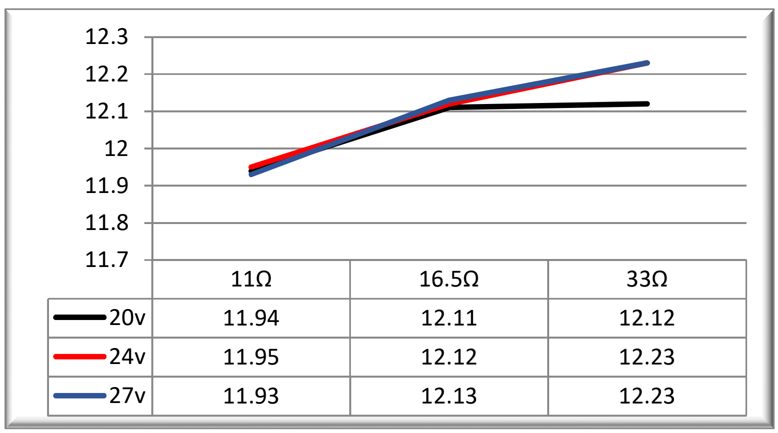
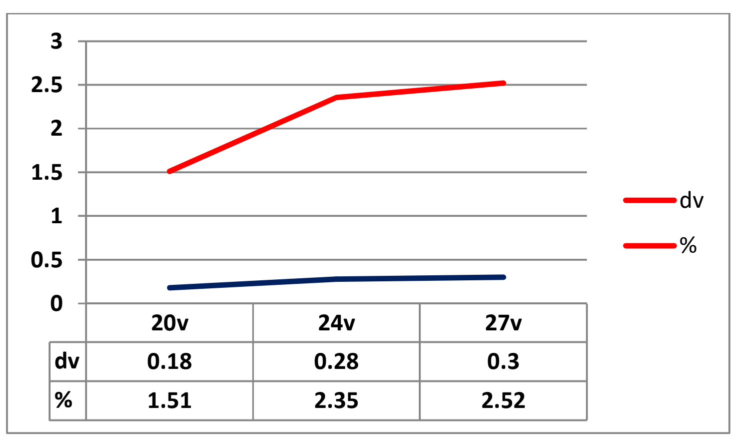


Figure 13. The buck DC-DC converter at 27v , 33Ω



	11Ω	16.5Ω	33Ω
20v	11.94	12.11	12.12
24v	11.95	12.12	12.23
27v	11.93	12.13	12.23

Figure 14. The Plots of Vo against R_L for the buck converter at minimum, nominal and maximum Vi .



	20v	24v	27v
dv	0.18	0.28	0.3
%	1.51	2.35	2.52

Figure 15

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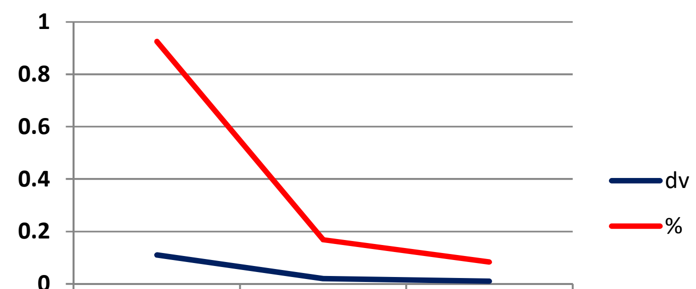

	33Ω	16.5Ω	11Ω
dv	0.11	0.02	0.01
%	0.92	0.17	0.08

Figure 16

4. Conclusion

This paper presents digital controller using minimum variance method with disturbance estimator which increase the accuracy. The whole procedure is based on the input/output measurements in the cases where external disturbances and parameter variations are acting on the closed-loop dynamics, taking into account that the minimum variance components of control algorithms may be considered as the counterpart of the equivalent control. Using MVC with disturbance estimator enable many advantages such as: having the same effect in case of adding integral part in front of such controlling method , reaching an accuracy of $O(T^2)$, suppression of the chattering phenomenon through the discrete-time integrator, theoretically if $T \to 0$, the discrete-time integrator tends to the continuous –time analog, and the control becomes free of chattering [9] .

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