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Quasi-Sliding Mode Based Generalized Minimum Variance Control of DC-DC Boost Converter

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***Abstract* - The paper presents the combination of generalized minimum variance control and discrete time quasi-sliding control applied to control a DC-DC boost converter that provides stable output voltage. The control algorithm is realized by measuring only sensed output voltage and comparing it with the reference voltage in order to achieve zero error signal. The shortcomings of generalized minimum variance control are significantly alleviate, while the implementation of quasi-sliding control based on input/output plant model results in high output voltage accuracy in the presence of parameters perturbations. The proposed control concept is verified by digital simulation.**

***Key words:* boost converter, quasi-sliding mode (QSM) control, generalized minimum variance (GMV) control, pulse width modulation (PWM).**

I. INTRODUCTION

DC–DC converters are widespread applications that convert one level of DC voltage into another, using switching action. They can be used in personal computers, battery charging, DC motor drive and welding machine due to their efficiency and reliable operation. The switching action is achieved by using appropriate controller. The main task of the controller is to drive the main switching device with a duty cycle, such that the dc component of the output voltage is equal to its reference input.

Sliding mode (SM) control is a nonlinear control method that alters the dynamics of a nonlinear system by applying a discontinuous control signal that forces the system to "slide" along a predefined hyper-surface [1]. A system motion with SM control has two main modes: reaching mode and sliding mode. In reaching mode, the system phase trajectory, starting from anywhere in the phase space moves toward a sliding hyper-surface and reaches it in finite time. This is followed by sliding mode in which the phase trajectory asymptotically tends to the origin (equilibrium) of the phase space. The sliding hyper-surface determines the closed loop dynamics of the system [1]-[3]. The main advantage of SM control is its insensitivity to parameter variations, external disturbances and modeling errors [6], [7]. With the advent of digital computers and its widespread use in control systems, considerable efforts have been made in the study of discrete-time quasi-sliding mode (QSM) control techniques [3]-[5].

On the other hand, the application of minimum variance (MV) control techniques in the design of QSM control enables expanding the idea of synthesis of control systems with output feedback to the variable structure control systems, which are mainly based on the use of plant model in state-space [8], [9]. On the other hand, the control methods based on the theory of variable structure control systems significantly increases the accuracy and robustness of MV control in the presence of external disturbances and parameters variations. In addition, MV control allows us to design QSM control using input-output plant model, and has the role of the so-called digital equivalent control u_{eq} replacement. It is obvious that these two control concepts complement each other, which results in giving an effective QSM based MV control.

One of the drawbacks of MV control implementation in designing of QSM control is the shortening of plant zeros. Another drawback is the saturation of the control signal at low values of sampling time T, when control signal assumes a high value. The implementation of generalized minimum variance (GMV) control principles in the synthesis of QSM control, instead of the concept based on MV control, can overcome this problem [9], [12].

This paper considers the design of DC-DC boost converter with QSM based GMV control. The controller design procedure is presented in details and the proposed control algorithm is implemented on the concrete converter. A digital simulation is performed then, and the obtained results are presented.

II. PROBLEM FORMULATION

System modeling is probably the most important phase in any form of system control design and the choice of a circuit model depends on the objectives of the simulation. The state-space description of the converter model in terms of the desired control variables (i.e., voltage and/or current) is the first step in the QSM controller design.

Based on Fig.1, representing DC-DC boost converter with QSM based GMV control, the continuous-time model of converter in state-space is:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\bar{u}(t) \\ y(t) &= Cx(t) \end{aligned}, \qquad (1)$$

where $\mathbf{x}(t) \in R^n$ is a state-space vector, $\bar{u}(t) \in R$ is an input, $y(t)$ is an output and:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{1}{R_L C} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \dfrac{\beta(v_i - v_0)}{LC} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \quad (2)$$

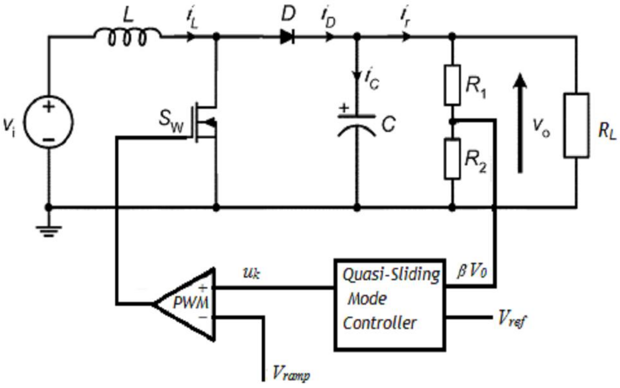

Fig.1.DC-DC boost converter with QSM based GMV controller.

Here C, L, R_L denote the capacitance, inductance, and instantaneous load resistance of the converter, respectively; i_c, i_L, i_r are the instantaneous capacitor, inductor, and load currents, respectively; V_{ref}, V_i, V_o represent the reference, instantaneous input, and instantaneous output voltages, respectively; β denotes sensor gain; $-$ is the inverse logic of u, $u(t)$ is 0 or 1, representing the switching state of power switch S_W.

The transfer function of DC-DC boost converter can be written from (1) and (2) as:

$$W_{bbc}(s) = \frac{Y(s)}{U(s)} = \frac{\dfrac{\beta(v_i - v_0)}{LC}}{s^2 + \dfrac{1}{R_L C} s}, \quad (3)$$

where $Y(s) = X_1(s) = \beta V_0(s)$. Under the assumption that control signal is a constant during the sampling period T, $u(t) = u(kT)$, $kT < t < (k+1)T$, the discrete-time model of DC-DC boost converter in state-space is given by:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{\Phi}\mathbf{x}(k) + \gamma u(k) \\ y(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned}, \quad (4)$$

where:

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & \dfrac{1}{a}(1 - e^{-aT}) \\ 0 & e^{-aT} \end{bmatrix}, \quad (5a)$$

$$\gamma = \begin{bmatrix} \dfrac{b}{a}(T + \dfrac{1}{a}(e^{-aT} - 1)) \\ -\dfrac{b}{a}(e^{-aT} - 1) \end{bmatrix}, \quad (5b)$$

$$a = \frac{1}{R_l C}, b = \frac{\beta(v_i - v_0)}{LC}. \quad (5c)$$

The input-output model of DC-DC boost converter in z-domain can be directly derived from (4) and it is given by:

$$y(k) = \frac{z^{-1} B(z^{-1})}{A(z^{-1})} u(k), \quad (6)$$

$$A(z^{-1}) = a_0 + a_1 z^{-1} + a_2 z^{-2}, \quad (7)$$

$$B(z^{-1}) = b_0 + b_1 z^{-1}, \quad (8)$$

where z^{-1} is the unit delay i.e. $z^{-1} = e^{-pT}$, p - denotes a complex variable and:

$$a_0 = 1, a_1 = -(1 + e^{-aT}), a_2 = e^{-aT}, \quad (9)$$

$$b_0 = \frac{b}{a} T + \frac{b}{a^2}(e^{-aT} - 1), \quad (10)$$

$$b_1 = -\frac{b}{a} T e^{-aT} - 2\frac{b}{a^2} e^{-2aT} + 3\frac{b}{a^2} e^{-aT} - \frac{b}{a^2}. \quad (11)$$

The main aim of design is to maintain the sensed output voltage $y(k) = \beta V_o(k)$ stable, constant and equal to some reference voltage $V_r(k) = V_{ref}$, despite the variations of load resistance R_L.

III. CONTROLLER DESIGN

In order to achieve the design task, QSM based GMV control algorithm is used [10], [12] in control of DC-DC boost converter. It is defined by:

$$u(k) = -\frac{F(z^{-1}) y(k) - C(z^{-1}) V_r(k+1) + \dfrac{\alpha T}{1 - z^{-1}} \operatorname{sgn}(s(k))}{E(z^{-1}) B(z^{-1}) + Q(z^{-1})}, \quad (12)$$

under an assumption that the reference input signal in subsequent time points ($V_r(k+1)$, $V_r(k+2)$,…) is known. $s(k)$ represents the switching function given by:

$$s(k+1) = C(z^{-1})\big(y(k+1) - V_r(k+1)\big) + Q(z^{-1}) u(k), \quad (13)$$

with:

$$C(z^{-1}) = c_0 + c_1 z^{-1} + c_2 z^{-2}, \quad (14)$$

$$C(z^{-1}) = (1 - e^{-2\pi f_c T} z^{-1})^2, \quad (15)$$

where f_c is a cut-off frequency. $C(z^{-1})$ is a polynomial with all zeros inside the unit disk of z-plane, while the polynomial $Q(z^{-1})$ must satisfy the following equality in the steady state:

$$Q(1) = 0, \quad (16)$$

The polynomials $E(z^{-1})$ and $F(z^{-1})$ are the solutions of so-called Diophantine equation:

$$C(z^{-1}) = E(z^{-1}) A(z^{-1}) + z^{-1} F(z^{-1}), \quad (17)$$

and in our case:

$$E(z^{-1}) = e_0 = c_0 / a_0 \ , \tag{18}$$

$$F(z^{-1}) = f_0 + f_1 z^{-1}, \ f_0 = c_1 - e_0 a_1, \ f_1 = c_2 - e_0 a_2 \ , \tag{19}$$

Notice that $s(k)$=0 is an equation of sliding hyper-surface in general case. Substituting (12) in (6), taking into account (13) and (17), one gets:

$$s(k+1) = s(k) - \alpha T \operatorname{sgn}(s(k)) \ , \tag{20}$$

that determines the dynamics of switching function. If $\alpha > 0$, a quasi-sliding motion is established in Δ-vicinity of $s(k)$=0, i.e. $|s(k)| \leq \Delta$ is always satisfied, where Δ is a function of αT.

When QSM is reached, the system over all stability depends on the roots of equation:

$$B(z^1)C(z^1) + A(z^{-1})Q(z^{-1}) = 0 \ , \tag{21}$$

which have to be inside the unit disk in the z-plane, whereas the pairs ($B(z^{-1})$, $Q(z^{-1})$),($C(z^{-1})$, $A(z^{-1})$) and ($C(z^{-1})$, $Q(z^{-1})$) do not have a common zero outside this disk.

IV. SIMULATION RESULTS

The values of DC-DC boost converter parameters are given in Table I. A digital simulation was performed to validate the use of QSM based GMV control in design of this converter. The load changes are also applied to test the functionalities of such system. The following controller parameters were set: T=50µs, α=10, β=0.166 and V_{ref}=8V.

The figures, given below, show the sensed output voltage waveform with the variation of load resistance R_L: Fig. 2 with load resistance equal to 24 Ω, Fig. 3 with 240 Ω and Fig. 4 with 132 Ω. We notice that when R_L has its maximum value 240 Ω, the proposed QSM voltage controller provides robust boost converter output voltage response as in the case of nominal load. Unfortunately, extreme minimal value of R_L (24 Ω), corresponding to the maximal load, causes significant output oscillation around the reference input, but still gives the desired output value.

TABLE I

Boost Converter Parameter Values

Input Voltage	V_i	24V
Capacitance	C	230µF
Inductance	L	300µH
Switching frequency	f_s	200KHz
Inductor resistance	r_L	0.14Ω
Capacitance ESR	r_C	69mΩ
Average Load resistance	R_L	132.5Ω
Minimum Load resistance	R_{L_min}	24Ω
Desired Output Voltage	V_o	48V
Maximum Load resistance	R_{L_max}	240Ω

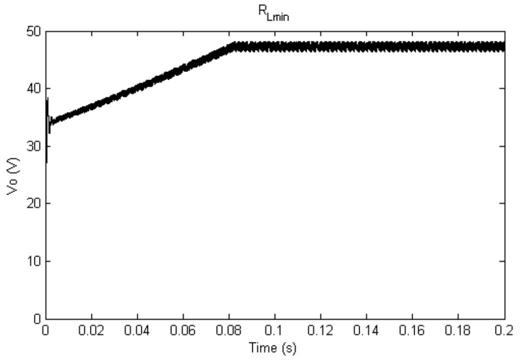

Fig.2. Sensed voltage at the output with load resistance R_{Lmin}

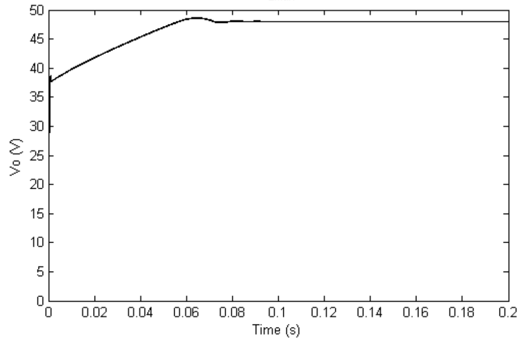

Fig3. Sensed voltage at the output with load resistance R_{Lmax}

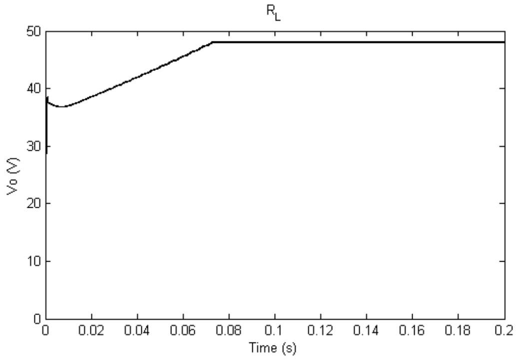

Fig.4. Sensed Voltage at the output with load resistance R_L

V. CONCLUSION

The proposed control strategy is based on measuring the output voltage signal and its comparison with the reference voltage to adopt zero error signal. Using the discrete-time quasi-sliding mode controller to control DC-DC boost converter has proved to be adequate for digitally controlled power converter and for getting the desired system requirements during parameter variations, gaining high output voltage accuracy in the presence of parameters perturbations. Another goal is achieved by filtering the switching control component using the discrete-time integrator which reduces the chattering phenomenon. In addition to significant changes in the parameters of the system under control, the digital quasi-sliding mode control using generalized minimum variance method, based on the input-output signal measurements, enables a high accuracy of the system under the condition that the range of the change of these parameters is known in advance.

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