Influence of the Operating Conditions of Two-Degree-of-Freedom Planetary Gear Trains on Tooth Friction Losses

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In a planetary gear train (PGT), the power loss by tooth friction is a function of the potential power developed within the gear train elements rather than that being transmitted through it. In the present work, we focus on the operating conditions of two-degree-of-freedom (two-DOF) PGTs. Any operating condition induces its own internal power flow pattern; this implies that tooth friction loss depends on the mechanism of power loss developed in the gearing that differs from one case to another over the entire range of operating conditions. The approach adopted in this paper stems from a unification of the kinematics and tooth friction losses of PGTs and is based on potential powers and power ratios. The range of applicability of the power relations is investigated and clearly defined, and tooth friction loss formulas obtained by their use are tabulated. A short comparison with formulas currently available in the literature is also made. The simplicity of the proposed method for analyzing two-input or two-output planetary gear trains is helpful in the design, optimization, and control of hybrid transmissions. It assists particularly in choosing correctly the appropriate operating conditions to the involved application. [DOI: 10.1115/1.4039452]

Introduction

Most of the one-degree-of-freedom (DOF) planetary gear trains (PGTs) are suitable for use in aerospace, automobile, and renewable energy technologies where smaller size and higher reduction ratios are required. Other applications include vehicle power transmissions [1], winches [2], capstans [3], and aircraft hoists and actuators [4,5]. Two-degree-of-freedom PGTs are gaining increasing interest in hybrid transmissions [6–9], turbine-generator control [10], and windmills [11]. In the present work, we shall concentrate only on two-DOF PGTs.

When the two-DOF PGT is operating as a differential, with power flow at all three members, many different combinations are possible depending on whether the member is an input or an output link. One phenomenon in multi-input multi-output PGTs is that for any given combination of input and output members, the system can operate only under certain operating conditions. This phenomenon was studied for single gear pair (GP) entities in Ref. [12].

In the design of a transmission mechanism, mechanical efficiency is one of the most important consideration factors. The mechanical efficiency depends primarily on the losses due to tooth friction in gear pairs. It is called tooth friction loss or mesh loss. In this study, the tooth friction losses due to the gear tooth meshing friction are the only consideration. Tooth friction loss (l) is equal to the carrier-potential power in a gear pair (P_x^c) multiplied by the tooth friction loss coefficient ($\xi = \eta_c - 1$). That is

$$l = (\eta_c - 1)P_x^c \tag{1}$$

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Macmillan was the first to observe that the torques acting on the links and power losses are independent of the observer's motion. The following quote is taken from Ref. [13]: "…our analysis is based upon an important principle relating to torques and the power lost in friction; this is the fact that magnitudes of the torques acting upon the various members of the gear are quite independent of the motion of the observer who measures them. In addition, the power lost, being determined solely by the internal torques and the relative motions of the wheels within the gear, is also independent of the observer's motion."

Equation (1) is valid whether the carrier is stationary or not [14–16]. Hence, torque or power distribution has a significant effect on the tooth friction loss of a PGT. A few significant articles related to torque and power flow analyses can be found in Refs. [17–23]. Various approaches for the derivation of mechanical efficiency in PGTs can be found in Refs. [24–36]. In particular, several formulas for the estimation of tooth friction loss due to tooth friction can be found in Refs. [37–42]. More complicated models of tooth friction loss were proposed by Chiu [43] and Dorey and McCandlish [44]. The effect of elastohydrodynamic lubrication is considered by Chiu [43], while both load-dependent and speed-dependent losses according to the variant quantities of the input power and speeds are considered by Dorey and McCandlish [44]. In this work, a simple loss factor is used to estimate the tooth friction loss. It is assumed that the loss factor is not velocity or load dependent.

Of particular interest in this work are two-DOF PGTs which are fundamental trains in many applications [7–11,45–51]. A review of the existing efficiency formulas for two-DOF systems is found in Ref. [52]. According to Pennestrì and Valentini [52], all of the nonisomorphic cases were considered by Maggiore [45] and the formulas added by Monastero [48] may only ease the computations. With reference to Fig. 1, it will be seen later that the work of Monastero [48] completes that of Maggiore [46].

With regard to the efficiency formulas of a simplified model of a two-DOF system, Chen and coworkers [16] obtained similar analytical relations deduced by Maggiore [46] and Monastero [48] but by means of a different approach. A recent study on the complete system [53] showed that the mechanical efficiency may drop to a value lower than 0.33, which is usually much lower than that of a simple gear train. They attributed the efficiency jump, when the input torque is increased to different local loss factors induced at gear meshes with different loads. The efficiency of gear trains determined using graph and screw theories was presented by Laus et al. [54].

Figure 1 shows a PGT in which there are three links where power can input or output. In this gear train, there are two external GPs; GP1 containing gears 1 and 3 and GP2 containing gears 4 and 3′.

In such a PGT, many different combinations of power flow can be obtained by attaching different links to an input power source

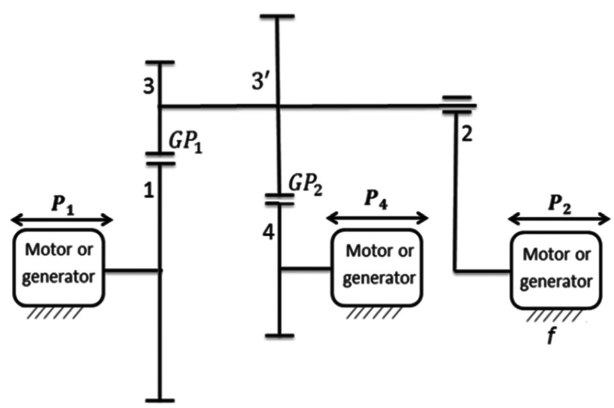

Fig. 1 The two-DOF PGT

or to an output power source. There are two main groups: the first includes the cases where the gear train operates with two inputs and one output, while the second contains the cases where the gear train operates with one input and two outputs.

The potential power in a gear pair can be expressed as the product of the torque acting on gear x by gear y and the angular velocity of gear x with reference to link j (link j is any link in the PGT, not limited to the planet-carrier as it is usually defined in the previous literature)

$$P_x^j = T_x(\omega_x - \omega_j) \tag{2}$$

In the carrier-moving reference frame, this power is called the carrier-potential power [12]. It is known as the virtual power in Ref. [14] or the latent power in Ref. [27]. The concept of potential power [12] is a generalization of the latent power to all of the kinematic inversions of a PGT.

The efficiency of planetary gear trains is frame dependent and can be written in terms of the potential power that can be developed in the gearing when link j is relatively fixed (will be called here as link-j-potential power) as

$$\eta_{j(x-y)} = -\frac{P_y^j}{P_x^j} \tag{3}$$

where j is any link in the PGT, and x and y are, respectively, the input and output links in the link-j-moving reference frame. A detailed explanation can be found in Ref. [12].

By looking at the gear train in a rotating reference frame attached to the planet carrier, the PGT reduces to a simple gear train. This allows the use of Eq. (1) to calculate tooth friction losses with the potential power (P_x^c) in place of the actual power (P).

It is shown in Ref. [12] that these tooth friction losses in the carrier-rotating reference frames are the same as in a grounded or inertial reference frame.

A comprehensive review of existing formulas for this system can be found in Refs. [16] and [52]. In the present work, we focus on the operating conditions of the two-degree-of-freedom PGTs to realize satisfactory train operation. After these operating conditions are specified and in order to complete the picture, the tooth friction loss for the two-DOF PGTs is derived over the entire range of operating conditions. The range of applicability of the power relations is investigated and clearly defined, and tooth friction loss formulas obtained by their use are tabulated.

In what follows, we will first review a general method to solve the kinematic equations of any PGT [55]. Then, the limitations on angular velocities and torques on the links of a PGT are analyzed in order to determine the input(s) and output(s) links for each case. Finally, the tooth friction loss formulas of the PGT under the specified operating conditions are derived.

Velocity Ratio Analysis

The planet gear ratio $N_{p,q}$ can be written for an external gear pair p and q as

$$N_{p,q} = -\frac{Z_p}{Z_q} = \frac{\omega_q - \omega_c}{\omega_p - \omega_c} \tag{4}$$

where ω_q, ω_p, and ω_c denote the speeds of gears p and q, and their carrier c, respectively, with the minus sign corresponding to the rotation of the external gears in the opposite direction.

The velocity ratio is used to denote the velocity ratio between two links of the PGT with respect to the third. Let the symbol $R_{w,u}^v$ denote the velocity ratio between links u and w with respect to link v. Then, the velocity ratio $R_{w,u}^v$ is

$$R_{w,u}^v = \frac{\omega_w - \omega_v}{\omega_u - \omega_v} = \frac{N_{p,w} - N_{p,v}}{N_{p,u} - N_{p,v}} \tag{5}$$

Potential-Power Ratio. The potential power ratio is defined as the ratio between the potential power that can be developed in link u relative to link j and the actual power transmitted through link u

$$\frac{P_u^j}{P_u} = \frac{T_u(\omega_u - \omega_j)}{T_u\omega_u} = \frac{(\omega_u - \omega_j)}{\omega_u - \omega_f} = R_{j,f}^u \tag{6}$$

where f is a hypothetical stationary link with zero angular velocity ($\omega_f = 0$).

Power-Flow Ratios in Planetary Gear Entity. Let $\eta_{c(x-y)}$ denote the efficiency associated with a planetary gear entity in which x and y denote, respectively, the input and output links in a moving reference frame in which the carrier c appears relatively fixed, then from Eq. (3), $\eta_{c(x-y)}$ can be written as

$$\eta_{c(x-y)} = -\frac{T_y(\omega_y - \omega_c)}{T_x(\omega_x - \omega_c)} = -\frac{T_y}{T_x}R_{y,x}^c \tag{7}$$

By multiplying Eq. (7) by ω_y/ω_x and using the facts that $P = T \cdot \omega$ and $R_{x,y}^f = \omega_x/\omega_y$, the power ratio can be written as

$$\frac{P_y}{P_x} = -\frac{\eta_{c(x-y)}}{R_{y,x}^c R_{x,y}^f} \tag{8}$$

From Eq. (7)

$$\frac{T_y}{T_x} = -\frac{\eta_{c(x-y)}}{R_{y,x}^c} \tag{9}$$

and since $T_x + T_y + T_c = 0$, then

$$1 + \frac{T_y}{T_x} + \frac{T_c}{T_x} = 0 \tag{10}$$

Substituting Eq. (9) into Eq. (10) and simplifying, we get

$$\frac{T_c}{T_x} = \frac{\eta_{c(x-y)}}{R_{y,x}^c} - 1 \tag{11}$$

Similarly, multiplying Eq. (11) by ω_c/ω_x and using the fact that $R_{c,x}^f = \omega_c/\omega_x$, and simplifying, we get

$$\frac{P_c}{P_x} = \left(\frac{\eta_{c(x-y)}}{R_{y,x}^c} - 1\right)R_{c,x}^f \tag{12}$$

Tooth Friction Losses and Total Efficiency. The tooth friction loss L can be estimated from Eqs. (1) and (6) as follows:

$$l = \left(\eta_{c(x-y)} - 1\right)R_{c,f}^x P_x \tag{13}$$

For ordinary gear trains, the velocity ratio $R_{c,f}^x = 1$, and hence, the actual power flowing in the train is the same as the potential power ($P_x = P_x^c$).

The efficiency of the planetary gear train can be written in terms of the actual powers as

$$\eta_p = -\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{1}{1 + \frac{l}{P_{\text{out}}}} = 1 + \frac{l}{P_{\text{in}}} \tag{14}$$

Determination of the Operating Conditions for a Planetary Gear Train

A two-DOF PGT can provide several operating modes depending on the assignment of the input and output links. These various operating modes need to be estimated in order to arrive at a proper design.

Given a PGT, we ask ourselves the following question: What are the range restrictions of torques and angular velocities which play a vital role in assigning a function to a link in a two-DOF PGT?

To answer this question, it will be necessary to develop a methodology for arranging the angular speeds of the links of the PGT in a sequence and to identify torques directions. The two-DOF PGT shown in Fig. 1 will be used to demonstrate and apply the approaches.

Arranging the Angular Speeds in a Sequence. By expressing the velocity ratio in terms of the gear ratios, the range of the angular speeds can be estimated without knowing the exact dimension of a mechanism. We now propose a step-by-step procedure for arranging the angular speeds of the links of the PGT shown in Fig. 1 in a sequence:

Step 1. Select any velocity ratio for the PGT and apply Eq. (5). Let us select $R^2_{4,1}$, then

$$R^2_{4,1} = \frac{N_{p,4}}{N_{p,1}} = \frac{|N_{p,4}|}{|N_{p,1}|} = \frac{(\omega_4 - \omega_2)}{(\omega_1 - \omega_2)} \tag{15}$$

Step 2. Specify the gear sizes, $N_{p,q}{}'$s, and estimate the value of $R^2_{4,1}$. From Eq. (4), we can write $N_{p,1} = -Z_3/Z_1$ and $N_{p,4} = -Z_{3'}/Z_4$. Since both $N_{p,1}$ and $N_{p,4}$ are negative, then both of $|N_{p,1}|$ and $|N_{p,4}|$ are positive. There are two cases:

(1) $\frac{|N_{p,4}|}{|N_{p,1}|} > 1$ gives $R^2_{4,1} = \frac{(\omega_4 - \omega_2)}{(\omega_1 - \omega_2)} > 1$

(2) $0 < \frac{|N_{p,4}|}{|N_{p,1}|} < 1$ gives $0 < R^2_{4,1} = \frac{(\omega_4 - \omega_2)}{(\omega_1 - \omega_2)} < 1$

Step 3. Each result obtained from step 2 can be further subdivided into two subcases which can be determined by the signs of the numerator and the denominator.

1.1 Both of the numerator and the denominator have a positive sign, then from the first case of step 2, we get $(\omega_4 - \omega_2) > 0$, $(\omega_1 - \omega_2) > 0$, and $(\omega_4 - \omega_2) > (\omega_1 - \omega_2)$. Therefore, from thesethreeinequalities, the angular velocities can be related to each other by the following inequality:

$$\omega_4 > \omega_1 > \omega_2$$

It is not difficult to show that this inequality includes inclusively the following four ranges:

1.1.1 $\omega_4 > \omega_1 > \omega_2 > 0$
1.1.2 $\omega_4 > \omega_1 > 0 > \omega_2$
1.1.3 $\omega_4 > 0 > \omega_1 > \omega_2$
1.1.4 $0 > \omega_4 > \omega_1 > \omega_2$

1.2 Both of the numerator and the denominator have a negative sign, then we get

$$\omega_2 > \omega_1 > \omega_4$$

It is not difficult to show that this inequality includes inclusively the following four ranges:

1.2.1 $\omega_2 > \omega_1 > \omega_4 > 0$
1.2.2 $\omega_2 > \omega_1 > 0 > \omega_4$
1.2.3 $\omega_2 > 0 > \omega_1 > \omega_4$
1.2.4 $0 > \omega_2 > \omega_1 > \omega_4$

Step 4. Similarly, eight velocity ranges are obtained for the case when $0 < |N_{p,4}|/|N_{p,1}| < 1$.

In detail, there are two cases, they are as follows:

2.1. Both of the numerator and the denominator have a positive sign, then from the second case of step 2, we get $(\omega_4 - \omega_2) > 0$, $(\omega_1 - \omega_2) > 0$, and $(\omega_1 - \omega_2) > (\omega_4 - \omega_2)$. Therefore, from thesethreeinequalities, the angular velocities can be related to each other by the following inequality:

$$\omega_1 > \omega_4 > \omega_2$$

It is not difficult to show that this inequality includes inclusively the following four ranges:

2.1.1 $\omega_1 > \omega_4 > \omega_2 > 0$
2.1.2 $\omega_1 > \omega_4 > 0 > \omega_2$
2.1.3 $\omega_1 > 0 > \omega_4 > \omega_2$
2.1.4 $0 > \omega_1 > \omega_4 > \omega_2$

2.2. Both of the numerator and the denominator have negative sign, then from the second case of step 2, we get $0 > (\omega_4 - \omega_2)$, $0 > (\omega_1 - \omega_2)$, and $(\omega_4 - \omega_2) > (\omega_1 - \omega_2)$. Therefore, from thesethreeinequalities, the angular velocities can be related to each other by the following inequality:

$$\omega_2 > \omega_4 > \omega_1$$

It is not difficult to show that this inequality includes inclusively the following four ranges:

2.2.1 $\omega_2 > \omega_4 > \omega_1 > 0$
2.2.2 $\omega_2 > \omega_4 > 0 > \omega_1$
2.2.3 $\omega_2 > 0 > \omega_4 > \omega_1$
2.2.4 $0 > \omega_2 > \omega_4 > \omega_1$

A total of 16 velocity ranges is reached.

However, it should be noted that the selection of the velocity ratio to estimate the velocity ranges is arbitrary.

Identification of Torques Directions. In this section, we will show how the torque ratios can be estimated in terms of the potential power that can be developed in the gearing when link j is relatively fixed. We now propose a step-by-step procedure for estimating the torques directions of the links of the PGT shown in Fig. 1:

Step 1. Back to the previous example, there is a case where link 1 may be an input link in the carrier-moving reference frame, then Eq. (3) can be written as

$$\frac{T_4}{T_1} = -\frac{\eta_{2(1-4)}}{R^2_{4,1}} \tag{16}$$

Also, there is a case where link 1 may be an input link in the sun-four-moving reference frame, then Eq. (3) can be written as

$$\frac{T_2}{T_1} = -\frac{\eta_{4(1-2)}}{R^4_{2,1}} \tag{17}$$

Step 2. As before, there are two cases:

(1) $\frac{|N_{p,4}|}{|N_{p,1}|} > 1$ gives $R^2_{4,1} > 0$ and $R^4_{2,1} > 0$

(2) $0 < \frac{|N_{p,4}|}{|N_{p,1}|} < 1$ gives $R^2_{4,1} > 0$ and $R^4_{2,1} < 0$

Step 3. Case when $|N_{p,4}|/|N_{p,1}| > 1$.

According to Eq. (16), and since $0 < \eta_{2(1-4)} < 1$ and $R^2_{4,1} > 0$, then T_1 and T_4 must have opposite signs. According to Eq. (17), and since $0 < \eta_{4(1-2)} < 1$ and $R^4_{2,1} > 0$, then T_1 and T_2 must have opposite signs. The torque ratios T_4/T_1 and T_2/T_1 become negative only when the torques have opposite signs. There are two cases

$$\left.\begin{array}{ll} 1. & T_1 > 0, T_2 < 0, \text{ and } T_4 < 0 \\ 2. & T_1 < 0, T_2 > 0, \text{ and } T_4 > 0 \end{array}\right\} \quad (18)$$

Step 4. Case when $0 < |N_{p,4}|/|N_{p,1}| < 1$.

According to Eq. (16), and since $0 < \eta_{2(1-4)} < 1$ and $R^2_{4,1} > 0$, then T_1 and T_4 must have opposite signs. According to Eq. (17), and since $0 < \eta_{4(1-2)} < 1$ and $R^4_{2,1} < 0$, then T_1 and T_2 must have the same sign. There are two cases

$$\left.\begin{array}{ll} 1. & T_1 > 0, T_2 > 0, \text{ and } T_4 < 0 \\ 2. & T_1 < 0, T_2 < 0, \text{ and } T_4 > 0 \end{array}\right\} \quad (19)$$

Estimating Proper Operating Conditions. A two-DOF PGT with power flow at all members can provide several assignments to its various links as inputs or outputs, depending on the torques and angular speeds. The relationship between power, torque, and angular velocity is

$$P = T \times \omega \quad (20)$$

Since the torques can assume two possibilities for each set of gear ratios, while the angular speeds assume eight ranges, 32 power-flow combinations can be derived from Eq. (20), some of them are similar.

These cases will not be detailed, so as not to confuse the reader. However, it should be noted that a detailed summary can be found in Table 1 accompanied by power loss formulas. Only one case will be detailed to illustrate how the operating conditions are related to their power loss formulas.

Estimating the Operating Conditions When Links 2 and 4 are Inputs. For example, if the PGT shown in Fig. 1 is to have links 2 and 4 as inputs and link 1 as output, then if the gear ratios are selected such that $(N_{p,4}/N_{p,1}) > 1$, then the velocity ranges must be selected such that they satisfy the following:

$$T_1\omega_1 < 0, T_2\omega_2 > 0 \text{ and } T_4\omega_4 > 0$$

For the first combination of torques from Eq. (18), there are two ranges that meet the condition, they are

1.1.4 $0 > \omega_4 > \omega_1 > \omega_2$ and
1.2.4 $0 > \omega_2 > \omega_1 > \omega_4$

Also for the second combination of torques from Eq. (18), there are two ranges that meet the condition, they are

1.1.1 $\omega_4 > \omega_1 > \omega_2 > 0$ and
1.2.1 $\omega_2 > \omega_1 > \omega_4 > 0$

Any two-DOF PGT will not work properly unless it is operated under the prescribed conditions. Therefore, we must first know the operating condition of a revolving drive before we can decide on a link to be an input or an output.

Tooth Friction Losses

Even for the same two-input PGT and for the same input and output links, we cannot write down the proper efficiency expression unless we know the sequence of the angular velocities. As will be described in the following, the originality of this method consists mainly in the way these expressions are related to their operating conditions.

The method of calculating the loss will be explained to only two cases. They are the cases described in Estimating the Operating Conditions When Links 2 and 4 are Inputs section, where

First case

(1) $(N_{p,4}/N_{p,1}) > 1$
(2) $T_1 > 0, T_2 < 0,$ and $T_4 < 0$
(3) $0 > \omega_4 > \omega_1 > \omega_2$

Since, for the case under consideration, $T_1 > 0$ and $(\omega_1 - \omega_2) > 0$, then $T_1(\omega_1 - \omega_2) > 0$, and link 1 is an input link in the carrier moving reference frame.

The tooth friction losses of GP1 and GP2 are given by Eq. (13) as

$$l = l_1 + l_2 = (\eta_1\eta_2 - 1)R^1_{2,f}P_1 \quad (21)$$

The efficiency of the PGT when links 2 and 4 are input links and link 1 is the output link can be written by substituting the tooth friction loss from Eq. (21) into Eq. (14) to yield

$$\eta_{(2,4-1)} = \frac{1}{1 + (\eta_1\eta_2 - 1)R^1_{2,f}} \quad (22)$$

Also, the total losses can be written as

$l = (\eta_{(2,4-1)} - 1)P_{\text{in}}$, and for $P_{\text{in}} = 1$, it becomes $l = (1/1 + (\eta_1\eta_2 - 1)R^1_{2,f} - 1)$

Second case

(1) $(N_{p,4}/N_{p,1}) > 1$
(2) $T_1 > 0, T_2 < 0,$ and $T_4 < 0$
(3) $0 > \omega_2 > \omega_1 > \omega_4$

Since, for the case under consideration, $T_1 > 0$ and $(\omega_4 - \omega_2) < 0$, then $T_1(\omega_1 - \omega_2) > 0$, and link 4 is an input link in the carrier moving reference frame.

The tooth friction losses of GP1 and GP2 are given by Eq. (13) as

$$l = (\eta_1\eta_2 - 1)R^4_{2,f}P_4 \quad (23)$$

From Eq. (3), the efficiency of the gear train in the carrier-moving reference frame is

$$\eta_{2(4-1)} = \eta_1\eta_2 = -\frac{P^p_1}{P^p_4} = -\frac{R^1_{2,f}P_1}{R^4_{2,f}P_4} \quad (24)$$

From Eq. (24), $R^4_{2,f}P_4 = -R^1_{2,f}P_1/\eta_1\eta_2$. Therefore, Eq. (23) can be rewritten as follows: $l = -(\eta_1\eta_2 - 1)\left(R^1_{2,f}P_1/\eta_1\eta_2\right)$.

By substituting the tooth friction loss l into Eq. (14), the efficiency of the PGT when links 2 and 4 are input links and link 1 is the output link can be written as

$$\eta_{(2,4-1)} = \frac{1}{1 - \dfrac{(\eta_1\eta_2 - 1)R^1_{2,f}}{\eta_1\eta_2}} \quad (25)$$

A similar procedure can be used to derive the equations for friction losses over the complete range of operating conditions of the gear train. Many different combinations of power flow are possible, depending on whether the links are input(s) or output(s) links. All the possible combinations are compiled in Table 1.

There are eight formulas proposed in Ref. [46], applied to exactly the same train discussed here. Monastero [48] added also, in addition to those given in Ref. [46], four formulas to cover the complete range of the working conditions of the PGT. Our solutions match exactly those reported in Refs. [46] and [48]. The present method value lies in the fact that before the formulas can be applied, it is necessary to determine the circumstances in which it is applicable, which are defined by the condition that the torques and angular velocities in which the PGT will operate shall occur. This does not appear to have been previously stated. Despite the apparent equivalence in the formulas presented, the symbolic

Table 1 Friction losses for different operating conditions of the PGT shown in Fig. 1

<table>
<tr><th rowspan="2">Input</th><th rowspan="2">output</th><th colspan="4">Tooth Friction Loss</th></tr>
<tr><th colspan="2">$0 < (N_{p,4}/N_{p,1}) < 1$</th><th colspan="2">$(N_{p,4}/N_{p,1}) > 1$</th></tr>
<tr><td rowspan="2">1,2</td><td rowspan="2">4</td><td>$T_1, T_2 > 0$ and $T_4 < 0$
$\omega_2 > \omega_4 > \omega_1 > 0$
Or $T_4 > 0$ and $T_1, T_2 < 0$
$0 > \omega_1 > \omega_4 > \omega_2$</td><td>$T_1, T_2 > 0$ and $T_4 < 0$
$\omega_1 > \omega_4 > \omega_2 > 0$
Or $T_4 > 0$ and $T_1, T_2 < 0$
$0 > \omega_2 > \omega_4 > \omega_1$</td><td colspan="2">$T_4 > 0$, $T_1 < 0$ and $T_2 > 0$
$\omega_2 > 0 > \omega_1 > \omega_4$
Or $T_4 < 0$, $T_1 > 0$ and $T_2 < 0$
$\omega_4 > \omega_1 > 0 > \omega_2$</td></tr>
<tr><td>$l_{(1,2-4)} = \left[1 + (\eta_1\eta_2 - 1)R^4_{2,f}\right]^{-1} - 1$</td><td colspan="3">$l_{(1,2-4)} = \left[1 - \frac{(\eta_1\eta_2 - 1)R^4_{2,f}}{\eta_1\eta_2}\right]^{-1} - 1$</td></tr>
<tr><td rowspan="2">2,4</td><td rowspan="2">1</td><td colspan="2">$T_4 > 0$, $T_1 < 0$ and $T_2 < 0$
$\omega_1 > \omega_4 > 0 > \omega_2$
Or $T_4 < 0$, $T_1 > 0$ and $T_2 > 0$
$\omega_2 > 0 > \omega_4 > \omega_1$</td><td>$T_4 > 0$, $T_1 < 0$ and $T_2 > 0$
$\omega_4 > \omega_1 > \omega_2 > 0$
Or $T_4 < 0$, $T_1 > 0$ and $T_2 < 0$
$0 > \omega_2 > \omega_1 > \omega_4$</td><td>$T_4 > 0$, $T_1 < 0$ and $T_2 > 0$
$\omega_2 > \omega_1 > \omega_4 > 0_r$
$T_4 < 0$, $T_1 > 0$ and $T_2 < 0$
$0 > \omega_4 > \omega_1 > \omega_2$</td></tr>
<tr><td colspan="3">$l_{(2,4-1)} = \frac{1}{1 - \frac{(\eta_1\eta_2 - 1)R^1_{2,f}}{\eta_1\eta_2}} - 1$</td><td>$l_{(2,4-1)} = \frac{1}{1 + (\eta_1\eta_2 - 1)R^1_{2,f}} - 1$</td></tr>
<tr><td rowspan="2">1,4</td><td rowspan="2">2</td><td colspan="2">$T_4 < 0$, $T_1 > 0$ and $T_2 > 0$
$\omega_1 > 0 > \omega_4 > \omega_2$
Or $T_4 > 0$, $T_1 < 0$ and $T_2 < 0$
$\omega_2 > \omega_4 > 0 > \omega_1$</td><td colspan="2">$T_4 > 0$, $T_1 < 0$ and $T_2 > 0$
$\omega_4 > 0 > \omega_1 > \omega_2$
Or $T_4 < 0$, $T_1 > 0$ and $T_2 < 0$
$\omega_2 > \omega_1 > 0 > \omega_4$</td></tr>
<tr><td colspan="2">$l_{(1,4-2)} = \frac{1}{1 - \frac{(\eta_1\eta_2 - 1)R^2_{1,f}}{\left(\frac{\eta_1\eta_2}{R^2_{4,1}} - 1\right)}} - 1$</td><td colspan="2">$l_{(1,4-2)} = \frac{1}{1 - \frac{(\eta_1\eta_2 - 1)R^2_{4,f}}{\left(\frac{\eta_1\eta_2}{R^2_{1,4}} - 1\right)}} - 1$</td></tr>
<tr><td rowspan="2">4</td><td rowspan="2">1,2</td><td>$T_4 < 0$, $T_1 > 0$ and $T_2 > 0$
$0 > \omega_1 > \omega_4 > \omega_2$
Or $T_1 < 0$, $T_4 > 0$ and $T_2 < 0$
$\omega_2 > \omega_4 > \omega_1 > 0$</td><td>$T_2 < 0$, $T_4 > 0$ and $T_1 < 0$
$\omega_1 > \omega_4 > \omega_2 > 0$
Or $T_2 > 0$, $T_4 < 0$ and $T_1 > 0$
$0 > \omega_2 > \omega_4 > \omega_1$</td><td colspan="2">$T_4 > 0$, $T_1 < 0$ and $T_2 > 0$
$\omega_4 > \omega_1 > 0 > \omega_2$
Or $T_1 > 0$, $T_4 < 0$ and $T_2 < 0$
$\omega_2 > 0 > \omega_1 > \omega_4$</td></tr>
<tr><td>$l_{(4-1,2)} = -\frac{(\eta_1\eta_2 - 1)R^4_{2,f}}{\eta_1\eta_2}$</td><td colspan="3">$l_{(4-1,2)} = (\eta_1\eta_2 - 1)R^4_{2,f}$</td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">1,4</td><td colspan="2">$T_1 < 0$, $T_4 > 0$ and $T_2 < 0$
$\omega_1 > 0 > \omega_4 > \omega_2$
Or $T_4 < 0$, $T_1 > 0$ and $T_2 > 0$
$\omega_2 > \omega_4 > 0 > \omega_1$</td><td colspan="2">$T_4 < 0$, $T_1 > 0$ and $T_2 < 0$
$\omega_4 > 0 > \omega_1 > \omega_2$
Or $T_4 > 0$, $T_1 < 0$ and $T_2 > 0$
$\omega_2 > \omega_1 > 0 > \omega_4$</td></tr>
<tr><td colspan="2">$l_{(2-1,4)} = -\frac{(\eta_1\eta_2 - 1)R^2_{4,f}}{(\frac{\eta_1\eta_2}{R^2_{1,4}} - 1)}$</td><td colspan="2">$l_{(2-1,4)} = -\frac{(\eta_1\eta_2 - 1)R^2_{1,f}}{(\frac{\eta_1\eta_2}{R^2_{4,1}} - 1)}$</td></tr>
<tr><td rowspan="2">1</td><td rowspan="2">2,4</td><td colspan="2">$T_1 > 0$, $T_4 < 0$ and $T_2 > 0$
$\omega_1 > \omega_4 > 0 > \omega_2$
Or $T_1 < 0$, $T_4 > 0$ and $T_2 < 0$
$\omega_2 > 0 > \omega_4 > \omega_1$</td><td>$T_4 < 0$, $T_1 > 0$ and $T_2 < 0$
$\omega_4 > \omega_1 > \omega_2 > 0$
Or $T_4 > 0$, $T_1 < 0$ and $T_2 > 0$
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Or $T_4 < 0$, $T_1 > 0$ and $T_2 < 0$
$\omega_2 > \omega_1 > \omega_4 > 0$</td></tr>
<tr><td colspan="3">$l_{(1-2,4)} = (\eta_1\eta_2 - 1)R^1_{2,f}$</td><td>$l_{(1-2,4)} = -\frac{(\eta_1\eta_2 - 1)R^1_{2,f}}{\eta_1\eta_2}$</td></tr>
</table>

solution of the efficiency of a two-DOFs PGT with associated applicable ranges is more precise.

Conclusions

The approach adopted in this paper stems from an extension and unification of earlier works [12,55], concerned with the kinematics and efficiency of PGTs.

In this study, we have addressed three subjects of importance to the design of planetary gear trains. First, a systematic methodology for the determination of the operating conditions of a planetary gear train is described. Second, the range restrictions of torques and angular velocities are used to identify the proper assignment of the links of a two-DOF planetary gear mechanism. Third, since the tooth friction losses associated with a carrier-moving reference frame are the same as in a grounded reference frame, the tooth friction loss relations associated with three links having two inputs and one output or one input and two outputs are derived. Finally, a procedure for estimating the total efficiency of the two-DOF planetary gear train is developed. The procedure is demonstrated by a typical planetary gear train.

It is hoped that the simplicity of relations (1)–(6) is particularly helpful in the analysis and design of PGTs, and consequently to their wider application, particularly for those with multi-inputs or multi-outputs. These relations should also assist the designer by helping him to correctly choose the appropriate operating conditions for the desired power flow case from which the efficiency can then be determined.

Nomenclature

L = overall tooth friction loss
Li = local tooth friction loss
$N_{p,q}$ = planet gear ratio
P_{in} = total input power
P_{out} = total output power
P^j_x = potential power that can be developed in gear x when link j is relatively fixed
$R^v_{w,u}$ = speed ratio between links w and u with respect to link v
T_{ik} = torque on link i which belongs to the kth gear pair entity
T_p = external torque acting on link p
Z_i = number of teeth on gear i
η_c = conventional gear train efficiency
$\eta_{c(p-q)}$ = mechanical efficiency of the gear pair entity when operating with a fixed carrier with link p as input and link q as output link
η_p = planetary (epicyclic) gear train efficiency
ξ = tooth friction loss coefficient
ω_j = angular speed of gear j

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