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KINEMATIC ANALYSIS OF INDUSTRIAL AUTOMATIC TRANSMISSIONS WITH PLANETARY GEAR TRAINS

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ABSTRACT
In this paper, a previous methodology for the kinematic analysis of planetary gear trains (PGTs) is extended to determine the velocity ratios (VRs) of various industrial automatic transmissions. One kinematic equation is derived to determine the VR no matter how complicated the mechanism is. By manipulating the ruling formula, all possible VRs can be determined. The methodology is based on the concept that a PGT can be fractionated into several gear train entities (GTEs). An algorithm for computing the VRs for any type of industrial automatic transmission is described. The reliability of the method is established by applying the ruling kinematic equation to and comparing the results with three commonly used industrial automatic transmissions. The first industrial automatic transmission consists of only one GTE. Both of the second and third industrial automatic transmissions consist of two GTEs. Two of their members are joined together; in the second transmission, the connection is permanent, while in the third the connecting members are joined by brakes or rotating clutches (RCs).

INTRODUCTION
PGTs are used in automatic transmissions to obtain different VRs by using RCs and brakes.

A compound PGT contains several coaxial members; the input member is joined to the power source, the output member is joined to the final drive, while the fixed member is joined to the housing.

Figure 1 shows a six-speed Lepelletier automatic transmission.

RCs are used to connect the various members to each other or to the input source. Brakes are used to connect other members to the housing. The RCs are indicated by C and the brakes are indicated by B.

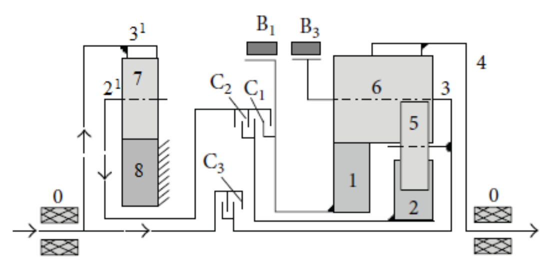


Figure 1 Six-speed Lepelletier automatic transmission having 41, 31, 61, 85, 23, 22, 15 and 31 teeth on gears 1, 2, 3, 4, 5, 6, 7 and 8, respectively [1].

Table 1 shows the corresponding clutching sequence for the six-speed Lepelletier automatic transmission, where the activated clutches Ci's and/or the brakes Bi's on the ith members of the gear train are denoted by **O**.

Table 1 Clutching sequence for Lepelletier automatic transmission [1].

	C_1	C_2	C_3	B_1	B_3	Velocity ratio
UD_1		O			O	4.135
UD_2		O		O		2.363
UD_3	O	O				1.508
UD_4		O	O			1.140
OD_1	O		O			0.860
OD_2			O	O		0.675
RD	O				O	-3.127

The output speeds are classified into three groups; under-drive (UD) when the velocity is between zero and the input velocity, over-drive (OD) when the velocity is more than the input velocity, reverse-drive (RD) when input velocity is less than zero, or direct-drive (DD) when the input velocity is equal to the output velocity.

Recently a new method is developed for kinematic analysis of PGTs [2]. The main result of it is that "for any planetary configuration, however complex, a single ruling kinematic equation is developed, which then is manipulated to determine all possible overall velocity ratios." In this paper, the results of [2] were extended to include all automatic transmission mechanisms with PGTs.

Many effective approaches are proposed for the analysis of PGTs kinematic[3-7]. The most important methods were mentioned in our previous work and can be quoted here as follows : "Freudenstein et al. [8] used efficiently the concept of fundamental circuit for the kinematic of EGTs. The method of the fundamental circuit needs the gear sizes in order to compute the VRs. Tian et al. [9] developed a matrix method to analyze EGMs. Kahraman et al. [10] introduced a methodology for determination of VR of EGT based on mechanism configuration. Talpasanu [11] presented metroid method for the kinematic analysis of EGTs. Tsai et al. [12] employed control technique of block diagrams to study kinematic relationship of EGT. Hsieh and Tsai [13] applied a lengthy computational technique to solve the overall VR analysis of EGMs. Hsu [14] presented an analytical method for the synthesis of the gear teeth numbers of EGMs. All above cited works use graphs to assemble system equations in their mathematical modeling. The graphs are based on the concept of fundamental circuit."

To find any VR, the transmission will be decomposed into two subsystems that are joined to each other either directly by two shared members or indirectly by RCs. Sometimes a shared member is formed by connecting two coaxial members to the gearbox housing either directly or indirectly by a brake.

The concepts of the gear pair entity (GPE) and the gear train entity (GTE) will be reviewed in the second and third paragraphs.

Then, we apply these concepts for the identification and comparison of the various VRs of three planetary–type automatic transmission. The first is a double-planet Ravigneaux gear train which is formed from only a single GTE. The second is type-6206 gear train; a compound gear train which consists of two GTEs rigidly joined by two of their members. The third is the Lepelletier gear train; a compound gear train which consists of two GTEs. Unlike the previous train, RCs are used to connect the different members of the single-planet simple and double-planet Ravigneaux GTEs of the Lepelletier gear train.

NOMENCLATURE

B	Brake
C	Clutch
GP	Gear Pair
GTE	Gear Train Entity
GPE	Gear Pair Entity
PGT	Planetary Gear Train
VR	Velocity Ratio
$N_{p,q}$	Planetary Gear ratio
RC	Rotating Clutch
$R^{z}_{x,y}$	Velocity Ratio between links x and y with respect to link z
Z_x	Number of teeth on gear x
ω_y	Angular Velocity of gear *y*

2 GEAR PAIR ENTITY

The gear pair entity (GPE) consists of two engaged gears and one carrier. A gear or a carrier can belong to many GPEs. There are two main types of GPEs: external and internal GPEs.

The Ravigneaux gear train shown in Fig. 1 contains of four GPEs (three of them is external and one is internal), they are: (5, 2, 3), (6, 5, 3), (6, 1, 3) and (6, 4, 3). The simple PGT in the same Figure contains of two GPEs (one is internal and the other is external) : (7, 3^1, 2^1) and (7, 8, 2^1).

The term "planet gear ratio" is used to denote the VR between links i and j of a GPE with respect to their carrier c. It can be as follows:

$$N_{i,j} = (\omega_j - \omega_c)/(\omega_i - \omega_c) \tag{1}$$

The gear ratio $N_{i,j}$ can also be written in terms the number of teeth as

$$N_{i,j} = \mp Z_i/Z_j \tag{2}$$

with the plus (minus) sign corresponding to the internal (external) GP.

When Willis explained in 1841 that the relationship between the speeds of three-shaft drives could be interpreted as the superposition of two very simple partial motions. It was then possible to derive Equation (1), which allows the identification of the speed-ratios of the revolving drive trains.

Chatterjee and Tsai [15] introduced the concept of fundamental geared entity to tackle the problem of finding the VRs of complex PGTs. The book by Tsai on Mechanism Design [16], or the book by Mueller on Epicyclic Gear trains [17], both address these topics extensively, along with several SAE papers, ASME papers, and IMechE papers which cannot be all cited in this brief introduction.

The many attempts to use the Willis equation to obtain VRs in complex PGTs are either complex or lengthy [15-17] and may lead to errors in application of the method. In this work, a single ruling formula will be derived for any planetary type v automatic transmission. By manipulating the ruling equation, all possible VRs can be determined easily in a single step.
In what follows, the concept of Gear train entity [3, 15] is reviewed.

3 GEAR TRAIN ENTITY

In Fig. 1, planet gears 5 and 6, along with the GPEs connecting them, represent a double-planet gear set, while planet 7 forms a single-planet set. The mechanism formed by a planet set, and all the GPEs joined directly to it, is called a gear train entity (GTE).

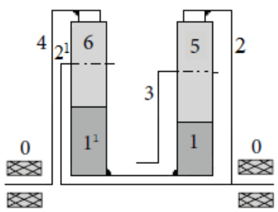
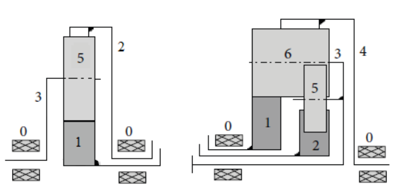


Figure 2 Schematic diagram of a compound PGT.

In order to distinguish different planet gear members and planetary gear entities, sub- and super-script will be used where necessary. In what follows we shall denote the planet gear member with a sub-script i. A superscript j denotes the planetary gear entity to which the planet gear member belong. Thus p_i^j denotes the i[th] planet gear member which belongs to the j[th] GTE. For convenience, we shall denote the first planet gear member by *p*. The labeling of GTEs is arbitrary.
By cutting through members 1 and 2, of the Simpson gear train shown in Fig. 2, two PGTs are formed, each one represents a GTE.
There are two frequently used GTEs, one contains only one planet and the other contains two engaging planets [16]. The functional schematics of examples of single- and double-planet GTEs are shown in **Fig**. (3) (a) and (b).



(a) (b)

Figure 3 : The functional schematics of (a) single-planet , and (b) double-planet GTEs.

The VR between members *x* and *y* with respect to member *z* can be written symbolically as :

$$R_{x,y}^{z} = (\omega_x - \omega_z)/(\omega_y - \omega_z) \tag{3}$$

4 PLANET GEAR RATIO

For a single-planet GTE, the planet gear ratio can be written as

$$\mathrm{R}_{\mathrm{x,p}}^{\mathrm{c}} = \frac{\omega_\mathrm{x} - \omega_\mathrm{c}}{\omega_\mathrm{p} - \omega_\mathrm{c}} = \mp Z_p / Z_x \tag{4}$$

Where Z_p is the number of teeth on the planet member *p* and Z_x is the number of teeth on a gear (*x*) engaging with the corresponding planet gear (*p*) .
In double planet GTE, the planet gear ratio can be written as

$$\mathrm{R}_{\mathrm{x,p}}^{\mathrm{c}} = \mathrm{R}_{p1,\mathrm{p}}^{\mathrm{c}} . \mathrm{R}_{\mathrm{x,p1}}^{\mathrm{c}} \tag{5}$$

Where (*p*) and (p_1) are the primary and secondary planets.
For a triple-planet GTE, the planet gear ratio can be written as

$$\mathrm{R}_{\mathrm{x,p}}^{\mathrm{c}} = \mathrm{R}_{p1,\mathrm{p}}^{\mathrm{c}} . \mathrm{R}_{p2,\mathrm{p1}}^{\mathrm{c}} . \mathrm{R}_{\mathrm{x,p2}}^{\mathrm{c}} \tag{6}$$

And so on [3].

5 OVERALL VELOCITY RATIO ANALYSIS

In compound PGTs, to find the overall VR for members related to different GTEs, we first fractionate the compound PGT into GTEs that are joined to each other either directly by two shared members or indirectly by RCs.

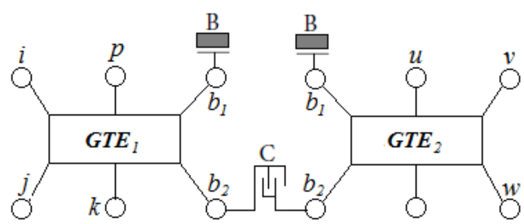


Figure 4 Shared members belonging to two different GTEs.

Sometimes a shared member is formed by connecting two coaxial members, belonging to two different GTEs, to the gearbox housing either directly or indirectly by a brake and is called a fixed shared member.
Let b_1 and b_2 be the two shared members, and x, y, and z be any three members of the PGT including the shared members, then by using these common members, we can write the VR $R_{x,y}^{z}$ as following :

$$R_{x,y}^{z} = \left[\frac{R_{x,b_2}^{b_1} - R_{z,b_2}^{b_1}}{R_{y,b_2}^{b_1} - R_{z,b_2}^{b_1}} \right] \tag{7a}$$

Where $R_{x,b_2}^{b_1}$, $R_{y,c_2}^{c_1}$ and $R_{z,b_2}^{b_1}$ are associated with either subsystem 1 or subsystem 2 depending on which subsystem the members x, y, and z belong to (see Fig. 4).

For any GTE in the PGT, the velocity ratio $R^z_{x,y}$ can be obtained in terms of common members p (planet gear) and c (planet carrier), by replacing c instead of b_1 and p instead of b_2 in Eq. (7). For any GTE , the VR can be written in terms of the common members *c* and *p* as follows

$$R^z_{x,y} = \left[\frac{R^c_{x,p} - R^c_{z,p}}{R^c_{y,p} - R^c_{z,p}}\right] \tag{7b}$$

The five steps to obtain the overall VR of any planetary gear mechanism (PGM) are:

- **STEP 1.** Use the definition of the GTE to identify the GTEs.
- **STEP 2.** Use Eq.(7) to obtain the overall VR of a PGT in terms of the VRs of two subsystems which are joined to each other by two common links.
- **STEP 3**. For any PGT, the VR $R^z_{x,y}$ can be written in terms of bridge members $b_1(c)$ and $b_2(p)$ as in Eqs. (7).
- **STEP 4.** The process continues until all of the VRs of Eq. (7) are written in terms of VRs of the GTEs.
- **STEP 5**. Using the following facts with Eq. (4) :

$$R^c_{p,p} = \frac{\omega_p - \omega_c}{\omega_p - \omega_c} = 1 \tag{8}$$

And

$$R^c_{c,p} = \frac{\omega_c - \omega_c}{\omega_p - \omega_c} = 0 \tag{9}$$

By this algorithm, the overall VR related to each possible assignment of the input, fixed, output and shared members of a PGT can be expressed in terms of the number of teeth of its coaxial gears.

To demonstrate the reliability of the method, the ruling formula was applied and results were compared with commonly used automatic transmissions. Due to the variety of PGTs, and the variety of ways to connect them, the paper goes into detail calculations of the various gear ratios of three commonly used automatic transmissions to confirm that the proposed ruling formula is valid for all types of transmissions.

6 VELOCITY RATIOS OF COMMONLY USED AUTOMATIC TRANSMISSIONS

The gear train shown in Figure 3 (b) correspond to type-6401 gear train. Three other commonly used EGTs are Lepelletier, Simpson and type-6401 gear trains. They are shown in Figures 1, 2 and 6.

6.1 TYPE-6401 GEAR TRAIN

Figure 5 shows the Borg-Warner automatic transmission [18]. It has four coaxial members, 1, 2, 3 and 4 incident to the housing of the transmission and two planet gears 5 and 6. Four drives and one reverse drive are obtained through connecting different members to clutches C1, C2 and C3, and brakes B1 and B3. Member 1 can be joined either to the input link by C1, or to the housing by B1 [19].

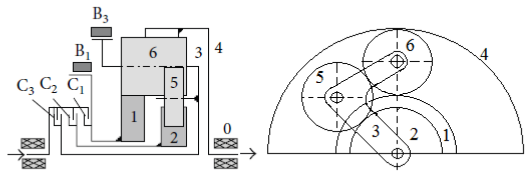


Figure 5 Borg-Warner four-speed automatic transmission having 36, 30, 72, 18 and 18 teeth on gears 1, 2, 4, 5 and 6, respectively [18].

Similarly, member 3 can be joined either to the input shaft by C3, or to the housing by B3. Member 2 can be joined to input shaft by the C2. Member 4 is designed as the output of the gear train. It is permanently joined to the final drive. The gear ratios for this train are

$$R^3_{4,6} = Z_6/Z_4 \tag{10}$$
$$R^3_{1,6} = -Z_6/Z_1 \tag{11}$$
$$R^3_{2,6} = R^3_{5,6}.R^3_{2,5} = Z_6/Z_2 \tag{12}$$

Depending on the activated clutches five drives (VRs) are feasible as indicated in **Table** 2.

6.1.1 First under-drive: In this drive C_2 and B_3 are joined while other clutches are disjoined. The planet carrier is used as the fixed member. Using Eq. (7b), the VR for members 2, 3 and 4 can be written as

$$R^3_{2,4} = \left[\frac{R^3_{2,6} - R^3_{3,6}}{R^3_{4,6} - R^3_{3,6}}\right] = \frac{Z_4}{Z_2} \tag{13}$$

Note that Eqs. (9), (10) and (12) have been applied in deducing Eq. (13).

Substituting Z_4=72 and Z_2=30 into Eq. (13), yields a VR of 2.4.

Table 2 Clutching sequence for the four-speed Ravigneaux transmission.

Range	Activated clutches					Velocity ratio
	C_1	C_2	C_3	B_1	B_3	
UD_1		O			O	$R^3_{2,4} = 2.4$
UD_2		O		O		$R^1_{2,4} = 1.4667$
DD		O	O			1.00
OD			O	O		$R^1_{3,4} = 0.6667$
RD	O				O	$R^3_{1,4} = -2.00$

6.1.2 second under-drive: C_2 and B_1 are joined while other clutches are disjoined. The large sun gear is used as the fixed member. Using Eq. (7b), the VR for members 1, 2 and 4 can be written as

$$R^1_{2,4} = \left[\frac{R^3_{2,6} - R^3_{1,6}}{R^3_{4,6} - R^3_{1,6}}\right] = \frac{Z_4(Z_1+Z_2)}{Z_2(Z_1+Z_4)} \qquad (14)$$

Note that Eqs. (10), (11) and (12) have been applied in deducing Eq. (14).
Substituting Z_4=72, Z_1=36 and Z_2=30 into Eq. (14), yields a VR of 1.4667.

6.1.3 Direct drive: Under this clutching condition C_2 and C_3 are engaged and the PGT works as a solid coupling with one to one VR.

6.1.4 Over-drive: by connecting C_3 and B_1 simultaneously, the transmission can be shifted into an over-drive. The large sun gear is used as the fixed member. Using Eq. (7b), the VR for the members 1, 3 and 4 can be written as

$$R^1_{3,4} = \left[\frac{R^3_{3,6} - R^3_{1,6}}{R^3_{4,6} - R^3_{1,6}}\right] = \frac{Z_4}{(Z_1+Z_4)} \qquad (15)$$

Note that Eqs. (9), (10) and (11) have been applied in deducing Eq. (15).
By substituting Z_1=36 and Z_4=72 into Eq. (15), yields a VR of 0.6667.

6.1.5 Reverse-drive: by connecting C_1 and B_3 simultaneously, the transmission is shifted into a reverse-drive. The planet carrier is used as the fixed member. Using Eq. (13), another VR for members 1, 3 and 4 can be written as

$$R^3_{1,4} = \left[\frac{R^3_{1,6} - R^3_{3,6}}{R^3_{4,6} - R^3_{3,6}}\right] = -\frac{Z_4}{Z_1} \qquad (16)$$

Note that Eqs. (9), (10) and (11) have been applied in deducing Eq. (16).
Substituting Z_4=72 and Z_1=36 into Eq. (16), yields a VR of –2. The negative sign means that the output shaft is rotating opposite to the input shaft.

6.2 TYPE-6206 GEAR TRAIN

Figure 6 shows a schematic drawing of a PGT used in AL4 four-speed automatic transmission. Four forward and one reverse VRs are obtained through connecting different members to clutches C_1 and C_3, and brakes B_1 and B_3. The clutching sequence for the transmission shown in Figure 6 is given in Table 3. The gear ratios for this train are

$$R^2_{3^1,6} = Z_6/Z_{3^1} \qquad (17)$$
$$R^2_{4,6} = -Z_6/Z_4 \qquad (18)$$
$$R^3_{2,5} = Z_5/Z_2 \qquad (19)$$
$$R^3_{1,5} = -Z_5/Z_1 \qquad (20)$$

By cutting through members 2 and 3 of Fig. 6, two PGTs are formed, each one represents a GTE.

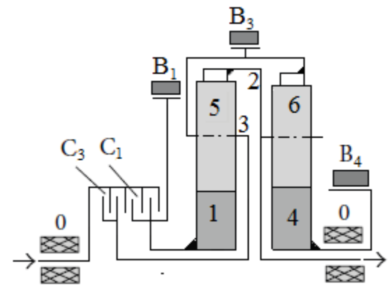


Figure 6 Four-speed AL4 automatic transmission having 33, 81, 80, 40, 24 and 20 teeth on gears 1, 2, 3, 4, 5 and 6, respectively [20].

Table 3 Clutching sequence for a four-speed AL4 automatic transmission.

Range	Activated clutches					Velocity ratio
	C_1	C_3	B_1	B_3	B_4	
UD_1	O				O	$R^4_{1,2} = 2.72$
UD_2		O			O	$R^4_{3,2} = 1.5$
DD	O	O				1.00
OD		O	O			$R^1_{3,2} = 0.71$
RD	O			O		$R^3_{1,2} = -2.45$

6.2.1 First under-drive: In this drive C_1 and B_4 are joined. The two shared members 2 and 3 will be used to fractionate type-6206 gear train into two GTEs; GTE_1 which contains members 1, 2, 3, and 5 and GTE_2 which contains members 2, 3, 4, and 6. Then by using the common members 2 and 3 as common links, we can express the VR using Eq. (7a) as follows :

$$R^4_{1,2} = \left[\frac{R^2_{1,3} - R^2_{4,3}}{R^2_{2,3} - R^2_{4,3}}\right] \qquad (21)$$

The VR $R^2_{1,3}$ of GTE_1 can be written, using Eq. (7b) as

$$R^2_{1,3} = \left[\frac{R^3_{1,5} - R^3_{2,5}}{R^3_{3,5} - R^3_{2,5}}\right] = \left[\frac{R^3_{1,5} - R^3_{2,5}}{-R^3_{2,5}}\right] = \frac{(Z_1+Z_2)}{Z_1} \qquad (22)$$

The VR $R^2_{4,3}$ of GTE_2 can be written, using Eq. (7b) as

$$R^2_{4,3} = \left[\frac{R^2_{4,6} - R^2_{2,6}}{R^2_{3,6} - R^2_{2,6}}\right] = \left[\frac{R^2_{4,6}}{R^2_{3,6}}\right] = -\frac{Z_{3^1}}{Z_4} \qquad (23)$$

Note that $R^2_{2,3} = R^3_{3,5} = R^2_{2,6} = 0$ and Eqs. (19) and (20) have been applied in deducing Eqs. (22) and Eqs. (17) and (18) have been applied in deducing Eqs. (23)
Substituting Eqs. (22) and (23) into Eq. (21), and after simplification, we get:

$$R^{4}_{1,2} = \left[\frac{(Z_1+Z_2)Z_4}{Z_1Z_3} + 1\right] \quad (24)$$

Substituting Z_1=33, Z_2=81, Z_3=80 and Z_4=40 into Eq. (24), yields a VR of 2.7272.

6.2.2 second under-drive: the rotating clutch C_3 and the clutch B_4 are joined while other clutches are disjoined. Hence, using Eq. (7b), the VR for members 2, 3 and 4 can be written as

$$R^{4}_{3,2} = \left[\frac{R^{2}_{3,6} - R^{2}_{4,6}}{R^{2}_{2,6} - R^{2}_{4,6}}\right] = \frac{Z_3+Z_4}{Z_3} \quad (25)$$

Note that $R^{2}_{2,6} = 0$ and Eqs. (17) and (18) have been applied in deducing Eq. (25). Substituting Z3=80 and Z_4=40 into Eq. (25), yields a VR of 1.5.

6.2.3 Direct drive: both C_1 and C_3 are joined. The power is transmitted between the input and output shafts with a $1/1$ velocity ratio.

6.2.4 Over-drive: C_3 and B_1 are joined while other clutches are disjoined. Using Eq. (8), the VR $R^{1}_{3,2}$ for members 1, 2 and 3 can be written as

$$R^{1}_{3,2} = \left[\frac{R^{3}_{3,5} - R^{3}_{1,5}}{R^{3}_{2,5} - R^{3}_{1,5}}\right] = \frac{Z_2}{Z_1+Z_2} \quad (26)$$

Note that $R^{3}_{3,5} = 0$ and Eqs. (19) and (20) have been applied in deducing Eqs. (26). Substituting Z_2=81 and Z_1=33 into Eq. (26), yields a VR of 0.7105.

6.2.5 Reverse-drive: by connecting C_1 and B_3 simultaneously, the transmission can be shifted into a reverse-drive. Using Eq. (13), the VR for members 1, 2 and 3 can be written as

$$R^{3}_{1,2} = \left[\frac{R^{3}_{1,5} - R^{3}_{3,5}}{R^{3}_{2,5} - R^{3}_{3,5}}\right] = -\frac{Z_2}{Z_1} \quad (27)$$

Note that $R^{3}_{3,5} = 0$ and Eqs. (19) and (20) have been applied in deducing Eq. (27). Substituting Z_2=81 and Z_1=33 into Eq. (27), yields a VR of –2.4545. The negative sign means that the output shaft is rotating opposite to the input shaft.

6.3 LEPELLETIER GEAR TRAIN

Lepelletier automatic transmission is shown in **Figure 1** and its clutching sequence is tabulated in **Table 1**.

The planet gear ratios and VRs of the Ravigneaux gear train are as in the previous section while those for the simple gear train are :

$$R^{2^1}_{8,7} = -Z_7/Z_8 \quad (28)$$

$$R^{2^1}_{3^1,7} = Z_7/Z_{3^1} \quad (29)$$

6.3.1 First under-drive: C_2 and B_3 are joined while other clutches are disjoined. GTE_1 contains members 2^1, 3^1, 7, and 8 while GTE_2 contains members 1, 2, 3, 4, 5, and 6.

Members 2 and 2^1 forms the first shared member between GTE_1 and GTE_2 while members 3 and 8 forms the fixed shared member.

We can express the VR using Eq. (7a) as follows :

$$R^{3}_{3^1,4} = \left[\frac{R^{8}_{3^1,2^1} - R^{3}_{3,2}}{R^{3}_{4,2} - R^{3}_{3,2}}\right] \quad (30)$$

Using Eq. (7b), the VR for members 2^1, 3^1 and 8 can be written as

$$R^{8}_{3^1,2^1} = \left[\frac{R^{2^1}_{3^1,7} - R^{2^1}_{8,7}}{R^{2^1}_{2^1,7} - R^{2^1}_{8,7}}\right] = \frac{Z_{3^1}+Z_8}{Z_{3^1}} \quad (31)$$

The VR of GTE_2 can be written, using Eq. (7b) as

$$R^{3}_{4,2} = \left[\frac{R^{3}_{4,6} - R^{3}_{3,6}}{R^{3}_{2,6} - R^{3}_{3,6}}\right] = \frac{Z_2}{Z_4} \quad (32)$$

Note that $R^{3}_{3,2} = R^{2^1}_{2^1,7} = R^{3}_{3,6} = 0$ and Eqs. (28) and (29) have been applied in deducing Eqs. (31), also Eqs. (10) and (12) have been applied in deducing Eqs. (32).

Substituting Eqs. (31) and (32) into Eq. (30), and after simplification, we get:

$$R^{3}_{3^1,4} = \frac{Z_4\left(Z_{3^1}+Z_8\right)}{Z_2 Z_{3^1}} \quad (33)$$

Substituting Z_4=85, Z_2=31, Z_8=31 and Z_3^1=61 into Eq. (33), yields a VR of 4. 135.

6.3.2 Second under-drive: C_2 and B_1 are joined while other clutches are disjoined. Members 2 and 2^1 forms the first shared member between GTE_1 and GTE_2 while members 1 and 8 forms the fixed shared member.

We can express the VR using Eq. (7a) as follows :

$$R^{1}_{3^1,4} = \left[\frac{R^{8}_{3^1,2^1} - R^{1}_{1,2}}{R^{1}_{4,2} - R^{1}_{1,2}}\right] \quad (34)$$

The VR of GTE_2 can be written, using Eq. (7b) as

$$R^{1}_{4,2} = \left[\frac{R^{3}_{4,6} - R^{3}_{1,6}}{R^{3}_{2,6} - R^{3}_{1,6}}\right] = \left[\frac{Z_2(Z_1+Z_4)}{Z_4(Z_1+Z_2)}\right] \quad (35)$$

Note that $R^{1}_{1,2} = 0$ and Eqs. (10), (11) and (12) have been applied in deducing Eqs. (35)

Substituting Eqs. (31) and (35) into Eq. (34), and after simplification, we get:

$$R^{1}_{3^1,4} = \frac{Z_4(Z_1+Z_2)\left(Z_{3^1}+Z_8\right)}{Z_2 Z_{3^1}(Z_1+Z_4)} \quad (36)$$

Substituting Z_8=31, $Z_{3}{}^{1}$=61, Z_1=41, Z_2=31 and Z_4=85 into Eq. (36), yields a VR of 2.363.

6.3.3 Third under-drive: When both C_1 and C_2 are joined, the Ravigneaux gear train will move as a one solid body. Under this clutching condition, the Ravigneaux VR is one. The overall VR is only that of the simple EGT. The VR of GTE_1 containing members 2, 3 and 8 can be written as given in Eq. (31)
Based on the data given above, we have Z_8=31 and $Z_{3}{}^{1}$=61. Substituting these values into Eq. (31) yields a third under-drive VR of 1.508.
Another VR of GTE_1 containing members 2, 3 and 8 can be written using Eq. (7b), as

$$R^{2^1}_{8,3^1} = \left[\frac{R^{2^1}_{8,7}-R^{2^1}_{2^1,7}}{R^{2^1}_{3^1,7}-R^{2^1}_{2^1,7}}\right] = -\frac{Z_{3^1}}{Z_8} \quad (37)$$

Note that $R^{2^1}_{2^1,7} = 0$ and Eqs. (28) and (29) have been applied in deducing Eq. (37).

6.3.4 Fourth under-drive: C_2 and C_3 are joined while other clutches are disjoined. Members 2 and 2^1 forms the first shared member between GTE_1 and GTE_2 while members 3 and 3^1 forms the second shared member.
We can express the VR using Eq. (7a) as follows :

$$R^{8}_{3^1,4} = \left[\frac{R^{3^1}_{3^1,2^1}-R^{3^1}_{8,2^1}}{R^{3}_{4,2}-R^{3^1}_{8,2^1}}\right] \quad (38)$$

Using Eq. (7b), the VR for members 2^1, 3^1 and 8 can be written as

$$R^{3^1}_{8,2^1} = \left[\frac{R^{2^1}_{8,7}-R^{2^1}_{3^1,7}}{R^{2^1}_{2^1,7}-R^{2^1}_{3^1,7}}\right] = \frac{Z_{3^1}+Z_8}{Z_8} \quad (39)$$

Note that $R^{3^1}_{3^1,2^1} = R^{2^1}_{2^1,7} = 0$ and Eqs. (28) and (29) have been applied in deducing Eqs. (39).
Substituting Eqs. (32) and (39) into Eq. (38), and after simplification, we get:

$$R^{8}_{3^1,4} = \left[\frac{\frac{Z_{3^1}+Z_8}{Z_8}}{\frac{Z_{3^1}+Z_8}{Z_8}-\frac{Z_2}{Z_4}}\right] \quad (40)$$

Substituting Z_8=31, $Z_{3}{}^{1}$=61, Z $_2$=31 and Z_4=85 into Eq. (40), yields a VR of 1.1401.

6.3.5 First over-drive: C_1 and C_3 are joined while other clutches are disjoined. Now, we consider $R^{8}_{3^1,4}$ as before but with the two shared members as 1-2^1 and 3-3^1.
We can express $R^{8}_{3^1,4}$ using Eq. (7a) as follows :

$$R^{8}_{3^1,4} = \left[\frac{R^{3^1}_{3^1,2^1}-R^{3^1}_{8,2^1}}{R^{3}_{4,1}-R^{3^1}_{8,2^1}}\right] \quad (41)$$

Using Eq. (7b), the VR for members 1, 3 and 4 can be written as

$$R^{3}_{4,1} = \left[\frac{R^{3}_{4,6}-R^{3}_{3,6}}{R^{3}_{1,6}-R^{3}_{3,6}}\right] = -\frac{Z_1}{Z_4} \quad (42)$$

Note that $R^{3^1}_{3^1,2^1} = R^{3}_{3,6} = 0$ and Eqs. (10) and (11) have been applied in deducing Eqs. (42)
Substituting Eqs. (39) and (42) into Eq. (41), and after simplification, we get:

$$R^{8}_{3^1,4} = \left[\frac{\frac{Z_{3^1}+Z_8}{Z_8}}{\frac{Z_1}{Z_4}+\frac{Z_{3^1}+Z_8}{Z_8}}\right] \quad (43)$$

Substituting Z_1=41, Z_8=31, $Z_{3}{}^{1}$=61 and Z_4=85 into Eq. (43), yields a VR of 0.86019.

6.3.6 Reverse-drive: by connecting C_1 and B_3 simultaneously, the transmission is shifted into a reverse-drive. Members 2^1 and 1 forms the first shared member between GTE_1 and GTE_2 while members 3 and 8 forms the fixed shared member.
We can express $R^{3}_{3^1,4}$ using Eq. (7a) as follows :

$$R^{3}_{3^1,4} = \left[\frac{R^{8}_{3^1,2^1}-R^{3}_{3,1}}{R^{3}_{4,1}-R^{3}_{3,1}}\right] \quad (44)$$

Note that $R^{3}_{3,1} = 0$.
Substituting Eqs. (31) and (42) into Eq. (44), and after simplification, we get:

$$R^{3}_{3^1,4} = -\frac{Z_4\left(Z_{3^1}+Z_8\right)}{Z_1 Z_{3^1}} \quad (45)$$

Substituting Z_1=41, Z_8=31, $Z_{3}{}^{1}$=61 and Z_4=85 into Eq. (45), yields a VR of –3.1267.

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