Proceedings of the ASME 2017 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC2017
August 6-9, 2017, Cleveland, Ohio, USA

DETC2017-67219

GRAPH-BASED ALGORITHM FOR POWER CIRCULATION AND EFFICIENCY ANALYSIS OF A PLANETARY GEAR REDUCER

Essam Lauibi Esmail
Assistant Professor
Al-Diwanyah, Al-Qadisiyah, Iraq
essam.esmail@qu.edu.iq

ANAHED HUSSEIN JUBER
Lecturer
Al-Diwanyah, Al-Qadisiyah, Iraq

ABSTRACT

The power developed within the elements of planetary gear train (PGT) may differ significantly from that being transmitted through it. This characteristic has a significant effect on the efficiency of split-power PGTs and requires a reliable, quick method to detect internal power circulation. By taking advantage of the usefulness of the graph-based approach, a simple preparatory power flow analysis, which provides a complete insight into the power flowing within the system elements without wasting time and effort in doing complicated mathematical computations, can be conducted. The graph-based approach is also useful in estimating the potential power which is related directly to the efficiency of PGTs. An analytical expression for the total efficiency of a PGR is derived using potential power analysis. Analysis of the power flowing through the PGR shows that there are internal power circulation and power amplification. The results are verified by an example. A brief comparison with existing efficiency formulas, which were applied to exactly the same system discussed here, is also given.

KEYWARDS: Efficiency; planetary gear train; power flow; split-power gears; power circulation; graph-based algorithm; power losses; potential power; planetary gear reducer; power amplification.

INTRODUCTION

Despite their unique properties of compact structure, smaller weight and size and high power capacity, PGTs may have dramatically low efficiency compared to simple GTs [2]. Therefore, the performance of PGTs becomes the most important design criteria nowadays.

Most of the studies for computing the efficiency of PGTS are based on the fundamental circuit technique [1-5, 16], on the concept of kinematic units [5, 6], and on Graph-based techniques [7-9, 12]

A review of efficiency formulas for two-DOF PGTs are given in [10, 15, and 17]. New methodologies are proposed for computing the power flow and efficiency of PGTs in Refs [11-14].

Of particular interest in design is the single-compound-planet PGT, shown schematically in **Fig. 1**. It can be used to provide high speed ratios with reasonably small number of gears. In this work, the relation between the reduction ratio and the efficiency is explored. A complete review of formulas for this PGT is available in Ref. [15]. A few cases are discussed to confirm the rightness of the proposed methodology and the final efficiency formulas.

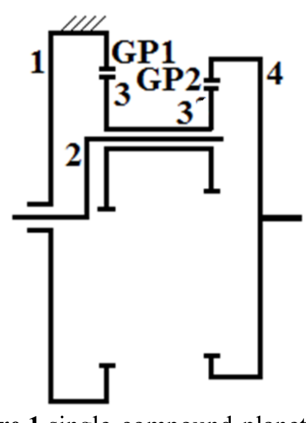

Figure 1 single-compound-planet PGR.

In this particular train, ring gear *1* is fixed and ring gear *4* is the output link. Planet carrier 2 is the input link. Z_1 and Z_3 are the numbers of teeth on gears 1 and 2, and $Z_{3'}$ and Z_4 being the numbers of teeth on gears $3'$ and 4. Gear pair GP_1 along with planet carrier 2 forms gear pair entity $1(GPE_1)$, while pair GP_2 along with planet carrier 2 forms gear pair entity $2(GPE_2)$. The power loss coefficients of GP_1 and GP_2, are ξ_1 and ξ_2, respectively.

The objective of this paper is to develop a graph-based algorithm for the investigation of the effectiveness of split-power GTs to transmit power. The analysis is based on graph technique. Details about monographs that allow fast and easy determination of velocity ratios, torques, powers and power flow directions, are found in Refs. [7, 19] and will not repeated here.

NOMENCLATURE

PGT	Planetary gear train
PGR	Planetary gear reducer
GT	Gear train
GPE	Gear pair entity
GP	Gear pair
GTE	Gear train entity
L	Friction power loss
$N_{p,q}$	Gear ratio
MRF	Moving reference frame
M-CRF	Moving-carrier reference frame
FRF	Fixed reference frame
P_j^v	Potential power that can be developed in the gearing when link *v* is relatively fixed.
P_{in}	Total input power
P_{out}	Total output power
$R_{w,u}^v$	Speed ratio between links *w* and *u* with respect to link *v*.
R_r	Reduction ratio
T_p	External torque acting on link *p*
T_{ik}	Torque on link *i* which belongs to the k^{th} gear pair entity
Z_i	Number of teeth on gear *i*
ω_j	Angular speed of link *j*
η_P	Planetary (Epicyclic) gear train efficiency
η	Conventional gear train efficiency
$\eta_{c(p-q)}$	Mechanical efficiency of the GPE when operating with a fixed carrier with link *p* as input and link *q* as output link
ξ	Power loss coefficient

KINEMATIC ANALYSIS

The GPE equation for gear pair (*p*, *x*) and carrier *c* is

$$\mathrm{N_{p,x}} = \frac{\omega_x-\omega_c}{\omega_p-\omega_c} = \mp\frac{Z_p}{Z_x} \tag{1}$$

The (-) sign is used for opposite rotation, and the (+) sign otherwise.

For the single-planet GTE in **Fig. 1**; $N_{p,c} = 0, N_{p,p} = 1, N_{p,r_1} = Z_p/Z_{r_1}$, and $N_{p,r_4} = Z_p/Z_{r_4}$. The gear ratios can be rewritten as follows: $N_{3,1} = Z_3/Z_1$, and $N_{3',4} = Z_{3'}/Z_4$.

The following formula will be used to deduce the speed ratio $R_{x,y}^z$ among any three links of the PGT [18]:

$$R_{x,y}^z = \frac{\omega_x-\omega_z}{\omega_y-\omega_z} = \frac{N_{p,x}-N_{p,z}}{N_{p,y}-N_{p,z}} \tag{2}$$

The reduction ratio $R_r = R_{c,r_4}^{r_1} = \frac{\omega_c}{\omega_4}$ can be written as

$$R_{c,r_4}^{r_1} = \frac{N_{p,c}-N_{p,r_1}}{N_{p,r_4}-N_{p,r_1}} = \frac{N_{3,1}}{N_{3,1}-N_{3',4}} \tag{3}$$

$N_{3',4}$ and $N_{3,1}$ are always larger than zero but smaller than one. If $N_{3,1} > N_{3',4}$, this will make $N_{3,1} - N_{3',4} > 0$, and therefore the reduction ratio $R_{c,r_4}^{r_1}$ is positive. If $N_{3,1} < N_{3',4}$, this will make $N_{3,1} - N_{3',4} < 0$, and therefore the reduction ratio $R_{c,r_4}^{r_1}$ becomes negative. Only the first case will be considered in this paper.

Figure 2 is the monograph created for this gear train, with the axes labeled with the corresponding speeds; ω_{r1}, ω_{r4}, ω_p, and ω_c.

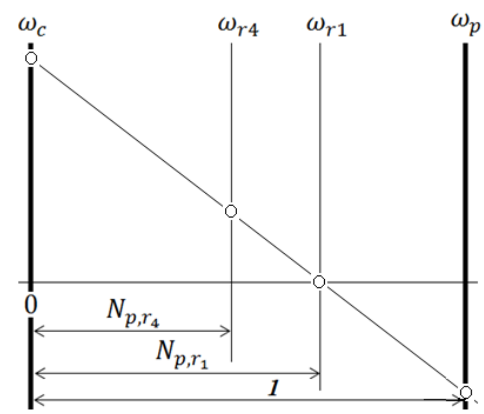

Figure 2: Monograph for the GT in **Fig. 1**.

GP EFFICIENCY

The efficiency, $\boldsymbol{\eta_{c(p-q)}}$, of the k^{th} gear pair $(\boldsymbol{p}, \boldsymbol{q})$, when carrier c is fixed, can be written as:

$$\eta_{c(p_k-q_k)} = 1 - \xi_k \tag{4}$$

For a train with more than one pair of gears in series,

$$\begin{aligned}\eta_c &= \eta_{c(p_1-q_1)}\eta_{c(p_2-q_2)} \ldots \eta_{c(p_k-q_k)} \\ &= (1-\xi_1)(1-\xi_2) \ldots (1-\xi_k)\end{aligned} \tag{5}$$

where subscripts 1, 2, …, k identify individual meshing pairs in the train.

PGT EFFICIENCY

The efficiency of the PGT can be written as:

$$\eta_P = -\frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out}+L} \tag{6}$$

GRAPH-BASED ALGORITHM FOR POWER FLOW

To perform the power flow analysis, torque vectors are drawn according to Esmail methodology; for more details see Ref. [19].

For the present case, **Fig. 2**, since the ring gear r_4 is an output link ($T_{r_4}\,\omega_{r_4} < 0$) and $\omega_{r_4} > 0$, therefore $T_{r_4} < 0$ which implies that $T_{r_4,2}$ is a vector pointing downward

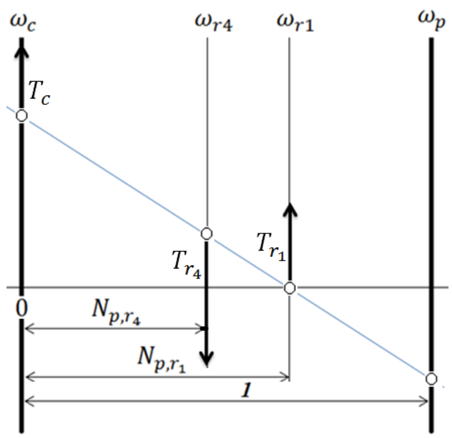

Figure 3: Torque and power flow through the GTR.

This implies that the torque vectors of both of the ring gear r_1 and the carrier c must point upward ($T_{r_1} > 0$ and $T_c > 0$). Notice that the value of ω_c is always positive if ω_{r_4} is positive ($\omega_c > \omega_{r_4} > (\omega_{r_1} = 0) > \omega_p$). This implies that the power at gear carrier c and ring gear r_4 have opposite signs ($T_{r_4}\,\omega_{r_4} < 0$ and $T_c\,\omega_c > 0$) ; gear carrier c is driver link at this GTR. **Figure 3** shows the monograph for the GTR with torques drawn as vectors.

Power Flow through Gear Pair Entity 2

Gear pair two (GP_2) along with planet carrier c forms gear pair entity two(GPE_2). Since the ring gear r_4 is an output link ($T_{r_4,2}\,\omega_{r_4} < 0$) and $\omega_{r_4} > 0$, therefore $T_{r_4,2} < 0$ which implies that $T_{r_4,2}$ is a vector pointing downward and $T_{p,2}$ and $T_{c,2}$ are vectors pointing upward.

Notice that $\omega_{r_1} = 0$ is between ω_c and ω_p, the vectors at c and p both point upward and the value of ω_p is always negative if ω_c is positive ($\omega_c > \omega_{r_4} > (\omega_{r_1} = 0) > \omega_p$). This implies that the power at gears c and p have opposite signs; the product of torque and angular velocity at p is negative and planet gear p is a driven link at GPE_2. **Figure 4** shows the monograph for the GPE_2 with torques drawn as vectors.

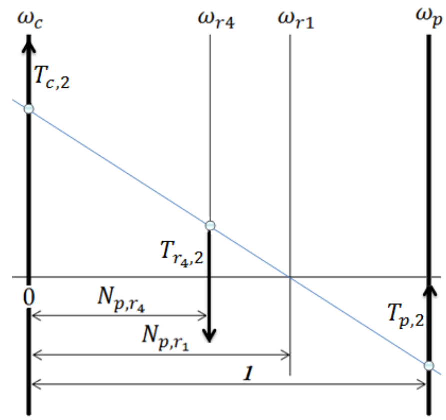

Figure 4: Torque and power flow through GPE_2.

Power Flow Through Gear Pair Entity 1

The first gear pair (GP_1) along with planet carrier c forms the first gear pair entity (GPE_1).

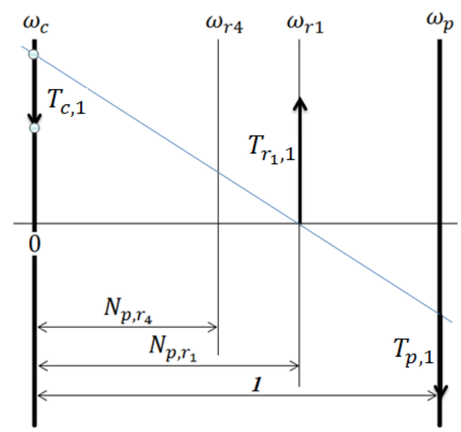

Figure 5: Torque and power flow through GPE_1.

By taking into account the fact that the planet gear is torque balanced, we have $T_{p1} = -T_{p2}$. Following the same reasoning as before, it can be shown that the planet gear p is a driver link at GPE_1 while the planet carrier c is a driven link.

Based on the results of **Figs. 4** and **5**, the power circulation can be visualized in **Figure 6.**

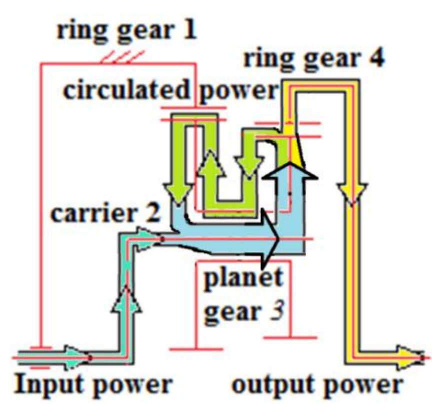

Figure 6: power circulation in the PGR.

Torque Ratios in GPEs

The torque ratios depend on the direction of the potential power flow. The potential power efficiency is the efficiency of GPE measured in any MRF [17]. It can be written for the k^{th} GPE as:

$$\eta_{c(p-q)} = -\frac{T_{qk}(\omega_q-\omega_c)}{T_{pk}(\omega_p-\omega_c)} = -\frac{T_{qk}}{T_{pk}} R^c_{q,p} \quad (7)$$

Equation (15) can be rewritten in an equivalent form as

$$\frac{T_{qk}}{T_{pk}} = -\frac{\eta_{c(p-q)}}{R^c_{q,p}} \quad (8)$$

This alternative form is often far more convenient to use, since the torque ratio is accounted for by focusing attention on the kinematic relations and on the conventional efficiency of the GP which is obtained from experiments.

The torque ratio between links *c* and *p* can be expressed as

$$\frac{T_{ck}}{T_{pk}} = \left[\frac{\eta_{c(p-q)}}{R^c_{q,p}} - 1\right] \quad (9)$$

The same equations may be applied for determining the torque ratios for any GTE and, consequently, for determining the efficiency of the entire PGT.

Power-Flow Ratios in GPEs

By multiplying Eq. (8) by $\frac{\omega_q}{\omega_p}$ and using the fact that $P = T.\omega$, the power ratio can be written as:

$$\frac{P_{qk}}{P_{pk}} = -\frac{\eta_{c(p-q)}}{R^c_{q,p}} R^f_{q,p} \quad (10)$$

where $\frac{\omega_q}{\omega_p} = R^f_{q,p}$ and *f* is the fixed link $\left(\omega_f = 0\right)$.

Similarly, multiplying Eq. (9) by $\frac{\omega_c}{\omega_p}$ and simplifying, we get:

$$\frac{P_{ck}}{P_{pk}} = \left[\frac{\eta_{c(p-q)}}{R^c_{q,p}} - 1\right] R^f_{c,p} \quad (11)$$

Power Loss

The power loss L is a function of the power developed in the gearing rather than of the actual power transmitted through the train. The power loss L can be estimated simply as follows:

$$L_k = -\left(P_{pk} + P_{qk} + P_{ck}\right) \quad (12)$$

GRAPH-BASED ALGORITHEM FOR EFFICIENCY ANALYSIS AND POWER LOSSES

The algorithm applied in this work to calculate the efficiency and power losses is based on the concept of the potential power developed in Ref. [17]. The procedure is as follows:

- Using the monograph technique, the output and input links in the MRF can be identified easily.
- In the MRF, the torque ratio of the input link to the output link, can be written from Equation (8) and/or (9).
- The speed ratio equation needed for Eq. (8) is obtained from Eq. (3).
- Equation (6) which depends on both the speed ratios and torque ratios is now used to find an expression for the PGT efficiency η_P.

Power Flow in the M-CRF

Assume now that the planet carrier c is relatively fixed by adding to the entire system an additional angular velocity $(-\omega_c)$ equal in absolute value to the angular velocity of the input shaft, but opposite in direction. The relative motions of all members of the PGT are unchanged. However, the entire train may be considered as a conventional gear train with relatively fixed axes of rotation.

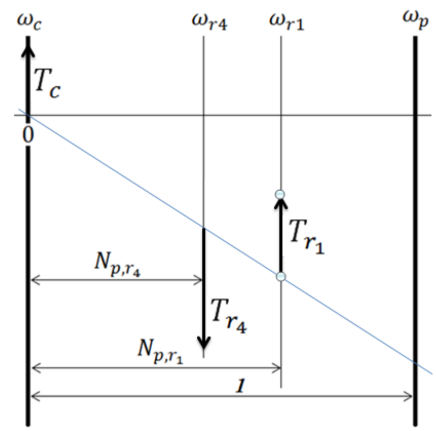

Figure 7: Torque and power flow through the GTR in the M-CRF.

To derive an expression for the efficiency of the PGT, it is vital to identify the driver gear in the M-CRF. Since the external torques applied to the system do not change, the actual torques acting on the links of the PGT in the FRF will continue to act on them in the moving reference frame.

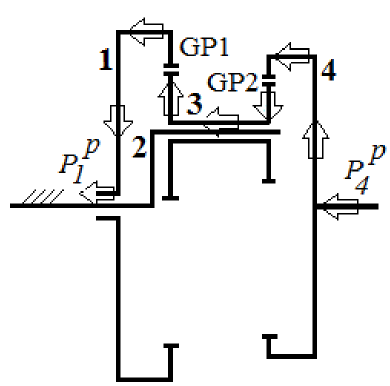

Figure 8: Potential Power flow direction.

The only change is in the angular velocities of the gears. The angular velocity of ring gear 4 in the moving reference frame, is equal to $(-\omega_c)$ while that of ring gear 4 equals $\omega_{r_4} - \omega_c < 0$, as shown in **Fig. (7)**. Notice that $T_{r_4} < 0$. Since $T_{r_4}(\omega_{r_4} - \omega_c) > 0$, therefore, ring gear 1 is the driven link in the M-CRF.

Figure 8 shows the potential power developed within the links of the PGR.

ANALYSIS OF TOTAL EFFICIENCY

From Equations (8) and (9), the torque ratios can be written as:

$$\frac{T_1}{T_4} = -\frac{\eta_{c(4-1)}}{R^c_{1,4}} \tag{13}$$

and

$$\frac{T_c}{T_4} = \frac{\eta_{c(4-1)}}{R^c_{1,4}} - 1 \tag{14}$$

The efficiency of the fixed-carrier reducer is

$$\eta_{c(4-1)} = \eta_{GP1}\,\eta_{GP2} \tag{15}$$

The PGT efficiency as from Eq. (6) is

$$\eta_P = -\frac{P_{out}}{P_{in}} = -\frac{T_4\omega_4}{T_c\omega_c} \tag{16}$$

But, from Eq. (3), the reduction ratio $R_r = \frac{\omega_c}{\omega_4}$.

By substituting the torque ratio $\frac{T_c}{T_4}$, from Eq. (14), and the reduction ratio R_r into Equation (16), and after some simplifications, the PGT efficiency can be written as:

$$\eta_P = \frac{1}{[1+(R_r-1)(1-\eta_{c(4-1)})]} \tag{17}$$

And the total power losses can be written in terms of the output power P_4, as:

$$L = [(R_r - 1)(1 - \eta_{c(4-1)})]P_4 \tag{18}$$

Equation (18) shows that as the speed ratio of this train increases, the tooth mesh losses in the train increase, while Equation (17) shows that the efficiency decreases.

RESULTS AND DISSCISION

Based on Eq. (3), the 3D drawing among $N_{3,1}$, $N_{3',4}$ and the reduction ratio $R^{r_1}_{c,r_4}$ is shown in **Figure 9**.

Figure **9** shows the relationship between the reduction ratio and the gear ratios. The left-hand portion of the curve represents the cases where speed ratio is positive, and the right-hand portion those with negative speed ratios.

It can be seen from **Fig. 9** that for this train a speed ratio of any magnitude, even infinity, can theoretically be obtained.

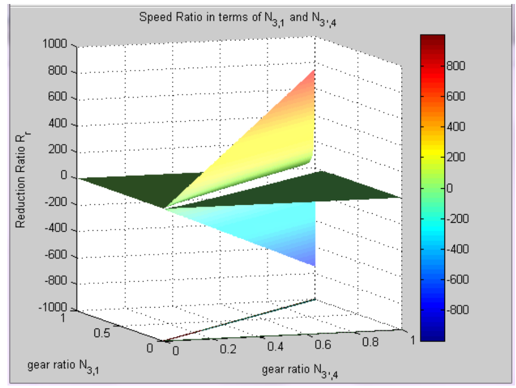

Figure 9: Speed Ratio $R^{r_1}_{c,r_4}$ in terms of $N_{3,1}$ and $N_{3',4}$.

Examples

If $Z_1 = 202$, $Z_4 = 200$, $Z_3 = 100$, and $Z_{3'} = 99$, then $N_{3,1} = 0.4950495$, and $N_{3',4} = 0.495$. According to (3), the reduction ratio is R_r =10 000.

If $Z_1 = 92$, $Z_4 = 90$, $Z_3 = 45$, and $Z_{3'} = 44$, then $N_{3,1} = 0.48913$, and $N_{3',4} = 0.48889$. According to (3), the reduction ratio is R_r =2 000.

If $Z_1 = 62$, $Z_4 = 60$, $Z_3 = 30$, and $Z_{3'} = 29$, then $N_{3,1} = 0.4839$, and $N_{3',4} = 0.4834$. According to (3), the reduction ratio is R_r =900.

If $Z_1 = 34$, $Z_4 = 32$, $Z_3 = 16$, and $Z_{3'} = 15$, then $N_{3,1} = 0.470588$, and $N_{3',4} = 0.46875$. According to (3), the reduction ratio is R_r =256.

If $Z_1 = 119$, $Z_4 = 112$, $Z_3 = 96$, and $Z_{3'} = 90$, then $N_{3,1} = 0.80672$, and $N_{3',4} = 0.80357$. According to (3), the reduction ratio is R_r =256.

Then, based on Eq. (17), the 3D drawing among η_c, R_r, and η_P can be expressed as in **Figure 10**.

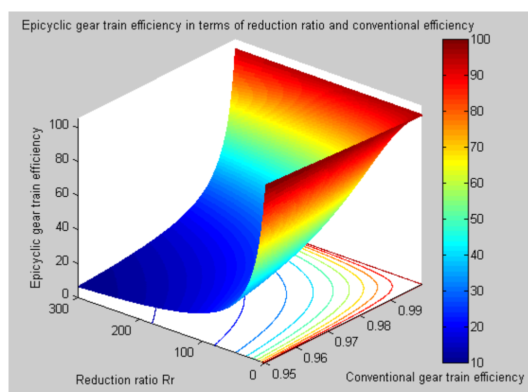

Figure 10 The efficiency of the PGR in terms of R_r and ξ.

Mainly, the efficiency decreases with the increase of R_r or with the increase of the loss factor ξ. Although the low efficiency of a PGR may not in itself be objectionable, dissipation of the power loss in heat may produce undesirably high temperature in the lubrication oil.

For specified loss factors (e.g., $\xi_1 = 1 - \eta_1 = \xi_2 = \xi =$ 0.005, 0.01, 0.015, 0.02, 0.025, or 0.03), the meshing efficiency between the reduction ratio R_r and η_P can be expressed as in **Figure 11**.

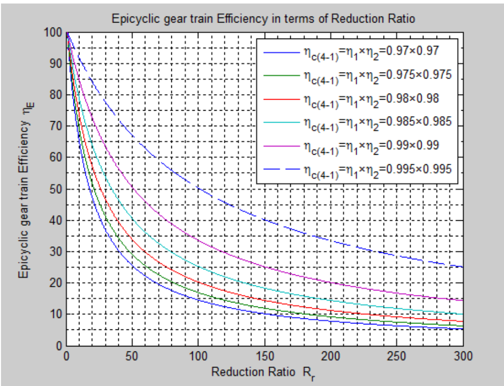

Figure 11 The meshing efficiency of the PGR with the reduction ratio R_r for different loss factors ξ.

Examples

The efficiency of the PGR for gears manufactured by shaving ($\xi = 0.01$) or by grinding ($\xi = 0.005$) is shown in **Figs. 12** and **13**, respectively.

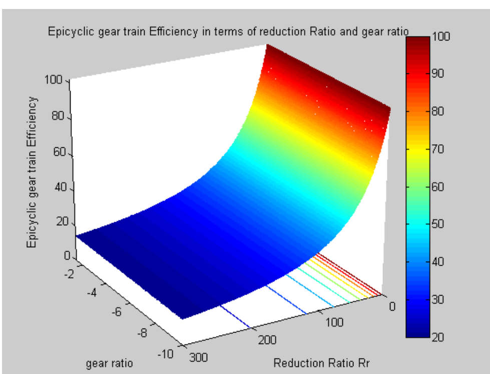

Figure 12 Efficiency for gears manufactured by shaving.

The PGR having gears manufactured by grinding is more efficient.

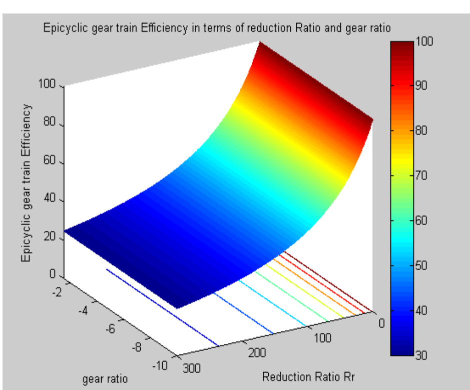

Figure 13 Efficiency for gears manufactured by grinding.

For a reduction ratio of 50, the PGT efficiency of a reducer with gears manufactured by shaving, is 50.63% and with gears manufactured by grinding is 67.17%.

SAMPLE CALCULATIONS

A PGT of the type shown in **Fig. 3** has the following design data: $Z_1 = 34$, $Z_4 = 32$, $Z_3 = 16$, and $Z_{3'} = 15$.

For gears *3* and *4*, $\eta_{c(4-3)} = 0.99292$, and for gears *3* and *1*, $\eta_{c(3-1)} = 0.99338$.

The efficiency of a conventional train with the same gears is $\eta_{c(4-1)} = \eta_{c(3-1)}\eta_{c(4-3)} = 0.98635$.

Using Equation (3), the reduction ratio is

$$R_r = R_{c,4}^1 = \frac{N_{3,1}}{N_{3,1} - N_{3',4}} = \frac{\frac{Z_3}{Z_1}}{\frac{Z_3}{Z_1} - \frac{Z_{3'}}{Z_4}} = 256\ ;$$

and from Equation (17),

$$\eta_P = \frac{1}{1 + \left(R_{c,4}^1 - 1\right)\left(1 - \eta_{c(3-1)}\eta_{c(4-3)}\right)} = \frac{1}{1 + (256 - 1)(1 - 0.99338 \times 0.99292)}$$

$$\eta_P = 22.3\%.$$

This result equals exactly that given in ref. [15].

Power Developed In the Gearing

For GPE$_2$, the power ratios between the power transmitted by ring gear 4 and the powers developed in gears 3 and 2 can be written from Eq. (10) and (11) as follows:

$$\frac{P_{32}}{P_{42}} = \frac{P_{32}}{P_{out}} = -\frac{\eta_{c(4-3)}}{R_{3,4}^2} R_{3,4}^1 \quad (19)$$

and

$$\frac{P_{22}}{P_{42}} = \frac{P_{22}}{P_{out}} = \left[\frac{\eta_{c(4-3)}}{R_{3,4}^2} - 1\right] R_{2,4}^1 \quad (20)$$

For GPE$_1$, $P_{31} = -P_{32}$.

The power developed in gear 3 and carrier 4 can be written from Eq. (11) as:

$$\frac{P_{21}}{P_{31}} = \left(\frac{\eta_{c(3-1)}}{R_{1,3}^2} - 1\right)\frac{1}{R_{3,2}^1} \quad (21)$$

The input power to carrier 2 can be found from:

$$P_{in} = P_2 = (P_{21} + P_{22}) \quad (22)$$

The velocity ratios needed for Eqs. (19), (20) and (21) are:

$$R_{3,4}^1 = \frac{Z_4(Z_1 - Z_3)}{Z_1 Z_{3'} - Z_3 Z_4} = \frac{32(34 - 16)}{34 \times 15 - 16 \times 32} = -288$$

$$R_{2,4}^1 = 256$$

$$R_{3,4}^{2}=\frac{Z_4}{Z_{3'}}=\frac{32}{15}=2.133$$

$$R_{1,3}^{2}=\frac{Z_3}{Z_1}=\frac{16}{34}=0.471$$

The power developed in a PGT of this type with $P_{out} = P_{42} = -1$, can be found as following:

From Eq. (19)

$$P_{32}=-\frac{\eta_{c(4-3)}}{R_{3,4}^{2}}R_{3,4}^{1}P_{out}=-134.044$$

But $P_{31} = -P_{32} = 134.044$

From Eq. (20)

$$P_{22}=\left[\frac{\eta_{c(4-3)}}{R_{3,4}^{2}}-1\right]R_{2,4}^{1}P_{out}=136.849$$

From Eq. (21)

$$P_{21}=\left(\frac{\eta_{c(3-1)}}{R_{1,3}^{2}}-1\right)\frac{1}{R_{3,2}^{1}}P_{31}=-132.368$$

And

$$P_{in}=(P_{21}+P_{22})=(136.849-132.368)=4.481$$

It is clear now that the circulated power is more than 134 and has a great effect on the power losses.

Power Losses

For the first gear pair entity, the power loss can be simply calculated from Eq. (12):

$$\begin{aligned}L_1 &= -(P_{11}+P_{21}+P_{31})\\ &= -(0-132.368+134.044)=-1.676\end{aligned}$$

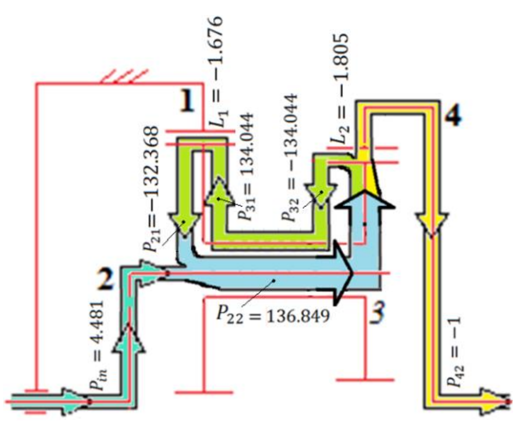

Figure 14 Powers flowing through the links and power losses.

For the second gear pair entity, the power loss is:

$$\begin{aligned}L_2 &= -(P_{22}+P_{32}+P_{42})\\ &= -(136.849-134.044-1)=-1.805\end{aligned}$$

The powers flowing through the links and the power losses can be visualized as in **Fig. 14**.

It is clear now that the internal power circulation is the causative factor of the low efficiency of the PGR.

REFERENCES

[1] Pennestri, E. and Freudenstein, F., A systematic approach to power-flow and static-force analysis in epicyclic spur-gear trains, ASME J. Mech. Des. 115, pp. 639–644, 1993.

[2] Pennestri, E. and Freudenstein, F., The Mechanical efficiency of epicyclic gear trains, ASME J. Mech. Des. 115, pp. 645-651, 1993.

[3] Yan, Hong-Sen and Hsieh, Long-Chang, Maximum mechanical efficiency of infinitely variable transmissions, Mechanism and Machine Theory, 29(5), pp. 777–784, 1994.

[4] Kahraman, A., Ligata, H., Kienzle, K., and Zini D., M., A kinematics and power flow analysis methodology for automatic transmission planetary gear trains, ASME Journal of Mechanical Design, 126, pp. 1071–1081, 2004.

[5] Liu, C.-P., and Chen, D.-Z., On the application of kinematic units to the topological analysis of geared mechanisms, ASME J. Mech. Des., 123, pp. 240 – 246, 2001.

[6] Tsai Mi-Ching, Huang Cheng-Chi and Lin Bor-Jeng, Kinematic analysis of planetary gear system using block diagrams, Transactions of the ASME Journal of Mechanical Design, 132, pp.065001-1 - 065001-10, 2010.

[7] Esmail E. L., Monographs for synthesis of epicyclic-type automatic transmissions., Meccanica, 48: pp 2037–2049, 2013. doi:10.1007/s11012-013-9721-z.

[8] Castillo, J M del., The analytical expression of the efficiency of planetary gear trains", Mechanism and Machine Theory, 37, pp. 197–214, 2002.

[9] Nelson, C. A., and Cipra, R. J., Simplified kinematic analysis of bevel epicyclic gear trains with application to power-flow and efficiency analyses, ASME J. Mech. Des., 127 (2), pp. 278–286, 2005,.

[10] Pennestrì, E., and Valentini, P. P., A review of formulas for the mechanical efficiency analysis of two degrees-of-freedom epicyclic gear trains, ASME J. Mech. Des., 125 (3) , pp. 602–608, 2003.

[11] Ciobotaru, T., Frunzeti D., and Jäntschi L., Method for analyzing epicyclic gearboxes, International Journal of Automotive Technology, 11(2), pp. 167− 172, 2010.

[12] Pennestrì E., et al., Efficiency evaluation of gearboxes for parallel hybrid vehicles: Theory and Applications, Mechanism and Machine Theory, 49, pp. 157–176, 2012.

[13] Chen, C., and Liang, T.T., Theoretic study of efficiency of two-dofs of epicyclic gear transmission via virtual power, ASME Journal of Mechanical Design, 133(3) pp. 031007-1 –031007-7, 2011.

[14] Gomà Ayats, J.R. et al., Power transmitted through a particular branch in mechanisms comprising planetary gear trains and other fixed or variable transmissions, Mechanism and Machine Theory, 46, pp. 1744–1754, 2011.

[15] Chen, C., and Angeles, J., Virtual-power flow and mechanical gear-mesh power losses of epicyclic gear trains, ASME Journal of Mechanical Design, 129, pp. 107–113, 2007.

[16] Esmail E. L. and Hassan S. S., An approach to power-flow and static force analysis in multi-input multi-output epicyclic-type transmission trains, Transactions of the ASME Journal of Mechanical Design, 132, pp. 011009-1 - 011009-10, 2010.

[17]Esmail, E. L. , 2016," Meshing Efficiency Analysis of Two Degree-of-Freedom Epicyclic Gear Trains", J. Mech. Des 138(8), 083301 (Jun 13, 2016) ;Paper No: MD-16-1034; doi: 10.1115/1.4033693

[18]Esmail, E.L. "A universal kinematic analysis of geared mechanisms", J Braz. Soc. Mech. Sci. Eng., Volume 39, Issue 6, pp. 2253-2258, (June 2017). doi:10.1007/s40430-017-0711-2.

[19] Esmail, E. L. , Hussen, H. A., 2009, "Monographs for Kinematic, Statics and Power Flow Analysis of epicyclic gear trains", ASME International Mechanical Engineering Congress and Exposition (IMECE09), November 13-19, Lake Buena Vista, Florida, USA, Volume 13, paper No. 10789, pp. 631-640