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Achieving an Optimum Slowing-down Energy Distribution Functions and Corresponding Reaction Rates for the ($D+^3He$ and $T+^3He$) Fusion Reactions

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Abstract. A new results for fusion reactivity and slowing-down energy distribution functions for controlled thermonuclear fusion reactions of the hydrogen isotopes are achieved to reach promising results in calculating the factors that covered the design and construction of a given fusion system or reactor. They are strongly depending upon their operating fuels, the reaction rate, which in turn, reflects the physical behavior of all other parameters characterization of the system design.

Keywords. Controlled thermonuclear fusion reactions; fusion cross section; slowing-down energy distribution functions; fusion reactivity; reaction rate.

INTRODUCTION

The collision cross section σ for a given reaction is a useful quantity to measure the probability of a fusion reaction between two reactant nuclei as a function of the relative velocity. If the reactants nuclei have a distribution of velocities, in other word, a thermal distribution, then it is useful to take an average over these distributions, by multiplying cross section and velocity [1]. This average is called the 'reactivity', and referred as $\langle\sigma v\rangle$. The reaction rate (fusions per volume per time) is $\langle\sigma v\rangle$ multiplied by the number density of the reactants [2].

The Maxwell-Boltzmann probability distribution is the trait to elucidate the essence of a thermonuclear fusion reaction. It is used frequently in the investigation of nuclear science and engineering, and it is significant to recognize its underlying assumptions. [3]

Thermonuclear fusion reactivity is an essential amount in the studies of fusion energy production, fusion ignition and energy break-even tests in magnetically and inertially confined systems.

Plasma state is the description of the fusion reactants. The importance of $\langle\sigma v\rangle$ as a function of kinetic temperature in a system with a specific energy confinement time is determined with respect to the Lawson criterion.

IMPORTANT PARAMETER

The fusion cross section is one of the most significant physical quantities in the engineering and designing of fusion studies. It is appropriate to utilize a modern formula covering all the probable controlled thermonuclear fusion reactions described in table (1).

TABLE 1. The Commonly Useful Fusion Reactions of the Hydrogen Isotopes.

	Thermonuclear Fusion Reaction	Probability	Q MeV
1	${}^2_1D + {}^2_1D \rightarrow {}^3_2\text{He} + n$	50%	3.269
2	${}^2_1D + {}^2_1D \rightarrow {}^3_1T + p$	50%	4.033
3	${}^2_1D + {}^3_1T \rightarrow {}^4_2\text{He} + n$	100%	17.589
4	${}^2_1D + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + p$	100%	18.353
5	${}^3_1T + {}^3_1T \rightarrow {}^4_2\text{He} + 2n$	100%	11.332
6	${}^3_1T + {}^3_2\text{He} \rightarrow {}^2_1D + {}^4_2\text{He}$	43%	14.320
7	${}^3_1T + {}^3_2\text{He} \rightarrow p + {}^5_2\text{He}$	6%	14.320

It is necessary to clarify that there are many theoretical and empirical relations to calculate the cross sections of the controlled thermonuclear fusion reaction. Through investigating the used relationship below, it gives accurate results

The formula of fusion cross section in lab coordinate for all these fusion reactions is given by [4, 5]

$$\sigma(E) = -16389C_3 \left(1 + \frac{M_a}{M_b}\right)^2 \{M_a E_{lab} [e^{31.4Z_1Z_2\sqrt{\frac{M_a}{E_{lab}}}} - 1] [(C_1 + C_2 E_{lab})^2 + (C_3 - \frac{2\pi}{e^{31.4Z_1Z_2\sqrt{\frac{M_a}{E_{lab}}}} - 1})^2]\}^{-1} \dots\dots\dots (1)$$

With parameters C_1 C_2 C_3 mentioned in ref [5], M_a and M_b are reactant masses, Z_1 Z_2 and reactant charges, E_{lab} energy of projectile in the laboratory system.

THE SLOWING-DOWN ENERGY DISTRIBUTION FUNCTION

The main influence of this distribution is that the particles in any material do not all have the same energy or velocity even if they have the same temperature. It is not very important whether these particles are atoms, molecules, or elementary particles such as protons, where Maxwell-Boltzmann probability distribution becomes equally well applicable to all of them.

The energy distribution of producing particles by a thermonuclear reaction has more interest right now because of the increasing possibility of producing these reactions in the laboratory. In order to calculate the lightest particles energy distribution (protons), it is supposed that the reacting mixture of gases is in thermodynamic equilibrium, which occurs at a temperature T and that the equilibrium is not troubled clearly by the nuclear reactions. It is supposed also that the cross section for producing the lightest particles is isotropic in the center of mass coordinate frame of two interacting particles. [3]

The slowing-down energy distribution functions $F(E)$ is the measure of probability that a particle is existed in the energy state E . The distribution function represents a generalization of the concepts of discrete probability to the situation, where energy can be dealt as a continuous variable.

The Maxwell-Boltzmann distribution is a function used to distribute quantity of energy between identical but distinguishable particles. [6]

As well as the supposition of distinguishability, classical and statistical physics suppose that:

- No restriction on the number of particles that can take up a given state.
- The distribution of particles between the obtainable energy states (At thermal equilibrium) will occupy the most probable distribution proportionate with the total available energy and entire number of particles.
- Every particular state of the system has an identical probability.

One of the inclusive thoughts contained in these suppositions is that it is unacceptable for any particle to get an energy much higher than the average (i.e., far more than its share). Energies that considered lower than the average are preferable because there are more ways to reach them. If one particle has energy exceeds or equals 10 times the average, for example, it will decrease the number of possibilities for the distribution of the remaining energy. Thus, it is unacceptable because the probability of occupying of a given state is proportional to the number of ways it can be acquired [7].

A great interest is exerted by both theorists and experimenters to research about particles interactions. These processes provide an illustration for the problems of nuclear structural and two-body interaction. For employment of probability to physical processes, the use of the distribution function is a very favorite decision. [8]

THE DENSITY OF STATES

A number of available states in a given energy interval affect the distribution of energy among the identical particles. This density of states is considered as a function of energy present the number of states per unit volume in an energy interval. The expression "statistical weight" is used occasionally as a synonym, especially in cases in terms of the available states described as a discrete. The physical limitations on the particles specifies the form of the density of states function.

The distribution of a constant magnitude of energy between a number of congruent particles count on the density of energy states that is available and the occupation probability in given state. The probability of a certain energy state to be occupied is provided through the use of the distribution function, but if there is extra available energy states in a given energy interval, posteriorly, these give a bigger weight to the probability for that interval of energy [9].

The spectrum of the produced particles from fusion reactions holds information about the energy distribution of the reactant particles, because the status of motion of the reactant in reaction affects the produced spectrum according the kinematic relation.

Particularly, high-energy combinations of the reactant energy distribution appear into high-energy combinations in the product spectrum [10].

An appropriate approximation to the factual formula to calculate slowing-down energy distribution functions for produced particles may be obtained by [11]

$$F(E) = \frac{v_1 v_2 2m_o dE}{\pi (kT)^2 \gamma_{12} \sqrt{2mm_1 m_2 \alpha Q}} \exp \left\{ \frac{-m_o}{mkT} \left[\sqrt{E} - \sqrt{\alpha(Q + E_r)} \right]^2 \right\} \int_0^{\infty} E_r dE_r \sigma(E_r) \exp \left(-\frac{E_r}{kT} \right) \dots \dots \dots (2)$$

Noting that $E \cong \alpha Q$ while $E_r \cong kT$ is very much less than E and $\sigma(E_r)$ is cross section in C.M coordinate.

Where m_1 and m_2 ions of mass respectively, v_1 and v_2 = numbers of ions per unit volume of, k = Boltzmann's constant, T = temperature, $m_o = m_1 + m_2$, m is the lightest particles mass, Q is Q-value.

$$\gamma_{12} = 1 \text{ if } m_1 \neq m_2 \text{ and } = 2 \text{ if } m_1 = m_2 .$$

It can be seen that the width of the produced particle energy distribution from equation (2) at e-1 of its maximum value is [11]

$$dE = \sqrt{\frac{mkT\alpha Q}{m_o}} \dots \dots \dots (3)$$

$$\alpha = \left(1 - \frac{m}{m_o} \right) \dots \dots \dots (4)$$

FUSION REACTIVITY AND REACTION RATE

A significant quantity is the reactivity $\langle \sigma v \rangle$, to determine the fusion rate reactions in plasma, one has to consider the velocity distributions of the reacting particles. $\langle \sigma v \rangle$ is obtained by multiplying the reaction cross-section $\langle \sigma v \rangle$, and the relative velocity (v) averaged over the velocity distributions of two reacting species. If the two species are classified by 1 and 2, with number densities n_1 and n_2 , respectively, then fusion reaction rate is obtained by [12]

$$R = \frac{1}{1 + \delta_{12}} n_1 n_2 \langle \sigma v \rangle \dots \dots \dots (5)$$

The term δ_{12} (known as $\delta_{12} = 1$ if $i = j$ and $\delta_{12} = 0$ other else) is present just to take into account the situation of reactions between identical particles (particles of the same species). Without the term δ_{12} , each reaction between identical particles is calculated twice. The reactivity can be written as: [12]

$$\langle \sigma v \rangle = \iint f_1(v_1) f_2(v_2) \sigma(v) v dv_1 dv_2 \dots \dots \dots (6)$$

Where $v = |v_1 - v_2|$, and $f_j(v_j)$ is the velocity distribution of species j ($j = 1, 2$) in the lab frame. By assumption that the velocities of two species are obey the Maxwell–Boltzmann distribution with a same temperature T

$$f_j(v_j)dv_j = \left(\frac{m_j}{2\pi kT}\right)^{3/2} \exp\left(-\frac{1}{2}m_j v_j^2/kT\right) dv_j \quad \dots\dots\dots (7)$$

Equation (6) written into explicit form:

$$\langle \sigma v \rangle = \left(\frac{m_1 m_2}{4\pi^2 k^2 T^2}\right)^{3/2} \iint \exp\left(-\frac{m_1 v_1^2 + m_2 v_2^2}{2kT}\right) \sigma(v) v dv_1 dv_2 \quad \dots\dots\dots (8)$$

It is appropriate to replace the variables of the integration from (v_1, v_2) to (v, v_r) , where v_r is the velocity of the center of mass of the two reacting particles. By these transformations, the $dv_1 dv_2 = dv dv_r$. Moreover, the total kinetic energy of reacting particles in the lab frame, $(1/2)m_1 v_1^2 + (1/2)m_2 v_2^2$, is changed by $(1/2)\mu v^2 + (1/2)(m_1 + m_2) v_r^2$, the total kinetic energy of the particles according to the center of mass in addition to the energy of motion of the center of mass. According to these results, which added to into Eq. (8), the integrals can be separated where: [12]

$$\begin{aligned} \langle \sigma v \rangle &= \left(\frac{m_1 m_2}{4\pi^2 k^2 T^2}\right)^{3/2} \int \exp\left(-\frac{1}{2}\mu v^2/kT\right) \sigma(v) v dv \times \int \exp\left(-\frac{m_1 + m_2}{2kT} v_r^2\right) v_r^2 dv_r \\ &= 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int_0^\infty \exp\left(-\frac{1}{2}\mu v^2/kT\right) \sigma(v) v^3 dv \quad \dots\dots\dots (9) \end{aligned}$$

Here, must assume the velocity distributions is isotropic. And by making another change to a variable from v to E ($=\mu v^2/2$), the energy in the C.M system, we get the latest form of the reactivity, to calculate the thermonuclear rates we have [13]

$$\langle \sigma v \rangle = (8/\pi m c^2 T^3)^{1/2} \int_0^\infty E \sigma(E_r) e^{-\frac{E}{T}} dE \quad \dots\dots\dots (10)$$

Where the cross-section can be represented in C.M units;

$E = 1/2\mu v^2$, $m = m_1 m_2 / (m_1 + m_2)$ and v is the relative velocity of the reacting particles that have masses m_1 and m_2 , It is worth mentioning that laboratory energies may be used if the replacement $E = (m/m_1) E_{lab}$ is made.

CALCULATIONS AND RESULTS

The fusion cross sections for the two reactions $D+^3\text{He}$ and $T+^3\text{He}$ (6%) that mentioned in table (1) are determined by using Eq.(1) for range of energies from 1 to 1000 KeV, and the calculations are clearly shown in fig.(2).

The slowing-down energy distribution function of protons in a thermonuclear reactions (reaction 4, 7 in table (1)) are calculated by using Eq. (2). It is supposed to have reacting mixture in thermodynamic equilibrium and the cross section for produced protons is isotropic in the C.M system.

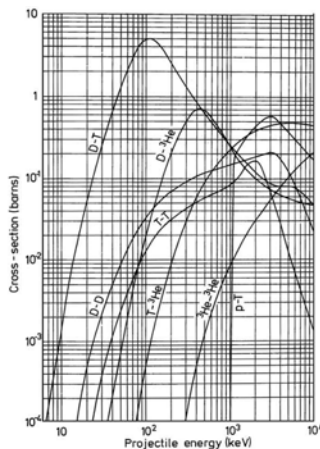


FIGURE 1. Standard cross-sections for the primary fusion reactions as a function of the projectile energy.

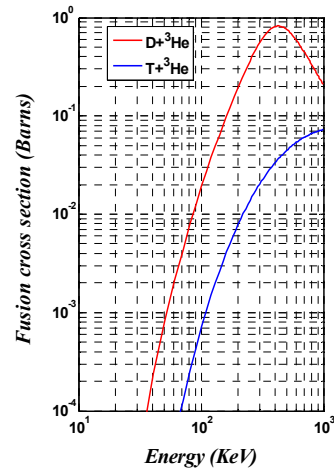


FIGURE 2. Cross-sections of the fusion reactions plotted according Eq. (1) in C.M coordinate

So we can calculate the distribution function using Eq. (2) through performing the integration over cross section $\sigma(E_r)$ Eq. (1) by *Maple* software. The (thermal) bulk of the produced particles populations was described by Maxwellian distribution and these calculations are achieved by fixing the following physical parameters: ion density $n_{\text{ion}} = 10^{20} \text{ m}^{-3}$, electron and ion temperatures of $T_e = T_i = 5 \text{ keV}$.

To deduce a distribution function for number of particles as a function of energy, one has to take into account the average population of each energy state. Therefore, the average of these states are calculated and their behavior described in figs (3, 4) as a function of energy.

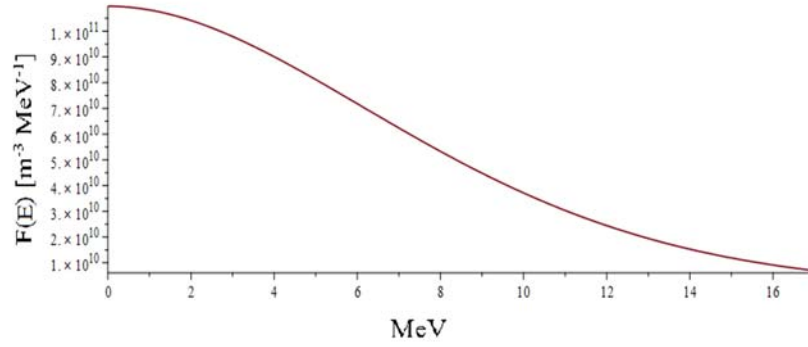


FIGURE 3. Slowing-down energy distribution of fusion product particles (protons) in D-³He plasmas.

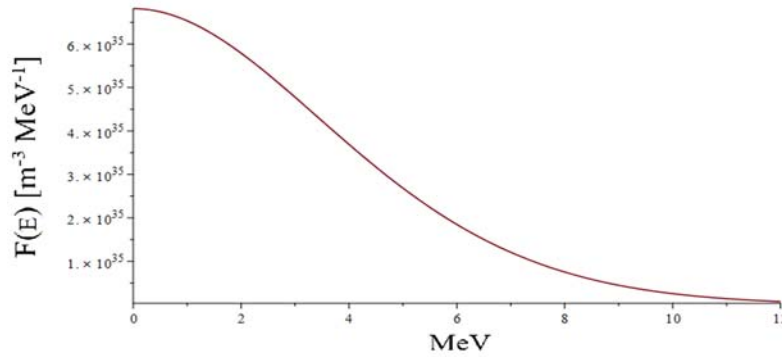


FIGURE 4. Slowing-down energy distribution of fusion product particles (protons) in T-³He plasmas.

The calculated fig. (3) and corresponding published fig. (5) in ref [14], reflects suitable agreement between their behaviors. This gives us the conviction that our choice of relations was right.

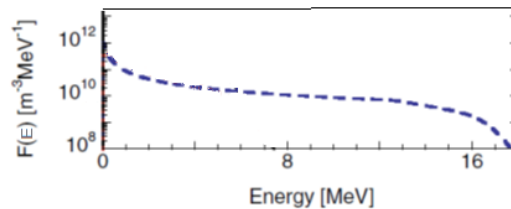


FIGURE 5. Slowing-down energy distributions in the fusion products particles (protons) in D-³He plasmas.

Calculations of plasma reactivity for these selected fusion reactions are required reaction cross-sections for wide ranges of energies. Therefore, it is necessary to extrapolate downwards calculations using the theoretical formula (10).

Provided that the cross-sections have the simple form Eq. (1) applicable at all energies, the integration can be performed in Eq. (10) over cross sections by maple software. The calculations involve the fusion reactivity that strongly related or depended upon the chosen cross section formula and concentrated on the common hydrogen isotopes fuels (Deuterium, Tritium) itself reactions, together, and with helium gas as described in table (1).

Our results about fusion reactivity are completely described pictorially in the following fig. (6,7) and table (2) for the $D-^3He$ and $T-^3He$ controlled thermonuclear fusion reactions.

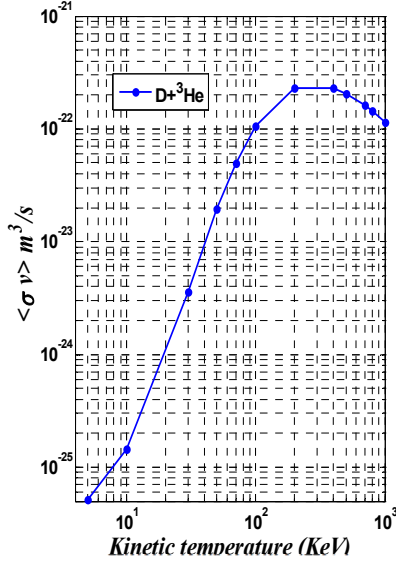


FIGURE 6. Fusion reactivity for $D-^3He$ reaction as function of ion temperature.

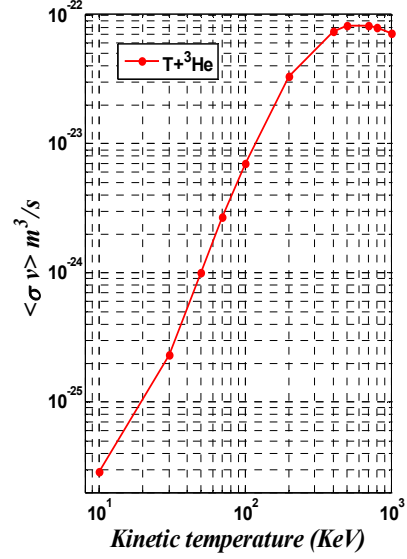


FIGURE 7. Fusion reactivity for $T-^3He$ reaction as function of ion temperature.

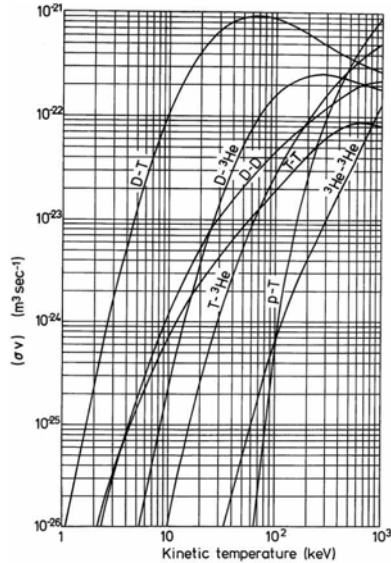


FIGURE 8. Standard fusion reactivity for $T-^3He$ and $T-^3He$ reactions as function of ion temperature.

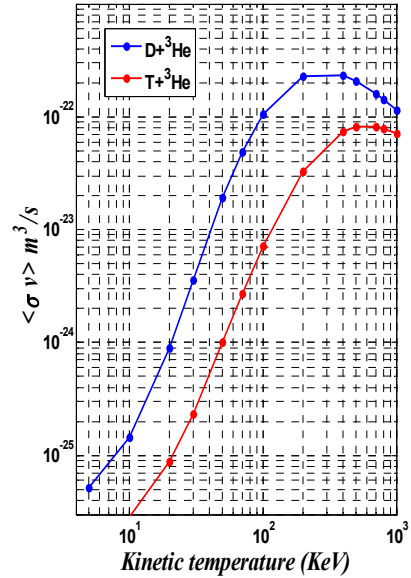


FIGURE 9. Fusion reactivity for $T-^3He$ and $T-^3He$ reactions as function of ion temperature.

TABLE 2. Fusion Reactivity for the Selected Fusion Reactions.

T KeV	Average Reactivity $\langle\sigma v\rangle$ m ³ /s	
	D+ ³ He	T + ³ He
1	1.4384e-026	4.9924e-027
10	1.4445e-025	2.8847e-026
30	3.5652e-024	2.3171e-025
50	1.9371e-023	1.0085e-024
70	4.9306e-023	2.6914e-024
100	1.0576e-022	7.0908e-024
200	2.2947e-022	3.2997e-023
400	2.3118e-022	7.4556e-023
500	2.0642e-022	8.1688e-023
700	1.6129e-022	8.2054e-023
800	1.4326e-022	7.9120e-023
1000	1.1503e-022	7.1480e-023

DISCUSSION AND CONCLUSION

The deduction of the slowing-down energy distribution functions is a massive mathematical problem, which uses the calculus of variations to derive the most probable distribution according to the explicit or understood rules or principles governing of constant energy and constant number of particles.

It is clear from the calculated Maxwellian average reactivity that is described in fig. (9) will appear or show compatible agreement with the published experimental results described clearly in fig. (8). We came to the fact that the mathematical results slightly differ from experimental and semi-experimental results, especially in some circumstances that occur in the experiment and losses in all parts of the plasma devices and due to parameterizations that used for modeling $\sigma(E)$ equation. One can interpret this case, that we choose a wide suitable formulism in calculating the fusion reaction cross section that plays an important role in adjusting all the global characteristics factors and this case leads to the conclusion that the physical behavior of such fusion interaction phenomena is very closed involves in the cross section formula. Finally, our result does not represent the optimum case but can represent a key for bringing a right decision by trying some updates choose empirical formulas for adjusting the most accurate cross section results, which in turn lead to fusion characteristics factors and finally we can reach a compatible fusion system design or construction.

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