Ministry of Higher Education and Scientific Research

University Al -Qadisiyah

College of Computer Science & Information Technology
Department of Mathematics

Applications of Conformal Mapping on a subclass of Meromorphic Univalent Funchions Defined by an operator.

A research

Submitted to the Department of Medical mathematics/College of Computer science δ Information Technoloy/ University of Al-Qadesiyah as partial fulfillment the requirement of the Degree of Bachelor of science in Medical mathematics.

By

The Student : Maryam Ismail Abis

Superrised by

Prof. Dr. Waggas Galib Atshan

2018م 1439هـ

بسم الله الرحمن الرحيم

(وَقُلِ اعْمَلُوا فَسَيَرَى اللَّهُ عَمَلَكُمْ وَرَسُولُهُ وَالْمُؤْمِنُونَ)

(سورة التوبة: (105))

الشكر والتقدير

في مثل هذه اللحظات يتوقف اليراع ليفكر قبل أن يخط الحروف ليجمعها في كلمات

... تتبعثر الأحرف وعبثاً أن يحاول تجميعها في سطور

سطوراً كثيرة تمر في الخيال ولا يبقى لنا في نهاية المطاف إلا قليلاً من الذكريات وصور تجمعنا برفاق كانوا إلى جانبنا.

فواجب علينا شكرهم ووداعهم ونحن نخطو خطوتنا الأولى في غمار الحياة
ونخص بالجزيل الشكر والعرفان إلى كل من أشعل شمعة في دروب عملنا

وإلى من وقف على المنابر وأعطى من حصيلة فكره لينير دربنا
إلى الأساتذة الكرام في كلية علوم الحاسوب وتكنولوجية معلومات
ونتوجه بالشكر الجزيل إلى

الدكتور

وقاص غالب عطشان
الذي تفضل بإشراف على هذا البحث فجزاه الله عنا كل خير فله منا كل التقدير والاحترام ..

اهداء

إلى من علمني النجاح والصبر

إلى من افتقده في مواجهة الصعاب

ولم تمهله الدنيا لأرتوي من حنانه .. أبي

وإلى من تتسابق الكلمات لتخرج معبرة عن مكنون ذاتها

من علمتني وعانت الصعاب لأصل إلى ما أنا فيه

وعندما تكسوني الهموم أسبح في بحر حنانها ليخفف من آلامي .. أمي

وإلى إخوتي وأسرتي جميعاً

ثم إلى كل من علمني حرفاً أصبح سنا برقه يضيء الطريق أمامي

Abstract

In this paper , we consider the subclass $D(k, \alpha, \lambda, \mathrm{q}, \mathrm{s})$ consisting of meromorphic univalent function defined by an operator. We obtain some geometric properties, like coefficient inquality, distortion bounds, extreme points, and Closure theorem. Also, some applications comformal mapping are obtained.

Key words: Univalent function, Coefficient inequality , Distortion bounds, Extreme points, Hadamard product , Conformal mapping .

AMS subject classification : 30C45.

Complex المحتويات

1-Introduction:

Let ∑ be the class of all function of the form:

$$f(z) = z^{-1} + \sum_{n=1}^{\infty} a_n \quad z^n \,, \qquad (1)$$

which are analytic meromorphic univalent in the punctured unit disk $U^* = \{z \epsilon \mathbb{C}: 0 < |z| < 1\}$. Let N denote the subclass of ∑ containing of function of the form:

$$f(z) = z^{-1} + \sum_{n=1}^{\infty} a_n \, z^n, (a_n \geq 0, n\epsilon\mathbb{N}). \qquad (2)$$

The Hadamard product of two function f is given by (2) and

$$g(z) = z^{-1} + \sum_{n=1}^{\infty} b_n z^n, (b_k \geq 0, n \in \mathbb{N}).$$

is defined by

$$(f * g) = \mathrm{z}^{-1} + \sum_{n=1}^{\infty} a_n \, b_\mathrm{n} \mathrm{z}^\mathrm{n}$$

For Positive real values of a1,……,aq and

b1,…,b bj ≠0,-1,-2,…; j=1,2,…,s) now, We need generalized hypergeometric function qFs(a1,…,aq;b1,…bs;z) here, defind by

$$qF_s\left(a_1, \dots, a_q; b_1, \dots, b_s; z\right) = \sum_{k=0}^{\infty} \frac{(a_1)k \dots (a_q)kz^k}{(b_1)k \dots (b_s)k \; k!},$$

$$(q \leq s+1, q, s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, z \in U),$$

Where (x)k is the pochhammer symbol defined by

$$(x)k = \begin{cases} 1 & , k = 0 \\ x(x+1)\dots(x+k-1), & k \in \mathbb{N} \end{cases}$$

Corresponding to the function $h(a1, \dots, a_q; b_1, \dots, b_s; z)$, defined by

$$h(a_1, \dots, a_q; b_1, \dots, b_s; z) = z^{-1} \; qF_s(a_1, \dots, a_q; b_1, \dots, b_s; z);$$

We consider a linear operator

$$H{}_{s}^{q}(a_1, \dots, a_q; b_1, \dots, b_s): N \to N,$$

which is defined by means of following Hadamard product
(or convolution):

$$H{}_{s}^{q}(a_1, \dots, a_q; b_1, \dots, b_s): f(z),$$

$$= \mathrm{h}(a_1, \dots, a_q; b_1, \dots, b_s; z) * f(z).$$

We observe that ,for a funuction f(z)of the form(2) ,we have

$$H{}_{s}^{q}(a_1, \dots, a_q; b_1, \dots, b_s) f(z) = z^{-1} + \sum_{n=1}^{\infty} s(n) a_n z^n,$$

where

$$s(n) = \frac{(a1)_{n+1} \dots (aq)_{n+1})}{(b1)_{n+1}, \dots, (bs)_{n+1}(n+1)!}$$

If ,for convenience, We write

$$H{}_{s}^{q}[a_1] = H{}_{s}^{q}(a_1, \dots, a_q; b_1, \dots, b_s).$$

The linear operator $H{}_{s}^{q}[a_1]$ Was investigated recently by Liu and Srivastava[11].Some Interesting subclasses of analytic function associated with The generalized hypergeometric function ,Were considered recently by (for example) Dziok et.al.[8] ,Atintas and Owa [2], Good man[9] ,Atshan and Kulkarni[6]and Liu[10].

For $\frac{1}{2} \leq \mathrm{k} < 1, 0 \leq \lambda < \frac{1}{3}$, $\alpha > 1$. We say that $f \in \mathbb{N}$ is in the

Class **D**$(k, \alpha, \lambda, q, s))$ If the following condition is satisfied:

$$\left| \frac{(1-2k)z\left(H_{s}^{q}[a_1]f(z)\right)'}{z^2\lambda\left(H_{s}^{q}[a_1]f(z)\right)'' + z(1-\lambda)\left(H_{s}^{q}[a_1]f(z)\right)'} \right|$$

$$< \alpha, (z \in U) \qquad (3)$$

Some authors studied meromorphic univalent functions for other classes , like, Aouf[3], Aouf et al.[4],Altintas et al.[1], Atshan _[5], Cho et al.[7]and Dziok et al.[8].

2.Coefficient Ineguality:

The following theorem ,gives a necessary and sufficient condition for a function f to be in the class D(k, α, λ, q, s).

Theorem (2.1):A function $f \epsilon N$ belongs to the class

D$(k, \alpha, \lambda, q, s)$ if and only if

$$\sum_{n=1}^{\infty}\left((1-2k) + \alpha(n\lambda - 2\lambda + 1)\right)S(n)a_n \qquad (4)$$

$$\leq \alpha(1-3\lambda) + (2k-1)$$

where

$$\frac{1}{2} \leq k < 1, 0 \leq \lambda < \frac{1}{3}, \alpha > 1, S(n) = \frac{(a_1)n + 1 \ldots (a_q)n + 1}{(b_1)n + 1 \ldots (b_s)n + 1(n+1)!}$$

The result (4)is sharp for the function

$$f(z) = z^{-1} + \frac{\alpha(1-3\lambda) + (2k-1)}{n((1-2k) + \alpha(n\lambda - 2\lambda + 1)S(n)} z^{\mathrm{n}},$$

n= 1,2,... .

Proof: Assume that the inequality (4)holds true and |z|=1,

then we have

$$\left|(1-2k)z(H\genfrac{}{}{0pt}{}{q}{s}[a_1]f(z))'\right|$$

$$-\alpha\left|\lambda z^2\left(H\genfrac{}{}{0pt}{}{q}{s}[a_1]f(z)\right)'' + (1+\lambda)z(H\genfrac{}{}{0pt}{}{q}{s}[a_1]f(z))'\right|$$

$$=\left|(1-2k)(-z^{-1}+\sum_{n=1}^{\infty} n\, s(n)a_n z^n)\right|$$

$$-\alpha\left|(\lambda(2z^{-2}\right.$$

$$+\sum_{n=1}^{\infty} n(n-1)s(n)a_n z^n) + ((1-\lambda)(-z^{-1}$$

$$\left.+\sum_{n=1}^{\infty} ns(n)a_n z^n))\right|$$

$$=\left|-(1-2k)z^{-1}+\sum_{n=1}^{\infty}(1-2k)n\, s(n)a_n z^n\right|$$

$$-\alpha\left|2\lambda z^{-1}+\sum_{n=1}^{\infty}(\lambda n(n-1)s(n)s(n)a_n z^n-(1-\lambda)z^{-1}\right.$$

$$\left.+\sum_{n=1}^{\infty}(1-\lambda)n\, s(n)a_n z^n\right|$$

$$\leq (1-2k)+\sum_{n=1}^{\infty}(1-2k)n\, s(n)a_n-(\alpha(1-3\lambda)$$

$$+\sum_{n=1}^{\infty}\alpha_n(n\lambda-2\lambda+1)s(n)a_n\,.$$

$$\sum_{n=1}^{\infty} n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)a_n-\big(\alpha(1-3\lambda)+(2k-1)\big)\leq 0$$

by hypothesis. Hence by maximum modulus principle $f\ \epsilon D(k,\alpha,\lambda,q,s)$.

Conversely ,suppose that f defined by (2) is in the Class D(k,α,λ,q,s).

Hence

$$\left|\frac{(1-2k)z\left(H_{s}^{q}[a_1]f(z)\right)'}{z^2\lambda\left(H_{s}^{q}[a_1]f(z)\right)''+z(1-\lambda)\left(H_{s}^{q}[a_1]f(z)\right)'}\right|$$

$$=\left|\frac{-(1-2k)z^{-1}+\sum_{n=1}^{\infty}(1-2k)ns(n)a_nz^n}{(3\lambda-1)z^{-1}+\sum_{n=1}^{\infty}n(\lambda n-2\lambda+1)s(n)a_nz^n}\right|$$

$$=\left|\frac{-(1-2\lambda)z^{-1}+\sum_{n=1}^{\infty}(1-2k)ns(n)a_nz^n}{(1-3\lambda)z^{-1}-\sum_{n=1}^{\infty}n(\lambda n-2\lambda+1)s(n)a_nz^n}\right|<\alpha.$$

Since Re(z)≤|z| for all z, we have

$$\mathrm{Re}\left\{\frac{-(1-2\lambda)z^{-1}+\sum_{n=1}^{\infty}(1-2k)ns(n)a_nz^n}{(1-3\lambda)z^{-1}-\sum_{n=1}^{\infty}n(\lambda n-2\lambda+1)s(n)a_nz^n}\right\}<\alpha.$$

We can choose the value of z on the real axis. Let z→1^-

through real values , we obtain the inequality (4).

Finally, sharpness if we take

$$f(z)=z^{-1}+\frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)}z^n,$$

$n=1,2,\ldots$

Corollary(2.1):. Let $f \epsilon D(k,\alpha,\lambda,q,s)$.Then

$$a_n \leq \frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)}, n = 1,2,\ldots \qquad (5)$$

3.Distortion bounds:

In the following theorems, we obtain the growth and distortion theorems for a function $f \epsilon D(k,\alpha,\lambda,q,s)$

Theorem(3.1): Let the function $f \epsilon D(k,\alpha,\lambda,q,s)$.Then

$$\frac{1}{|z|} - \frac{\alpha(1-3\lambda)+(2k-1)}{\big((1-2k)+\alpha(1-\lambda)\big)s(1)}|z| \leq |f(z)| \leq \frac{1}{|z|} +$$

$$\frac{\alpha(1-3\lambda)+(2k-1)}{\big((1-2k)+\alpha(1\lambda)\big)s(1)}|z|, |z| < 1.$$

The result is sharp and attained

$$f(z) = z^{-1} + \frac{\alpha(1-3\lambda)+(2k-1)}{\big((1-2k)+\alpha(1-\lambda)\big)s(1)}$$

Proof:

$$|f(z)| = \left|z^{-1} + \sum_{n=1}^{\infty} a_n z^n\right| \leq \frac{1}{|z|} + \sum_{n=1}^{\infty} a_n |z|^n$$

$$\leq \frac{1}{|z|} + |z| \sum_{n=1}^{\infty} a_n.$$

By Theorem (2.1), we get

$$\sum_{n=1}^{\infty} a_n \leq \frac{\alpha(1-3\lambda)+(2k-1)}{\big((1-2k)+\alpha(1-\lambda)\big)s(1)}.$$

Thus

$$|f(z)| \leq \frac{1}{|z|} + \frac{\alpha(1-3\lambda)+(2k-1)}{((1-2k)+\alpha(1-\lambda)s(1)}|z|.$$

Also

$$|f(z)| \geq \frac{1}{|z|} - \sum_{n=1}^{\infty} a_n |z|^n \geq \frac{1}{|z|} - |z| \sum_{n=1}^{\infty} a^n$$

$$\geq \frac{1}{|z|} - \frac{\alpha(1-3\lambda)+(2k-1)}{((1-2k)+\alpha(1-\lambda)s(1)}|z|$$

and this complex the proof.

Theorem(3.2): left $f \in D(k,\alpha,\lambda,q,s)$.Then

$$\frac{1}{|z|^2} - \frac{\alpha(1-3\lambda)+(2k-1)}{\left((1-2k)+\alpha(1-\lambda)\right)s(1)} \leq |f'(z)| \leq \frac{1}{|z|^2} + \frac{\alpha(1-3\lambda)+(2k-1)}{((1-2k)+\alpha(1-\lambda)s(1)},$$

With equality for the function

$$f(z) = z^{-1} + \frac{\alpha(1-3\lambda)+(2k-1)}{((1-2k)+\alpha(1-\lambda)s(1)} z.$$

Proof:

$$\left((1-2k)+\alpha(1-\lambda)\right)s(1) \sum_{n=1}^{\infty} n\, a_n$$

$$\leq \sum_{n=1}^{\infty} n\left((1-2k)+\alpha(n\lambda-2\lambda)+1\right)s(1)a_n$$

$$\leq \alpha(1-3\lambda)+(2k-1) \qquad (6)$$

From Theorem(2.1).Thus

$$\left|f'(z)\right| = \left| -z^{-2} + \sum_{n=1}^{\infty} n\, a_n z^{n-1} \right| \geq \frac{1}{|z|^2} - \sum_{n=1}^{\infty} n\, a_n |z|^{n-1} \Bigg|$$

$$\geq \frac{1}{|z|^2} - \sum_{n=1}^{\infty} n\, a_n$$

$$\geq \frac{1}{|z|^2}$$

$$-\frac{\alpha(1-3\lambda)+(2k-1)}{\big((1-2k)+\alpha(1-\lambda)\big)s(1)}. \qquad (7)$$

Also

$$|f'(z)| \leq \frac{1}{|z|^2} + \sum_{n=1}^{\infty} n\, a_n |z|^{n-1} \leq \frac{1}{|z|^2} + \sum_{n=1}^{\infty} n\, a_n$$

$$\leq \frac{1}{|z|^2} + \frac{\alpha(1-3\lambda)+(2k-1)}{\big((1-2k)+\alpha(1-\lambda)\big)s(1)} \qquad (8)$$

Combining (6),(7),we get requested result.

4.Extreme point:

In The following theorem ,we obtain the extreme point of the class $d(k,\alpha,\lambda,q,s)$

Theorem(4 .1): Let $f_0(z) = \frac{1}{z}$, and

$$f_{n(z)=\frac{1}{z}+\frac{\alpha(1-3\lambda)+(2k-1)}{n((1-2k)+\alpha(n\lambda-2\lambda+1)s(n)}z^n}, \quad n=1,2,\ldots,$$

Where $\frac{1}{2} < k < 1, 0 \leq \lambda < \frac{1}{3}, \alpha > 1$

Then the function $f \in D(k,\alpha,\lambda,q,s)$

If and only if it can be expressed in the form:

$$f(z)=\sum_{n=0}^{\infty}\mu_n f_n(z)$$

where

$$\mu_n\geq 0,\sum_{n=0}^{\infty}\mu_n=1$$

or

$$1=\mu_0+\sum_{n=0}^{\infty}\mu_n\,.$$

Proof: Let $f(z)$ can be expressed as in the form

$$f(z)=\sum_{n=0}^{\infty}\mu_n f_n(z)$$

Then

$$f(z)=\mu_0\frac{1}{z}+\sum_{n=1}^{\infty}\mu_n f_n(z)$$

$$=\mu_0\frac{1}{z}+\sum_{n=1}^{\infty}\mu_n\left(\frac{1}{z}+\frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)}z^n\right)$$

$$=\mu_0\frac{1}{z}+\sum_{n=1}^{\infty}\mu_n\frac{1}{z}$$
$$+\sum_{n=1}^{\infty}\mu_n\left(\frac{1}{z}+\frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)}z^n\right)$$

$$= \frac{1}{z}\left(\mu_0 + \sum_{n=1}^{\infty} \mu_n\right)$$

$$+ \sum_{n=1}^{\infty} \frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)} \mu_n z^n$$

$$= \frac{1}{z} + \sum_{n=1}^{\infty} \frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)} \mu_n z^n$$

$$= \frac{1}{z} + \sum_{n=1}^{\infty} \mu_n z^n$$

where

$$h_n = \frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)} \mu_n.$$

Thus

$$\sum_{n=1}^{\infty} h_n \cdot \frac{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)}{\alpha(1-3\lambda)+(2k-1)}$$

$$= \sum_{n=1}^{\infty} \frac{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)}{\alpha(1-3\lambda)+\alpha(2k-1)}$$

$$\cdot \frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)} \mu_n$$

$$\sum_{n=1}^{\infty} \mu_n = 1 - \mu_0 \le 1.$$

There for , we have $f \epsilon D(k, \alpha, \lambda, q, s)$.

Conversely , suppose that $f \epsilon D(k, \alpha, \lambda, q, s)$ .Then by (4),we have

$$a_n \leq \frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)}, (n=1,2,\dots)$$

We may set,

$$\mu_n = \frac{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)}{\alpha(1-3\lambda)+(2k-1)} a_n, (n=1,2,\dots)$$

and

$$\mu_0 = 1 - \sum_{n=1}^{\infty} \mu_n.$$

Then

$$f(z) = z^{-1} + \sum_{n=1}^{\infty} a_n z^n,$$

$$= z^{-1} + \sum_{n=1}^{\infty} \frac{\alpha(1-3\lambda)+(2k-1)}{n\big((1-2k)+\alpha(n\lambda-2\lambda+1)\big)s(n)} \mu_n z^n$$

$$= z^{-1} + \sum_{n=1}^{\infty} \mu_n \left(f_n(z) - z^{-1}\right)$$

$$= z^{-1}\left(1 - \sum_{n=1}^{\infty} \mu_n\right) + \sum_{n=1}^{\infty} \mu_n f_n(z)$$

$$= z^{-1}\mu_0 + \sum_{n=1}^{\infty} \mu_n f_n(z)$$

$$= \sum_{n=1}^{\infty} \mu_n f_n(z)$$

The proof complete.

5. Closure Theorem:

In the following theorem , we obtain the closure theorem of the function in the class $D(k,\alpha,\lambda,\mathrm{q},\mathrm{s})$.

Theorem(5.1): Let the function $f_i(z)$ defined by

$$f_i(z) = z^{-1} + \sum_{n=1}^{\infty} a_{n,j} z^n \,, (a_n, j \ \geq 0, j = 1,2, \dots, l)$$

be in the class $D(k,\alpha,\lambda,\mathrm{q},\mathrm{s})$. Then the function $h(z)$ defined by

$$h(z) = \sum_{j=1}^{i} d_j f_j(z)$$

and

$$\sum_{j=1}^{\iota} d_j = 1, d_j \geq 0$$

in the class $D(k,\alpha,\lambda,\mathrm{q},\mathrm{s})$.

Proof : By definition of $h(z)$, we have

$$h(z) = \sum_{j=1}^{\iota} d_j \, \mathrm{z}^{-1} + \sum_{\mathrm{n}=1}^{\infty} \left[\sum_{j=1}^{\iota} d_j a_n, j \right] z^n$$

$$= \mathrm{z}^{-1} + \sum_{j=1}^{\iota} \left(\sum_{j=1}^{\iota} d_j a_n, j \right) z^n.$$

Since $f_j(z)$ are in the class $D(k,\alpha,\lambda,\mathrm{q},\mathrm{s})$ for every $j = 1,2, \dots, \iota$, We obtain

$$\sum_{\mathrm{n}=1}^{\infty} \mathrm{n}\big((1-2\mathrm{k})+(\mathrm{n}\lambda-2\lambda+1)\big)\mathrm{s}(\mathrm{n})\mathrm{a}_{\mathrm{n}}$$

$$\leq \alpha(1-3\lambda)+(2\mathrm{k}-1).$$

Hence

$$\sum_{n=1}^{\infty} \mathrm{n}\big((1-2\mathrm{k})+\alpha(\mathrm{n}\lambda-2\lambda+1)\big)\mathrm{s}(\mathrm{n})\left[\sum_{j=1}^{\iota} d_j a_n, j\right]$$

$$=\sum_{j=1}^{\iota} d_j\left[\sum_{n=1}^{\infty} \mathrm{n}\big((1-2\mathrm{k})+\alpha(\mathrm{n}\lambda-2\lambda+1)\big)\mathrm{s}(\mathrm{n})a_n, j\right]$$

$$\leq \big(\alpha(1-3\lambda)+(2\mathrm{k}-1)\big)\sum_{j=1}^{\iota} d_j$$

$$=\alpha(1-3\lambda)+(2\mathrm{k}-1).$$

The $h(z) \epsilon D(k, \alpha, \lambda, \mathrm{q}, \mathrm{s})$.

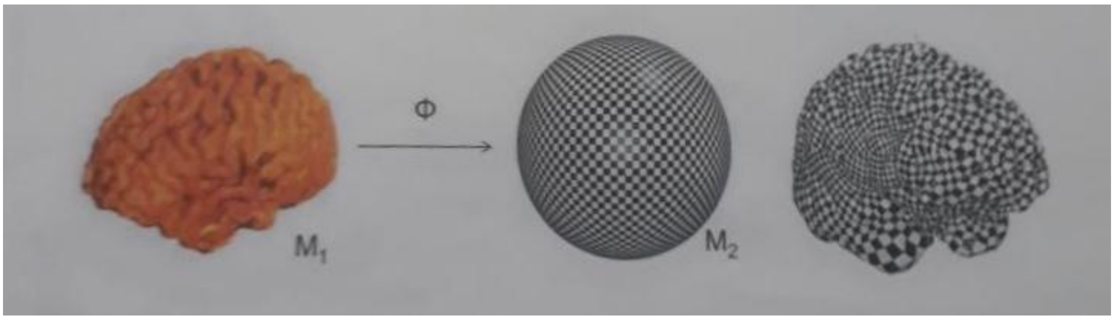
The proof is complete.

Applications

- Motivation
- Part I: Conformal Geometry & Applications
    - Basic Mathematical Background
    - Computational Algorithms
    - Applications
- Part II: Quasi-conformal Geometry & Applications
- Conclusion

Why conformal for Brain Mapping?

- Metric preserved up to scaling → Local geometry preserved
- Angle-preserving → inherits a natural orthogonal grid on the surface.
- simple (g) Matrix → simple differential operator expression on the parameter domain and simple projected equations.



What is Conformal map?

- Conformal map $f: M \rightarrow N$ = preserves inner product up to a scaling factor (the conformal factor 1).

Conformal factor

- Mathematically, $f * (ds_N^2) = \lambda(x_1, x_2) ds_M^2$ where $ds_M^2 = \sum_{i,j=1}^{2} g_{ij} dx^i \Delta \, dx^j$

References

[1]-O. Altintas , H. Irmark and H. M.Srivastava va , A family of meromorphically univalent functions with positive coefficients, Pan. Amer. Math .J. ,5(1)(1995),75-81.

[2]-O.Altintas and S.Owa, Neighborhoods of certain analytic functions with negative coefficients, Int. J.Math Sci,19(1996),797-800.

[3]- M.K.Aouf, On a certain class of meromorphic univalent function with positive coefficients, Rendicontidi Mathematica Serie VII, Vol.ll, Roma ,(1991), 209-219.

[4]- M.K> Aouf, N. Magesh, J.Jothibasu and S.Murthy, On a certain class of meromorphic functions with positive coefficients Stud. Uuiv Babes Bokyai Math ,58(1)(2013), 31-42.

[5]- W.G. Atshan, Subclass of meramorphic functions with positive coefficients defined by Ruschewegh derivative II Surveys in Mathematics and its Applications, 3(2008), 67-77.

[6]- W.G. Atshan and S.R.Kulkarin, On a class of P-Valent analytic functions with negative coefficients defined by Dziok-srivastava va linear operator ,
Inter J.Math. Sci &Engg. Appls.(IJMSEA),2(1)(2007), 173-182 .

[7]-N.E. cho, S. H. Lee and S.Owa, Aclass of meromorphic univalent functions with positive coefficients, Koebe J. Math ,4(1987), 43-50.

[8]- J. Dziok, G. Murugusundaramoorthy and J.Sokol, On certain class of meromorphic functions with positive coefficients, Acta Mathematica Scientia (2012), 32B(4), 1376-1390.

[9]-A W.Goodman, Univalent functions and non-analytic curves, Proc. Amer Math. Soc., 8(1957), 598-601.

[10]- J.L.Liu, Properties of some families of meromorphic p-valent functions, Math Japonica, 52(2000), 425-434.

[11] - J.L,Liu and H.M.Srivastava va Alinear operator and associated families of meromorphically multivalent functions J.Math. Anal. Appl. 259(2001), 566-581.

المستخلص

في هذا البحث تناولنه الصف الجزئي $D(k, \alpha, \lambda, \text{q}, \text{s})$ من الدوال احادية التكافؤ الميرومورفيت والمعرفة بواسطة مؤثر حيث حصلنا على بعض الخواص الهندسية مثل , متراجحة المعامل , حدود التشوية , النقاط المتطرفة , مبرهنة الانغلاق وايضا حصلنا على تطبيقات الدوال الحافظه للزوايا.

وزارة التعليم العالي والبحث العلمي

جامعة القادسية : كلية علوم الحاسوب وتكنولوجية المعلومات

القسم : رياضيات الطبية

تطبيقات الدوال الحافظة للزوايا على صنف جزئي من الدوال احادية التكافؤ الميرومورفية بواسطة مؤثر.

بحث مقدم الى قسم الرياضيات الطبية / **كلية علوم الحاسوب وتكنولوجيا المعلومات** / جامعة القادسية كاستكمال جزئي لمتطلبات نيل درجة بكالوريوس علوم في الرياضيات الطبية

من قبل الطالبة : مريم اسماعيل عبيس

اشراف : أ.د. وقاص غالب عطشان

1439 هـ 2018 م