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Properties of a class of Multivalent Functions Defined by Ruscheweyh Derivative

A Research submitted

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by

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بسم الله الرحمن الرحيم

(هُوَ الَّذِي جَعَلَ الشَّمْسَ ضِيَاءً وَالْقَمَرَ نُورًا وَقَدَّرَهُ مَنَازِلَ

لِتَعْلَمُوا عَدَدَ السِّنِينَ وَالْحِسَابَ مَا خَلَقَ اللَّهُ ذَٰلِكَ إِلَّا بِالْحَقِّ

يُفَصِّلُ الْآيَاتِ لِقَوْمٍ يَعْلَمُونَ)

صدق الله العلي العظيم

الإهداء

إلى سيدي ومولاي أمير المؤمنين

علي ابن ابي طالب

(عليه السلام)

شفاعة لرضا الخالق

سبحانه وتعالى

الشكر والعرفان

((إذا قصرت يدك عن المكافأة فليطل لسانك بالشكر))

الامام علي (عليه السلام)

الحمد لله الذي من على عباده بتمام الخلق ونعمة الدين وفتح لهم خزائن الدنيا واسرار اليسير من العلم ليكونوا له من الوارثين .

من دواعي الوفاء والعرفان بالجميل إن اثني ثناء الشاكرين على مشرفي الاستاذ (د . وقاص غالب عطشان) اطال الله بعمره وجعله معينا لسالكي طريق العلم والمعرفة لما بذله من جهد في قراءة البحث ولتوجيهاته التي أغنت البحث بالشيء الكثير

واتقدم بالامتنان المقرون بالدعاء من الله سبحانه وتعالى إن يمن على الاستاذ (د . قصي حاكم) بطول العمر والعزة ليبقى المنهل الذي يفيض بعلمه الغزير على طلبة قسم الرياضيات لما أبداه من رعاية ولطف أبويين

والتمس العذر من لم يجد اسمه في صفحة الشكر سواءً ساعدني بكلمة أم اطراء جميل او تمنٍ بدعاء لإكمال البحث واقول لهم وفقكم الله .

جمهورية العراق
وزارة التعليم العالي والبحث العلمي
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كلية علوم الحاسوب وتكنولوجيا المعلومات
قسم الرياضيات

خواص لصنف من الدوال متعددة التكافؤ المعرفة بواسطة مشتقة رشاوية

بحث مقدم
الى مجلس كلية علوم الحاسوب والرياضيات / قسم الرياضيات وهو جزء من متطلبات نيل درجة البكالوريوس علوم في الرياضيات

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Abstract

In the present, we introdced a new class $N(\gamma, P, \lambda, A, B, v)$

Of p *- valent functions in the unit disk* $U = \{z \in C : |z| < 1\}$
Defined by Ruscheweyh derivative. We obtain some certain properties, like, Coefficient inequality, distortion theorem , radii of starlikeness and convexity and convex linear combinations.

AMS subject classification: 30C45

Keywords: P-Valent function, Ruscheweyh derivative, distortion theorem , radius of starlikeness , convex linear combination.

Contents

المستخلص:

في البحث الحالي، قدمنا صنف جديد $N(\text{Ɣ}, P, \lambda, A, B, V)$ من الدوال متعددة التكافؤ في قرص الوحدة $\mathrm{U}: \{\mathrm{z} \in \mathrm{C}: |\mathrm{z}| < 1\}$ والمعرفة بواسطة مشتقة رشاوية حيث حصلنا على بعض الخواص الاكيدة، مثل، متراجحة المعامل، مبرهنة التشوية، انصاف اقطار النجمية والتحدب والتراكب الخطية المحدبة.

1. Introduction:

Let N_p *denote the class of function of the form :*

$$f(z) = z^P - \sum_{n=1}^{\infty} a_{n+p} z^{n+p}, \left(a_{n+p} \geq 0, p \in N = \{1,2,\ldots\}\right), \qquad (1.1)$$

which are analytic and p-valent in the unit disk U.

A function $f \in N_p$ *is said to be in the class* $C^*(\alpha)$ *if*

$$Re\left\{\frac{zf\acute{}(z)}{f(z)}\right\} > \propto, (0 \leq \propto \leq p, z \in U) \qquad (1.2)$$

The element of this class are called starlike functions of order $\propto$

A functions $f \in N_p$ *is said to be in the class* $S^*(\propto)$ *if*

$$Re\left\{1 + \frac{zf\acute{}(z)}{f(z)}\right\} > \propto, (0 \leq \propto \leq p, z \in U) \qquad (1.3)$$

The element of this class are called Convex functions of order $\propto$

We shall use Ruscheweyh derivative [6,7] $D^{\lambda+P-1}$ *of the function* f *given by (1,1) defined by*

$$D^{\lambda+p-1} f(z) = \frac{z^p}{(1-z)^{p+\lambda}} * f(z) =$$

$$z^p - \sum_{n=1}^{\infty} \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)} a_{n+p} z^{n+p} \qquad (1.4)$$

Definition (1.1): *let* $f \in N_p$ *given by (1.1). Then the class* $N(\gamma, P, \lambda, A, B, v)$ *is defined by :*

$$N(\gamma, P, \lambda, A, B, v) = \{f \in N_p : \left|\frac{p\left(\frac{(D^{\lambda+p} f(z)}{(D^{\lambda+p-1} f(z)} - 1\right)}{\gamma \frac{(D^{\lambda+p} f(z)}{(D^{\lambda+p-1} f(z)} + (A+B+\gamma)}\right| < v,$$

$0 \le \gamma < 1, 0 < A \le 1, 0 \le B < 1 \; and \; \lambda > -p \quad 0 < v < 1\}$ (1.5)

The p-valent functions studied by several authors for authors for another classes, like [1,2,3,4] and [5].

2. Coefficient Inequality:

The following theorem gives a sufficient and necessary condition for a function f to be in the class $N(\gamma, P, \lambda, A, B, v)$.

Theorem (2.1) : *let $f \in N_p$. Then let $f \in N(\gamma, P, \lambda, A, B, v)$ if and only if*

$$\sum_{n=1}^{\infty} \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)} \left(\frac{\lambda+n}{\lambda+p}(p+v\gamma) - p + v(A+B+\gamma) \right) a_{n+p} \le v(A+B+2\gamma), \qquad \mathbf{(2.1)}$$

$$where \; 0 \le \gamma < 1, 0 < A \le 1, 0 \le B < 1,$$
$$0 < v < 1, \lambda > -p \; and \; p \in \mathbb{N}$$

Proof: *suppose that the inequality (2.1) holds true and $|z| = 1$ .Then , we have*

$$\left| p\left(\frac{(D^{\lambda+p} f(z)}{(D^{\lambda+p-1} f(z)} - 1 \right) \right| - v \left| \gamma \frac{(D^{\lambda+p} f(z)}{(D^{\lambda+p-1} f(z)} + (A+B+\gamma) \right|$$

$$= \left| -\sum_{n=1}^{\infty} P\left(\frac{\lambda+n}{\lambda+p} - 1 \right) \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)} a_{n+p} z^{n+p} \right| -$$

$$v \left| (A+B+2\gamma) z^p - \sum_{n=1}^{\infty} \left(\gamma \frac{\lambda+n}{\lambda+p} + (A+B+\gamma) \right) \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)!} a_{n+p} z^{n+p} \right|$$

$$\le \sum_{n=1}^{\infty} P\left(\frac{\lambda+n}{\lambda+p} - 1 \right) \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)} a_{n+p} |z|^{N+P} - V(A+B+2\gamma)|z|^P$$

$$+ \sum_{n=1}^{\infty} \left(\gamma \frac{\lambda+n}{\lambda+p} + (A+B+\gamma) \right) \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)!} a_{n+p} |z|^{n+p}$$

$$= \sum_{n=1}^{\infty} \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)!} \left(\frac{\lambda+n}{\lambda+p}(p+v\gamma) - p + V(A+B+\gamma)\right) a_{n+p}|z|^{n+p}$$

$$-V(A+B+2\gamma)|z|^{P}$$

$$= \sum_{n=1}^{\infty} \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)!} \left(\frac{\lambda+n}{\lambda+p}(p+v\gamma) - p + V(A+B+\gamma)\right) a_{n+p} - V(A+B+2\gamma) \leq 0$$

By hypothesis. Hence by maximum modulus principle,
$f \in N(\gamma, P, \lambda, A, B, v)$

Conversely , suppose that , $f \in N(\gamma, P, \lambda, A, B, v)$*. Then from (1.5), we have*

$$\left| \frac{p\left(\frac{(D^{\lambda+p}f(z)}{(D^{\lambda+p-1}f(z)} - 1\right)}{\gamma\frac{(D^{\lambda+p}f(z)}{(D^{\lambda+p-1}f(z)} + (A+B+\gamma)} \right|$$

$$= \left| \frac{\sum_{n=1}^{\infty} P\left(\frac{\lambda+n}{\lambda+p} - 1\right)\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!} a_{n+p} z^{n+p}}{(A+B+2\gamma)z^{p} - \sum_{n=1}^{\infty} v\left(\gamma\frac{\lambda+n}{\lambda+p} + (A+B+\gamma)\right)\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!} a_{n+p} z^{n+p}} \right| < v$$

Since Re $(z) \leq |z|$ *for all* $z(z \in u)$*, we get*

$$Re\left\{ \frac{\sum_{n=1}^{\infty} P\left(\frac{\lambda+n}{\lambda+p} - 1\right)\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)!} a_{n+p} z^{n+p}}{(A+B+2\gamma)z^{p} - \sum_{n=1}^{\infty} v\left(\gamma\frac{\lambda+n}{\lambda+p} + (A+B+\gamma)\right)\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!} a_{n+p} z^{n+p}} \right\} < v$$

We choose the value of z on the real axis and letting $z \to 1-$ *through real values, we optain the inequality (2.1)*

Corollary (2.1): *Let* $f \in N(\gamma, P, \lambda, A, B, v)$*. Then*

$$a_{n+p} \leq \frac{v + (A+B+\gamma)}{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\gamma) - p + v(A+B+\gamma)\right)} \qquad (2.2)$$

The equality in (2.2) is attained for the function f given by the sharp for the function

$$f(z) = z^p - \frac{v(A+B+2Ɣ)}{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+vƔ)-p+v(A+B+Ɣ)\right)} z^{n+p} \qquad (2.3)$$

3. Distortion Theorem:-

In the following theorems, we obtain the distortion bounds for the function f in the class $N(Ɣ, P, \lambda, A, B, v)$.

Theorem (3.1): *let the function t $f \in N(Ɣ, P, \lambda, A, B, v)$. Then*

$$|z|^p - \frac{v(A+B+2Ɣ)}{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)!}\left(\frac{\lambda+1}{\lambda+p}(p+vƔ)-p+v(A+B+Ɣ)\right)} |z|^{p+1} \leq |f(z)| \leq |z|^p +$$

$$\frac{v(A+B+2Ɣ)}{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+1}{\lambda+p}(p+vƔ)-p+v(A+B+Ɣ)\right)} |z|^{p+1} \qquad (3.1)$$

The result is sharp for the function f given by

$$F(z) = |z|^p - \frac{v(A+B+2Ɣ)}{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+1}{\lambda+p}(p+vƔ)-p+v(A+B+Ɣ)\right)} |z|^{p+1}$$

Proof : *we have*

$F(z) = |z|^p - \sum_{n=1}^{\infty} a_{n+p} z^{n+p}$ *, then*

$|F(z)| \leq |z|^p - \sum_{n=1}^{\infty} a_{n+p} |z|^{n+p}$ *and since* $f \in N(Ɣ, P, \lambda, A, B, v)$.

Then by theorem (2.1), we have

$$\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+n)(n-p)!}\left(\frac{\lambda+1}{\lambda+p}(p+vƔ) - p + v(A+B+Ɣ)\right)\sum_{n=1}^{\infty} a_{n+p} \leq$$

$$\sum_{n=1}^{\infty} \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+vƔ) - p + v(A+B+Ɣ)\right) a_{n+p} \leq v + (A+B+2Ɣ)$$

Hence

$$\sum_{n=1}^{\infty} a_{n+p} \leq \frac{v(A+B+2\text{Ɣ})}{\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+p)(1-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\text{Ɣ})-p+v(A+B+\text{Ɣ})\right)} \qquad (3.2)$$

Thus

$$|F(z)| \leq$$

$$|Z|^p + |z|^{p+1}\sum_{n=1}^{\infty} a_{n+p} \leq |z|^p - \frac{v+(A+B+2\text{Ɣ})}{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\text{Ɣ})-p+v(A+B+\text{Ɣ})\right)}|z|^{p+1} \qquad (3.3)$$

Similarly, we have

$$|F(z)| \geq |z|^p - \sum_{n=1}^{\infty} a_{n+p}|Z|^{n+p} \geq |z|^p - |z|^{p+1}\sum_{n=1}^{\infty} a_{n+p}$$

$$\geq |z|^p \frac{v(A+B+2\text{Ɣ})}{\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+p)(1-p)!}\left(\frac{\lambda+1}{\lambda+p}(p+v\text{Ɣ})-p+v(A+B+\text{Ɣ})\right)}|z|^{p+1} \qquad (3.4)$$

From combining(3.3) and (3.4), we get (3.1).

4. Radii of starlikeness and Convexity:

In the following theorems, we obtain the radii of starlikeness and convexity of the class $N(\text{Ɣ}, P, \lambda, A, B, v)$.

Theorem (4.1): *let* $f \in N(\text{Ɣ}, P, \lambda, A, B, v)$. *Then f is multivalent starlike of order* $\beta, (0 \leq \beta < p)$ *in the disk* $|z| < r = r_1(\text{Ɣ}, P, \lambda, A, B, v, \beta)$.

Where

$$r_1(\text{Ɣ}, P, \lambda, A, B, v, \beta) = \inf_n \left\{\frac{(p+\beta)\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\text{Ɣ})-p+v(A+B+\text{Ɣ})\right)}{(n+p-\beta)\, v(A+B+2\text{Ɣ})}\right\}^{\frac{1}{n}}, \qquad (\mathbf{4.1})$$

N=1,2, …

The result is sharp for the function f given by (2.3).

Proof: *It is sufficient to show that*

$$\left|\frac{zf'(z)}{f(z)}-p\right|\leq p-\beta, \text{ for } |z|<r_1. \qquad (4.2)$$

But

$$\left|\frac{zf'(z)}{f(z)}-p\right|=\left|\frac{zf'(z)-pf(z)}{f(z)}\right|=\left|\frac{-\sum_{n=1}^{\infty}a_{n+p}z^n}{1-\sum_{n=1}^{\infty}a_{n+p}z^n}\right|\leq\frac{\sum_{n=1}^{\infty}a_{n+p}|z|^n}{1-\sum_{n=1}^{\infty}a_{n+p}|z|^n}$$

Thus, (4.2) will be satisfied if

$$\frac{\sum_{n=1}^{\infty}na_{n+p}|z|^n}{1-\sum_{n=1}^{\infty}na_{n+p}|z|^n}\leq p-\beta,$$

Or if

$$\sum_{n=1}^{\infty}\left(\frac{n+p-\beta}{p-\beta}\right)a_{n+p}|z|\leq 1\,.$$

Since $f\in N(\gamma,P,\lambda,A,B,v)$, *we have*

$$\sum_{n=1}^{\infty}\frac{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\gamma)-p+v(A+B+\gamma)\right)}{v(A+B+2\gamma)}a_{n+p}\leq 1.$$

Hence, (4.3) will be trove if

$$\frac{n+p-\beta}{p-\beta}|z|^n\leq\frac{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\gamma)-p+v(A+B+\gamma)\right)}{v(A+B+2\gamma)},$$

Or equivalently

$$|z|\leq\left\{\frac{(p+\beta)\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\gamma)-p+v(A+B+\gamma)\right)}{(n+p-\beta)\,v(A+B+2\gamma)}\right\}^{\frac{1}{n}},n\geq 1.$$

which follows the result.

Theorem (4.1): *let* $f\in N(\gamma,P,\lambda,A,B,v)$. *Then* f *is multivalent convex of order* $\beta,(0\leq\beta\leq P)$ *in* $|z|<r=r_2(\gamma,P,\lambda,A,B,v,\beta)$. *Where*

$$r_2(\gamma, P, \lambda, A, B, v, \beta) = \inf_n \left\{ \frac{p(p-\beta)\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\gamma)-p+v(A+B+\gamma)\right)}{(n+p)(n+p-\beta)\, v(A+B+2\gamma)} \right\}^{\frac{1}{n}}, n \geq 1$$

The result is sharp for the function f given by (2.3).

Proof: *It is sufficient to show that*

$$\left|\frac{zf'(z)}{f(z)} + 1 - p\right| \leq p - \beta, \text{ for } |z| < r_2.$$

But

$$\left|\frac{zf'(z)}{f(z)} + 1 - p\right| = \left|\frac{zf'(z)+(1-p)f'(z)}{f(z)}\right| = \left|\frac{-\sum_{n=1}^{\infty} n(n+p)a_{n+p}z^{n+p}}{Pz^p - \sum_{n=1}^{\infty}(n+p)a_{n+p}z^{n+p}}\right| \leq$$

$$\frac{\sum_{n=1}^{\infty} n(n+p)a_{n+p}|z|^n}{P-\sum_{n=1}^{\infty}(n+p)a_{n+p}|z|^n}$$

Thus, (4.4) will be satifed if

$$\frac{\sum_{n=1}^{\infty}(n+p)a_{n+p}|z|^n}{P-\sum_{n=1}^{\infty}(n+p)a_{n+p}|z|^n} \leq p - \beta,$$

Or if

$$\sum_{n=1}^{\infty}\left(\frac{(n+p)(n+p-\beta)}{p(p-\beta)}\right) n a_{n+p}|z|^n \leq 1 .$$

Since $f \in N(\gamma, P, \lambda, A, B, v)$, we have

$$\sum_{n=1}^{\infty} \frac{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)}\left(\frac{\lambda+n}{\lambda+p}(p+v\gamma)-p+v(A+B+\gamma)\right)}{N(A+B+2\gamma)} a_{n+p} \leq 1.$$

Hence , (4.5) will be true if

$$\left(\frac{(n+p)(n+p-\beta)}{p(p-\beta)}\right)|z|^n \leq \frac{\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)}\left(\frac{\lambda+n}{\lambda+p}(p+v\gamma)-p+v(A+B+v)\right)}{v(A+B+2\gamma)}.$$

or equivalently

$$|z| \leq \left\{ \frac{p(p-\beta)\frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!}\left(\frac{\lambda+n}{\lambda+p}(p+v\gamma)-p+v(A+B+\gamma)\right)}{(n+p)(n+p-\beta)\,v(A+B+2\gamma)} \right\}^{\frac{1}{n}}, n \geq 1,$$

which follows the result.

5. *Convex linear combination:*

Theorem (5.1): the class $N(\gamma, P, \lambda, A, B, V)$ *is closed under convex liner*

Combinations.

Proof: *let f and g be the arbitrary elements of* $N(\gamma, P, \lambda, A, B, V)$. *Then for every t(0<t<1), we show that (1-t)* $f(z) + tg(z) \in N(\gamma, P, \lambda, A, B, V)$ *,*
Thus we have

$$(1-t)f(z) + tg(z) = z^p - \sum_{n=1}^{\infty} [(1-t)a_{n+p} + tb_{n+p}] z^{n+p}$$

Therefore

$$\sum_{n=1}^{\infty} \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+n)(n-p)} \left(\frac{\lambda+n}{\lambda+p}(p+v\gamma) - p + v(A+B+\gamma)\right) [(1-t)a_{n+p} + tb_{n+p}]$$

$$= (1-t) \sum_{n=1}^{\infty} \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)} \left(\frac{\lambda+n}{\lambda+p}(p+v\gamma) - p + v(A+B+\gamma)\right) a_{n+p}$$

$$= t \sum_{n=1}^{\infty} \frac{\Gamma(\lambda+n)}{\Gamma(\lambda+p)(n-p)!} \left(\frac{\lambda+n}{\lambda+p}(p+v\gamma) - p + V(A+B+\gamma)\right) b_{n+p}$$

$$\leq (1-t)V(A+B+2\gamma) + tv(A+B+2\gamma) = V(A+B+2\gamma)$$

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