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College of Computer Science and Mathematics

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Some Geometric Properties of a Subclass of Univalent Functions with Applications

A Research submitted

To the Department of Medical Mathematics, College of Computer Science and Mathematics, University of Qadisiyah,

As partial fulfillment of requirements of the degree of B.sc. in medical mathematics

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بسم الله الرحمن الرحيم

(هُوَ الَّذِي جَعَلَ الشَّمْسَ ضِيَاءً وَالْقَمَرَ نُورًا وَقَدَّرَهُ مَنَازِلَ لِتَعْلَمُوا عَدَدَ السِّنِينَ وَالْحِسَابَ مَا خَلَقَ اللَّهُ ذَٰلِكَ إِلَّا بِالْحَقِّ ۚ يُفَصِّلُ الْآيَاتِ لِقَوْمٍ يَعْلَمُونَ)

صدق الله العلي العظيم

الإهداء

إلى فخر الكائنات البشير النذير السراج المنير، الرسول الكريم محمد صلى الله عليه واله الطاهرين

إلى الذي اعطاني الصبر والعزيمة

والدي العزيز

إلى رمز الحب والحنان

والدتي العزيزة

إلى من اشدد بهم أزري وأشركهم بأمري

أخواني

إلى الشموع التي أضاءت لي طريق العلم

أساتذتي

إلى رموز الوفاء ولإخلاص . . . . . أصدقائي

إلى كل من مد لي يد العون ولو بنصيح

لهم مني كل الخير

الشكر والتقدير

لا بد لنا ونحن نخطو خطواتنا الاخيرة في الحياة الجامعية من وقفه نعود الى اعوام قضيناها في رحاب الجامعة مع اساتذتنا الكرام الذين قدموا لنا الكثير باذلين بذلك جهودا كبيره في بناء جيل الغد لتبعث الامه من جديد وقبل ان نمضي نقدم اسمى آيات الشكر والامتنان والتقدير والمحبة الى الذين عملوا أقدس رسالة في الحياة . . . . . الى الذين مهدوا لنا طريق العلم والمعرفة . . . . الى جميع اساتذتنا الافاضل . . . .

"كن عالما فأن لم تستطع فكن متعلم، فان لم تستطع فأحب العلماء ، فأن لم تستطع فلا تبغضهم "

واخص بالشكر والتقدير واتقدم بخالص الشكر الجزيل والعرفان بالجميل والاحترام والتقدير لمن غمرني بالفضل واختصني بالنصح وتفضل علي بقبول الاشراف على بحثي استاذي ومعلمي أ . د . وقاص غالب عطشان

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كلية علوم الحاسوب والرياضيات

قسم الرياضيات الطبية

بعض الخواص الهندسية لصنف جزئي من الدوال أحادية التكافؤ مع التطبيقات

بحث مقدم

الى مجلس كلية علوم الحاسوب والرياضيات / قسم الرياضيات الطبية وهو جزء من متطلبات نيل درجة البكالوريوس علوم في الرياضيات الطبية

من قبل الطالب

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بإشراف

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Abstract

The object of this paper is to study some geometric properties of a subclass of univalent functions with applications. Coefficient bounds, closure theorems, distortion bounds, extreme points, integral operator with some applications about conformal mappings are obtained for this class.

Keywords: Univalent function, Ruscheweyh derive, coefficient bounds, distortion bounds, closure theorems, Extreme points, Integral operator, conformal mappings.

2000 mathematics subject classification: 30C45.

Contents

المستخلص:

الهدف من هذا البحث هو دراسة بعض الخواص الهندسية لصنف جزئي من الدوال أحادية التكافؤ مع التطبيقات. حصلنا على حدود المعامل، مبرهنات الانغلاق، حدود التشوية، النقاط المتطرفة، مؤثر تكاملي مع بعض التطبيقات حول الدوال الحافظة للزوايا.

1. Introduction:-

Let N be class of function of the form:-

$$f(z) = Z - \sum_{n=2}^{\infty} a_n z^n (a_n \geq 0, n \in N = \{1,2,\dots\}), \qquad (1.1)$$

which are analytic and univalent in the unit disk U={$\{z \in C: |z| < 1\}$.

Let f be a function given by (1.1) and g be a function defined by:

$$g(z) = z - \sum_{n=2}^{\infty} b_n z^n (b_n \geq 0). \qquad (1.2)$$

The convolution (or Hadamard product) for the functions f and g denoted by $f * g$ is defined as following :-

$$(f * g) = z - \sum_{n=2}^{\infty} a_n * b_n z^n. \qquad (1.3)$$

Definition 1: *A function $f \in N$ is in the class $\mathcal{A}(\beta, \sigma, A, B, \lambda)$ if it satisfies the condition :*

$$\left| \frac{(D^{\lambda} f(z))\grave{} - \sigma(D^{\lambda} f(z))}{(\sigma B + (A-B)\beta)(D^{\lambda} f(z) - B\sigma z(D^{\lambda} f(z))\grave{}} \right| < 1, \qquad (1.4)$$

where $-1 \leq B < A \leq 1, 0 < \beta < 1$ *and* $\sigma > 0$ *and* $D^{\lambda} f(z)$ *is the Ruscheweyh derivative [7] ,[9] of f of order λ defined as follows:*

$$D^{\lambda} f(z) = z + \sum_{n=2}^{\infty} s_n(\lambda)\, a_n z^n.$$

Some authors Studied univalent functions for another classes, like, Altintas[1] *Goodman* [3], *Kim and RØnning [4] ,Ravichandm [6] and Shams and Kulkani[8].*

2. Coefficient bounds:-

The following theorem gives a sufficient and necessary condition for a function $f(z)$ to be in the class $\mathcal{A}(\beta,\sigma,A,B,\lambda)$.

Theorem 1: Let $f \in N$. Then $f \in \mathcal{A}(\beta,\sigma,A,B,\lambda)$ **if and only if**

$$\sum_{n=2}^{\infty}[\sigma(n-1)(1-B)+(A+B)\beta]\, s_n(\lambda)a_n < (A-B)\beta, \qquad (2.1)$$

where -1$-1 \leq B < A \leq 1,0 < 1$ *and* $\sigma > 0$.

The inequality (2.1) is sharp.

Proof: $f \in \mathcal{A}(\beta,\sigma,A,B,\lambda)$, *then*

$$\left|\frac{(D^{\lambda}f(z))\grave{} - \sigma(D^{\lambda}f(z))}{(\sigma B+(A-B)\beta)(D^{\lambda}f(z)-B\sigma z(D^{\lambda}f(z))\grave{}}\right| < 1.$$

Since

$D^{\lambda}f(z) = z + \sum_{n=2}^{\infty} s_n(\lambda)\, a_n z^n$ *and* $Re(z) < |z|$, *then we can write*

$$Re\left\{\frac{\sum_{n=2}^{\infty}\sigma(n-1)s_n(\lambda)a_n z^n}{(A-B)\beta z-\sum_{n=2}^{\infty}[\sigma B(1-n)+(A+B)\beta]s_n(\lambda)a_n z^n}\right\} < 1.$$

Hence

$$Re\left\{\frac{\sum_{n=2}^{\infty}\sigma(n-1)s_n(\lambda)a_n z^{n-1}}{(A-B)\beta z-\sum_{n=2}^{\infty}[\sigma B(1-n)+(A+B)\beta]s_n(\lambda)a_n z^{n-1}}\right\} < 1.$$

$$\left\{\frac{\sum_{n=2}^{\infty}\sigma\,(n-1)s_n(\lambda)a_n}{(A-B)\beta z-\sum_{n=2}^{\infty}[\sigma B(1-n)+(A+B)\beta]s_n(\lambda)a_n}\right\} < 1.$$

So we can write

$$\sum_{n=2}^{\infty}[\sigma(n-1)(1-B)+(A+B)\beta]\, s_n(\lambda)a_n < (A-B)\beta.$$

Conversely, suppose that the condition (2.1) hold true, we must show that $f \in \mathcal{A}(\beta,\sigma,A,B,\lambda)$. *Equivalently, we prove that.*

$$\left|\frac{\sigma z(D^{\lambda}f(z))\grave{} - \sigma(D^{\lambda}f(z))}{(\sigma B+(A-B)\beta)(D^{\lambda}f(z) - B\sigma z(D^{\lambda}f(z))\grave{}}\right| < 1.$$

But, we have

$$\left|\frac{\sigma z(D^{\lambda}f(z))\grave{} - \sigma(D^{\lambda}f(z))}{(\sigma B+(A-B)\beta)(D^{\lambda}f(z) - B\sigma z(D^{\lambda}f(z))\grave{}}\right|$$

$$= \left|\frac{\sum_{n=2}^{\infty}\sigma\,(n-1)s_n(\lambda)a_n z^n}{(A-B)\beta z - \sum_{n=2}^{\infty}[\sigma B(1-n)+(A+B)\beta]s_n(\lambda)a_n z^n}\right|$$

$$< \left\{\frac{\sum_{n=2}^{\infty}\sigma\,(n-1)s_n(\lambda)a_n}{(A-B)\beta z - \sum_{n=2}^{\infty}[\sigma B(1-n)+(A+B)\beta]s_n(\lambda)a_n}\right\} < 1.$$

The last inequality is true by (2.1), so $f \in \mathcal{A}(\beta,\sigma,A,B,\lambda)$. *The function*

$$f_n(z) = z - \frac{(A-B)\beta}{[\sigma(1-n)(1-B)+(A-B)\beta]s_n(\lambda)}z^n, n \geq 2,$$

Show that inequality (2.1) is sharp

Corollary 1:

Let $f \in \mathcal{A}(\beta,\sigma,A,B,\lambda)$. *Then, we have*

$$a_n < \frac{(A-B)\beta}{[\sigma(1-n)(1-B)+(A-B)\beta]s_n(\lambda)}, n \geq 2, \qquad (2.2)$$

3. Closure theorems:-

In the following theorems, we find that convex combination for the class $\mathcal{A}(\beta,\sigma,A,B,\lambda)$.

Theorem 2: *let* $f_q(z) = z - \sum_{n=2}^{\infty} a_{n.q} z^n \ (a_{n.q} \geq 0, q = 1,2 \ldots\ldots, \ell)$ *be in the class* $\mathcal{A}(\beta,\sigma,A,B,\lambda)$. *Then the function* $w(z) = \sum_{q=1}^{\ell} g_q f_q(z)$, *where* $\sum_{q=1}^{\ell} g_q = 1$, *is also in the class* $\mathcal{A}(\beta,\sigma,A,B,\lambda)$.

Proof:

We have

$$w(z) = \sum_{q=1}^{\ell}(z - \sum_{n=2}^{\infty} a_{n.q} z^n) = z - \sum_{q=1}^{\ell} g_q \left(\sum_{n=2}^{\infty} a_{n.q} z^n\right),$$

$$= z - \sum_{n=2}^{\infty}(\sum_{q=1}^{\ell} g_q\, a_{n.q}) z^n.$$

Hence, we obtain

$$\sum_{n=2}^{\infty}\left[\frac{[\sigma(n-1)(1-B)+(A+B)\beta]s_n(\lambda)}{(A-B)\beta}\right](\sum_{q=1}^{\ell} g_q\, a_{n.q})$$

$$\sum_{q=1}^{\ell}\left[\sum_{n=2}^{\infty}\left[\frac{[\sigma(n-1)(1-B)+(A+B)\beta]s_n(\lambda)}{(A-B)\beta}\right] a_{n.q}\right] g_q < \sum_{q=1}^{\ell} g_q = 1.$$

This completes the proof.

Theorem 3:- *The class* $\mathcal{A}(\beta,\sigma,A,B,\lambda)$ *is a convex set.*

Proof:

We want to show the function

$$G(z) = (1-\lambda) f_1(z) + \lambda f_2(z), (0 \leq \lambda \leq 1) \qquad (3.1)$$

Is in the class $\mathcal{A}(\beta,\sigma,A,B,\lambda)$, *where* $f_1(z), f_2(z) \in \mathcal{A}(\beta,\sigma,A,B,\lambda)$ *and*

$$f_1(z) = z - \sum_{n=2}^{\infty} a_{n,1} z^n , f_2(z) = z - \sum_{n=2}^{\infty} a_{n,2} z^n.$$

By (2.1), we have

$$\sum_{n=2}^{\infty} \big[[\sigma(n-1)(1-B) + (A+B)\beta] s_n(\lambda)\big] a_{n,1} < (A-B)\beta.$$

and

$$\sum_{n=2}^{\infty} [\sigma(n-1)(1-B) + (A+B)\beta] s_n(\lambda)\, a_{n,2} < (A-B)\beta. \qquad (3.2)$$

Therefore $G(z) = (1-\lambda) f_1(z) + \lambda f_2(z)$

$$(1-\lambda)(z - \sum_{n=2}^{\infty} a_{n.1} z^n + \lambda(z - \sum_{n=2}^{\infty} a_{n.2} z^n)$$

$$= z - \sum_{n=2}^{\infty} [(1-\lambda) a_{n.1} + \lambda a_{n.2}] z^n.$$

We must show G(z) with the coefficient $((1-\lambda)a_{n.1} + a_{n.2})$ *satisfy in the relation (2.1) also the coefficient* $((1-\lambda)a_{n.1} + a_{n.2})$ *satisfy the inequality (2.2) Further.*

$$\sum_{n=2}^{\infty} \big[[\sigma(n-1)(1-B) + (A+B)\beta]\big] s_n(\lambda) \big[(1-\lambda) a_{n,1} + a_{n,2})\big]$$

$$= \sum_{n=2}^{\infty} \big[[\sigma(n-1)(1-B) + (A+B)\beta]\big] s_n(\lambda) \, [(1-\lambda) a_{n,1}$$

$$+ \sum_{n=2}^{\infty} [\sigma(n-1)(1-B) + (A+B)\beta] s_n(\lambda)(1-\lambda)\, a_{n,2}$$

$$\leq [A+B]\beta(1-\lambda) + [A-B]\lambda = (A-B)\beta.$$

Therefore it follow that $G(z)$ *in the class* $\mathcal{A}(\beta, \sigma, A, B, \lambda)$.

4. Distortion Bounds:-

In the following theorem, we obtain distortion bounds for the class $\mathcal{A}(\beta,\sigma,A,B,\lambda)$

Theorem 4: *If the function* $f(z)z = z-\sum_{n=2}^{\infty} a_n z^n$ *be in the class* $\mathcal{A}(\beta,\sigma,A,B,\lambda)$*.Then for* $|z|<1, |z|=r$

$$r-\frac{(A-B)\beta}{[\sigma(1-n)(1-B)+(A-B)\beta](\lambda+1)}\, r^2 \leq |f(z)|$$

$$\leq r+\frac{(A-B)\beta}{[\sigma(1-B)+(A-B)\beta](\lambda+1)}\, r^2, \qquad (4.1)$$

where equality holds for the function

$$f(z)=z-\frac{(A-B)\beta}{[\sigma(1-B)+(A-B)\beta](\lambda+1)} z^2 \text{ at } z=ir, z=r.$$

Proof: *Let* $f(z)\in \mathcal{A}(\beta,\sigma,A,B,\lambda)$*.Then by Theorem 1, we have*

$$\sum_{n=2}^{\infty} a_n \leq \frac{(A-B)\beta}{[\sigma(1-B)+(A-B)\beta](\lambda+1)}. \qquad (4.2)$$

Thus, for $0<r<1, |z|=r$

$$|f(z)|=\left|z-\sum_{n=2}^{\infty} a_{n.2} z^n\right| \leq |z|+\sum_{n=2}^{\infty} a_n |z|^n$$

$$\leq r+r^2\sum_{n=2}^{\infty} a_n$$

$$\leq r+r^2\frac{(A-B)\beta}{[\sigma(1-B)+(A-B)\beta](\lambda+1)}. \qquad (4.3)$$

This gives the right hand inequality of (4.1) by (4.3).

Similarly, we have $|f(z)| \geq |z| - \sum_{n=2}^{\infty} a_n |z|^n \geq r - r^2 \sum_{n=2}^{\infty} a_n$

$$\geq r - r^2 \frac{(A-B)\beta}{[\sigma(1-B)+(A-B)\beta](\lambda+1)}. \tag{4.4}$$

This gives the left hand side of (4.1) by using (4.4), hence the result (4.1) follows.

5. Extreme Points:-

In the following theorem, we introduce the extreme points for the class $\mathcal{A}(\beta,\sigma,A,B,\lambda)$

Theorem 5: *Let* $f_1(z) = z$ *and*

$$f(z) = z - \frac{(A-B)\beta}{[\sigma(n-1)(1-B)+(A-B)\beta]s_n(\lambda)}\ z^n, n \geq 2$$

$n \in N$. *Then the function* $f(z) \in \mathcal{A}(\beta,\sigma,A,B,\lambda)$ *if and only if it can expressed in the form*

$$f(z) = \sum_{n=2}^{\infty} \mu_n f_n(z), \text{ where } \mu_n \geq 0 \text{ and } \sum_{n=2}^{\infty} \mu_n = 1. \qquad (5.1)$$

Proof:

Assume that f *can be expressed in the form (5.1). then, we have*

$$f(z) = \sum_{n=2}^{\infty} \mu_n f_n(z)$$

$$= \mu_1 f_1(z) + \sum_{n=2}^{\infty} \mu_n f_n(z)$$

$$= z - \sum_{n=2}^{\infty} \frac{(A-B)\beta\mu_n}{[\sigma(n-1)(1-B)+(A-B)\beta]s_n(\lambda)}\ z^n$$

$$h_n = \frac{(A-B)\beta\mu_n}{[\sigma(n-1)(1-B)+(A-B)\beta]s_n(\lambda)} z^n.$$

By theorem 1, we have $f(z) \in \mathcal{A}(\beta,\sigma,A,B,\lambda)$ *if and only if*

$$\sum_{n=2}^{\infty} \frac{[\sigma(1-n)(1-B)+(A-B)\beta]s_n(\lambda)}{(A-B)\beta}\ h_n < 1.$$

For $f(z) = z - \sum_{n=2}^{\infty} h_n z^n$. *Hence*

$$\sum_{n=2}^{\infty}\frac{[\sigma(1-n)(1-B)+(A-B)\beta]s_n(\lambda)}{(A-B)\beta}\ \ h_n$$

$$\sum_{n=2}^{\infty}\frac{[\sigma(n-1)(1-B)+(A-B)\beta]s_n(\lambda)}{(A-B)\beta}\times\frac{(A-B)\beta\mu_n}{[\sigma(1-B)+(A-B)\beta]s_n(\lambda)}$$

$$\sum_{n=2}^{\infty}\mu_n=1-\mu_n\leq 1.$$

Therefore, we conclude the result.

Conversely, let $f(z)\in\ \mathcal{A}(\beta,\sigma,A,B,\lambda)$ *Then by (5.1), we may set*

$$\mu_n=\frac{[\sigma(n-1)(1-B)+(A-B)\beta]s_n(\lambda)}{(A-B)\beta}a_n, n\geq 2.$$

We have $\mu_n\geq 0$ *and if we set* $\mu_1=1-\sum_{n=2}^{\infty}\mu_n$ *, then we can write*

$$f(z)=z-\sum_{n=2}^{\infty}a_n z^n$$

$$=z-\sum_{n=2}^{\infty}\frac{(A-B)\beta}{[\sigma(n-1)(1-B)+(A-B)\beta]s_n(\lambda)}\mu_n z^n$$

$$f(z)=z-\sum_{n=2}^{\infty}\mu_n(z-f_n(z))$$

$$=z\left(1-\sum_{n=2}^{\infty}\mu_n\right)-\sum_{n=2}^{\infty}\mu_n f_n(z)=\sum_{n=1}^{\infty}\mu_n f_n(z).$$

This complete the proof

Remark1: *The extreme point of the class* $\mathcal{A}(\beta,\sigma,A,B,\lambda)$ *are the function* $f_1(z), f_n(z), n\geq 2$ *as in Theorem 5.*

6. Integral operator:-

Now, we introduce an integral operator due to Bernard: [2],

$$l_k(f(z) = \frac{1+k}{z^k}\int_0^z f(t)\,t^{k-1}dt, (k > -1)$$

and we study the effect of this operator on the class $\mathcal{A}(\beta, \sigma, A, B, \lambda)$

Theorem 6: *If* $f \in \mathcal{A}(\beta, \sigma, A, B, \lambda)$ *then* $l_k(f(z))$ *is also in the class* $\mathcal{A}(\beta, \sigma, A, B, \lambda)$

Proof:

$f(z) = z - \sum_{n=2}^{\infty} a_n z^n$, *then*

$$l_k(f(z)) = \frac{1+k}{z^k}\int_0^z \left(t - \sum_{n=2}^{\infty} a_n z^n\right) t^{k-1}dt, = z - \sum_{n=2}^{\infty}\left(\frac{1+k}{n+k}\right) a_n z^n.$$

Since $n > 1$ *so* $\frac{1+k}{n+k} < 1$ *, then, we can write*

$$\sum_{n=2}^{\infty}\left[\frac{[\sigma(n-1)(1-B) + (A-B)\beta]s_n(\lambda)}{(A-B)\beta}\right]\left(\frac{1+k}{n+k}\right) a_n$$

$$\leq \sum_{n=2}^{\infty}\left[\frac{[\sigma(n-1)(1-B) + (A-B)\beta]s_n(\lambda)}{(A-B)\beta}\right] a_n < 1.$$

Thus $l_k(f(z) \in \mathcal{A}(\beta, \sigma, A, B, \lambda)$ *this completes the proof.*

Applications

- Motivation
- Part I: Conformal Geometry & Applications
- Part II: Quasi-conformal Geometry & Applications
  - Basic Mathematical Background
  - Computational Algorithms
  - Applications
- Conclusion

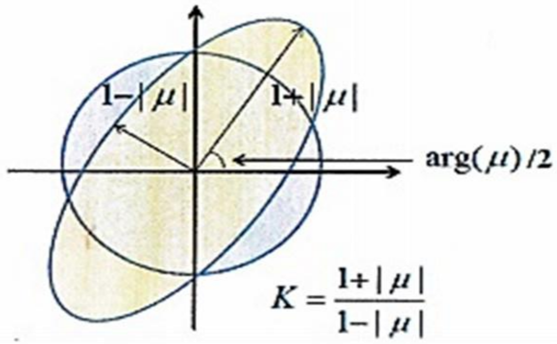
What is Quasi-conformal map?

- Generalization of conformal maps (angle-preserving);
- Orientation preserving homeomorphism between Riemann surfaces;
- Bounded conformality distortion;
- Intuitively, map infinitesimal circle to ellipse;
- Mathematically, it satisfies: $\frac{\partial f}{\partial \bar{z}} = \mu(z) \frac{\partial f}{\partial z}$ ← Beltrami coefficient

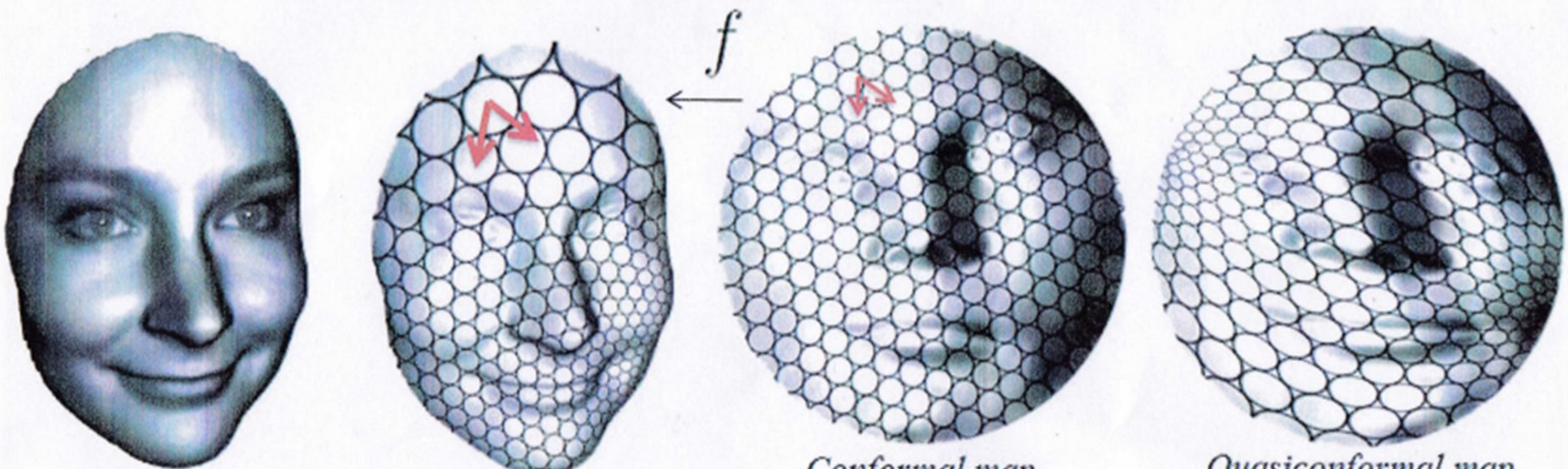
$$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right);$$
$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right)$$

Conformal $\Leftrightarrow$ $\mu = 0$ $\Leftrightarrow$ $\frac{\partial f}{\partial \bar{z}} = 0$

- Beltrami coefficient:
  Measure conformality distortion;
  Invariant under conformal



Example: Quasi-conformal



Original face — *Circle packing* — *Conformal map (Circles to circles)* — *Quasiconformal map (Circles to ellipses)*

Conformal

$$f^*(ds_E^2) = \lambda |dz|^2$$

Quasi-conformal

$$f^*(ds_E^2) = |\frac{\partial f}{\partial z}|^2 |dz + \mu(z) d\overline{z}|^2$$

Why Quasi-conformal?

*1. **Natural deformations** are unlikely to be rigid, or Isometric, or even conformal. The search space for The mapping should include all diffeomorphisms.*

*2. **Quasi-conformal Geometry** studies the . deformation pattern between shapes. It effectively measures the conformality distortion under the deformation.*

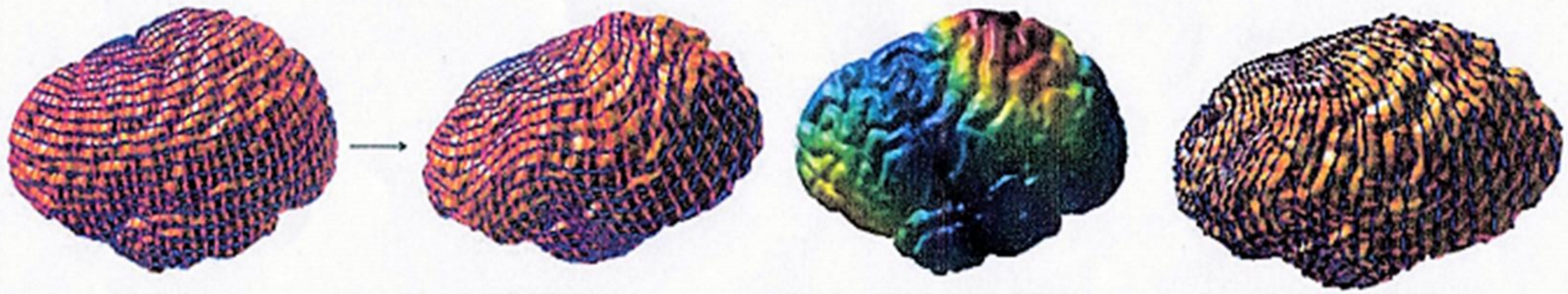
Surface Diffeomorphism v.s Beltrami Coefficient

Goal:

Look for a simple representation of surface diffeomorphisms, called Beltrami representation. (Lui & Wong et al. 2009)

Theorem:

Let S_1 and S_2 be two surfaces. Suppose $f : S_1 \to S_2$ is a surface diffeomorphism from S_1 and S_2 . Given 3 points correspondence $\{p_1, p_2, p_3 \in S_1\} \leftrightarrow \{f(p_1), f(p_2), f(p_3) \in S_2\}$. f can be represented by a unique Beltrami coefficient $\mu : S_1 \to \mathbb{C}$.



Brain 1 **Brain 2** **Beltrami Coefficient** **Reconstructed**

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