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On soft Banach Algebra

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by

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

قُلِ اللَّهُمَّ مَالِكَ الْمُلْكِ تُؤْتِي الْمُلْكَ مَنْ تَشَاءُ وَتَنْزِعُ الْمُلْكَ

مِمَّنْ تَشَاءُ وَتُعِزُّ مَنْ تَشَاءُ وَتُذِلُّ مَنْ تَشَاءُ

بِيَدِكَ الْخَيْرُ إِنَّكَ عَلَى كُلِّ شَيْءٍ قَدِيرٌ

صدق الله العلي العظيم

(سورة) ال عمران – اية ٢٦

الاهداء

بسم الله الرحمن الرحيم

(و قل إعملوا فسيرى الله عملكم ورسوله والمؤمنون)

صدق الله العظيم

إلهي لايطيب الليل إلا بشكرك ولايطيب النهار إلا بطاعتك .. ولاتطيب اللحظات إلا بذكرك .. ولا تطيب الآخرة إلا بعفوك.. ولا تطيب الجنة إلا برؤيتك

الله جل جلاله

إلى من بلغ الرسالة وأدى الأمانة .. ونصح الأمة .. إلى نبي الرحمة ونور العالمين

سيدنا محمد صلى الله عليه واله وسلم

الى من جرع الكأس فارغا ليسقيني قطرة حب الى من كلت انامله ليقدم لنا لحظة سعادة الى من حصد الاشواك عن دربي ليمهد لي طريق العلم الى القلب الكبير والدي العزيز إ

إلى من ركع العطاء أمام قد ميها وأعطتنا من دمها وروحها وعمرها حبا وتصميما ودفعا لغدٍ أجمل

إلى الغالية التي لا نرى الأمل إلا من عينيها

أمي الحبيبة

الى القلوب الطاهرة والنفوس البريئة الى رياحين حياتي

اخوتي واخواتي

ا الى من علمونا حروفا من ذهب و كلمات من درر و عبارات من اسمى

و اجلى عبارات في العلم

الى اساتذتنا الكرام

Abstract

In a soft normed linear space we have been able to define a new concept of convergence of a sequence of soft elements, which we call soft convergence. We have defined a soft topology [15] generated by a soft norm and which is also known to be a topology of sets [10]. We are also able to introduce a definition of soft Banach algebra and study some of its propertie

INTRODUCTION

Banach algebra is an important field of functional analysis, which has many ap- plications

in various branches of mathematics. Many examples of classical Banach

algebras are known, among them are $B(X)$, the space of bounded linear operators

on X and $C(X)$, the space of continuous functions on X. When X is a Hilbert

space, the space of bounded linear operators play a key role in quantum mechanics

and differential equations. We have introduced fuzzy Banach algebra in [16, 17].

Thus it is a natural query to extend the concept of Banach algebra in soft setting.

In this paper we introduce a definition soft Banach algebra and study some of its

properties. In section 2, preliminary results are given. In section 3, we introduce a

new concept of convergence of a sequence of soft elements. With this convergence we

have shown that the condition of finiteness of parameter set is not required in many

cases like completeness of finite dimensional soft normed linear spaces etc [3, 6]. In

this section it is also shown that the norm axiom $N(5)$ is redundant, which was used

frequently to prove most of the theorems on soft normed linear spaces [3, 6] and we are also able to define a soft topology generated by a soft norm in a soft normed linear space. In section 4, we introduce the concept of soft Banach algebra and some of its preliminary properties are studied. Section 5 concludes the paper.

جامعة القادسية

كلية التربية

قسم الرياضيات

On Soft Banach Algebra

بحث مقدم الى مجلس قسم الرياضيات جامعة القادسية كلية التربية وهو جزء من متطلبات نيل درجة البكلوريوس علوم في الرياضيات

من قبل

حنين سلمان عيدان

٢٠١٨

الملخص

في الفضاء الخطي المعياري اللين كنا قادرين على دراسة مفهوم جديد من تقارب المتتابعات اللينة العناصر التي نسميها لينة التقارب .وكذالك تمكنا من دراسة التبولوجيا اللينة التي تم انشاؤها بواسطة السوفت والمعرفة بتبولوجيا المجموعات . نحن أيضا كنا قادرين على تقديم تعريف للينة باناخ الجبرا ودراسة بعض المبرهنات.

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CHAPTER ONE

Definition 1.1.1[1](linear algebra): We shall say that R is a linear algebra if R is a linear space. an operation of multiplication (which in general is not commutative) is in R satisfying the following conditions:-

1) $x(xy) = (xx)y = x(\alpha y)$
2) $x(yz) = (xyz)$
3) $(x + y)z = xz + yz$
For arbitrary $x\epsilon R$ and any number α.

In the sequel we shall consider only linear algebras and the term "algebra" will a linear algebra.

Definition 1.1.2[1]: In elements x, y in the algebra R are said to commute if $xy = yx$ an algebra is said to be commutative if any two of its elements commute. In the sequel we shall in general that the algebra under consideration are commutative a subset $R_1 \subseteq R$ is called a sub algebra of the algebra R if the application of the addition scalar multiplication and multiplication to element of R_1 elements in R_1

Definition 1.1.3[1]: A commutative sub algebra is said to be maximal if it is not contained in any a commutative sub algebra. It follows from the preceding discussion that.

Theorem 1.1.4[1]: Every commutative sub algebra is contained in a maximal commutative sub algebra.

Proof:- The set $\sum$ of all all commutative sub algebra of the algebra R, which can in a given commutative sub algebra. Is a partially ordered set. Ordered by in which satisfies the condition of zeros lemma: namely. The least upper bound of any linear ordered set of these sub algebra is simply their union on the basis of the Zorn lemma. $\sum$ contains a maximal element which will then be the maximal commutative sub algebra containing x. Since every element x is contained in the commutative sub algebra $R_a(x)$, it follows from proposition I that.

Theorem 1.1.5[1]: Every element x is contains in a maximal commutative sub algebra.

Example 1.1.6[1]: We denote the set of all continuous complex- value function on the topological space x by $C(x)$ in $C(X)$ we define operations of addition- scalar multiplication and multiplication respectively as the addition of function, the multiplication of function by a number and the multiplication of function clearly $C(x)$ will then be an algebra this algebra is commutative.

Example 1.1.7[1]: Suppose x is an arbitrary linear, we denote the set of all linear operators in x with domain x by $A(X)$. In $A(X)$ we define operation of addition, scalar multiplication, and multiplication as the corresponding operation on operations (see subsection 6.) then $A(x)$ is an algebra $A(X)$ is commutative only in the case when X is one-dimensional.

Definition 1.1.8[1]: (Algebra with identity)

An algebra R iscalled an algebra with identity if R contains an elemente which satisfies the condition: $ex = xe = x$ for all $x\epsilon R$.

The element e itself which satisfies condition (1) is called an identity of the algebra R.

Theorem 1.1.9[1]: Every algebra R without identity can be considered as a sub algebra of an algebra R with identity.

Theorem 1.1.10[1]: A maximal commutative sun algebra R, of the algebra R with identity is also an algebra with identity.

Theorem 1.1.11[1]: If x^{-1} exists and if x, y commute, then x^{-1} and y also commute.in fact multiplication both members of the equality $xy = yx$ on the left and right by x^{-1}, we obtain $yx^{-1} = x^{-1}y$.

Theorem 1.1.12[1]: If x is the maximal commutative sub algebra which contains x and x^{-1} exists then $x^{-1}\epsilon X$.

Theorem 1.1.13[1]: If every element $x \neq 0$ in the algebra R with identity has a left inverse, then R is a division algebra.

Definition 1.1.14[1]: An element $y\epsilon R$ is called a left quasi- inverse of the element $e + x$ in R, $e + y$ is a left inverse of the element $e + x$ in R, that mean if $(e + y)(e + x) = e$.

Example 1.1.15[1]: The algebra $C(x)$ is an algebra with identity. The identity of this algebra is the function which is identically equal to unity on x.

Example 1.1.16[1]: The algebra $A(x)$ and $A(x)$ are algebra with identity which is the identity operator.

Definition 1.1.17[1]: The center of algebra R is the set of those element $a\epsilon R$ which commutative with all the elements of R. The center a commutative sub algebra of the algebra R.

Definition 1.1.18[1]: A set I_1 of elements of the algebra R is called a left ideal R if

1) $I_1 \neq R$.
2) I_1 is a sub space of the linear space R.
3) If $x\epsilon I$, $A\epsilon R$ then $ax\epsilon I$.

Theorem 1.1.19[1]: An element $x$of an algebra with identity has a left (right) inverse if and only if it is not contained in any left (right)ideal.

Theorem 1.1.20[1]: Every left (right) ideal of the algebra R with identity is contained in a maximal left (right) ideal.

Theorem 1.1.21[1]: An element x of an algebra with identity has a left (right) inverse if and only if it is not contained in any maximal left(right) ideal.

Theorem 1.1.22[1]: Every two- sided ideal of an algebra with identity is contained in a maximal two- sided ideal.

Theorem 1.1.23[1]: Every regular (right, left, two- sided) ideal can be extended to a maximal (right, left, respectively, two- sided) ideal (which is obviously regular also).

Theorem 1.1.24[1]: An element x in the algebra R has a left quasi-inverse if and only if for arbitrary maximal regular left ideal M, there exists element such that $x + y + yx \, \epsilon M$.

Theorem 1.1.25[1]: An element x in the algebra R dose not have a left a quasi- inverse if and only if $I_1 = \{z + z_x\}, z\epsilon R$.

Definition 1.1.26[1]: An element x_o in the algebra R with identity is said to be generalized nilpotent if $(e + yx_o)^{-1}$ exists for an arbitrary element $y\epsilon R$.the set of all generalized nilpotent element in the algebra R is called its (Jacobson) radical.

Theorem 1.1.27[1]: The radical of an algebra with identity coincides with intersection of all its maximal left ideal.

Theorem 1.1.28[1]: An element x_0 belong to the radical of an algebra with identity if and only if a two- sided inverse $(e + ax_o)^{-1}$ exists for every element a of the algebra.

Theorem 1.1.29[1]:

The intersection of all maximal left ideals coincides with the intersection of all maximal right ideals and is the radical of the algebra.

Definition 1.1.30[1]: An algebra is said to be semi simple if it is radical consist of only the zero element suppose now that R is an algebra without identity and that R' is the algebra obtained form R by adjoining the identity.

Definition 1.1.31[1]: An element x_o is said to be generalized nilpotent $xx_o + zx_o$ has a left quasi- inverse for arbitrary $z\epsilon R$ and arbitrary numbers x in this definition R is no large necessarily an algebra with identity.

Theorem 1.1.32[1]: In a non- radical algebra, the radical is the intersection of all maximal regular left ideal and also the intersection of all maximal regular right ideal and therefore it is two sides ideal.

Theorem 1.1.33[1]: The quotient algebra module the radical is a semi simple algebra.

Theorem 1.1.34[1]: Every irreducible algebra R_1 different form (0), of linear operators in the vector space x is a semi simple algebra.

Definition 1.1.35[1]: A mapping $x \rightarrow x'$ of the algebra R into an arbitrary algebra R' if $x \rightarrow x', y \rightarrow y'$ imply that $yx \rightarrow y'x', x + y \rightarrow x' + y'$, $xy \rightarrow x'y'$ if R is the image of the algebra R, then the homomorphism is called a homomorphism of R onto R'.

Definition 1.1.36[1]: Two algebras R and R' are said to be isomorphic if there exists isomorphism of R onto R'.

Theorem 1.1.37[1]: Under a homomorphism of the algebra R into the algebra R', the inverse image I of the zero of R is a two sides ideal in R.

Theorem 1.1.38[1]: Under a homomorphism mapping of the algebra R. The inverse image I of the zero element is a two-sided ideal of this algebra and the homomorphic image itself is isomorphic to the quotient algebra R modulo I.

Theorem 1.1.39[1]: The quotient algebra R/I is simple if and only if I is a maximal two-sided ideal in R.

Definition 1.1.40[1]: Algebra is the so- called left regular representation of the algebra each element $a\epsilon R$ is assigned the operator A_a of left multiplication by a $A_a x = ax$.

Theorem 1.1.41[1]: Every primitive algebra is isomorphic to an irreducible algebra of linear operators in some vector space.

Theorem 1.1.42[1]: Every primitive algebra is semi simple.

Theorem 1.1.43[1]: If $I \neq \{0\}$ is a two sided ideal in the primitive algebra R and if a is an arbitrary nonzero element of the algebra R, then $I_a \neq \{0\}$.

Definition 1.1.44[1]: (topological algebra)

R is called a topological algebra if :

1) R is an algebra
2) R is a locally convex topological linear space.
3) The product xy is a continuous function of each of the factors x, y provided other factor is fixed.

Definition 1.1.45[1]: A mapping $x \to x'$ of the topological algebra R into the topological algebra R' is called a continuous homomorphism if:

1) $x \to x'$ is a homomorphism of the algebra R into the algebra R'.
2) $x \to x'$ is a continuous mapping of the topological space R into the topological space R'.

Definition 1.1.46[1]: A subset $R_1 \subseteq R$ is said to be a closed sub algebra of the algebra R if

1) R_1 is a sub algebra of the algebra R.
2) R_1 is a closed subspace of the topological space R.

Theorem 1.1.47[1]: If R_1 is a sub algebra of the algebra R_1 then it's closer $\overline{R_1}$ is a closed sub algebra of R.

Theorem 1.1.48[1]: The algebra $R_1(s)$ is the closer of the algebra $R_a(s)$: $R_a(s) = \overline{R_a(s)}$.

Theorem 1.1.49[1]: The closer of a commutative sub algebra of a topological algebra is commutative.

Theorem 1.1.50[1]: A maximal commutative sub algebra of a topological algebra is closed.

Theorem 1.1.51[1]: The set R_s of all elements x of a topological algebra R_1 which commute with all elements of some set $S \subseteq R_1$ is a closed sub algebra of the algebra R.

Theorem 1.1.52[1]: The center z of a topological algebra R is a closed commutative sub algebra in R.

Theorem 1.1.53[1]: The closer of a 9left, right, two- sided)ideal in a topological algebra, which does not coincide with the entire algebra, is also (left, right, two sided) ideal in this algebra.

Definition 1.1.54[1]: A topological algebra R with identity is called an algebra with continuous inverse if there exists an neighborhood $U_o(e)$ posseting the following properties:

1) Every element $x \epsilon U_o(e)$ has an inverse x^{-1}
2) $x^{-1}$is a continuous function of x at the point $x = e$.

Definition 1.1.55[1]: (normed algebra): R is called normed algebra if

1) R is an algebra
2) R is a normed space
3) for any two elements $x, y \epsilon R$ $\left|xy\right| = \left|x\right| \left|y\right|$ ……….(1)
4) if R is contains an identity e, then $\left|e\right| = 1$. The norm in a normed algebra R defines a topology in R in a natural manner recall that in this topology, the open balls $\left|x - x_o\right| < r$ with center at x_o from a neighborhood basis of the element $x_o \epsilon R$.

Proposition 1.1.56[1]: In the norm topology, the product xy is a continuous function of the variables x, y simultaneous.

In fact, in virtue of (1)

$$|xy - x_o y_o| = |(x - x_o)(y - y_o) + (x - x_o)y_o + x_o(y - y_o)|$$

$$\le |x - x_o||y - y_o| + |x - x_o||y_o| + |y - y_o||x_o|.$$

Now, the assertion follows directly from this since a normed space R is a topological linear space, we conclude from proposition ( )

Proposition 1.1.57[1]: In the topology define by the norm, a normed algebra is a topological algebra a normed algebra R is said to be complete if R is a complete normed algebra will also be called a Banach algebra.

Proposition 1.1.58[1]: Every non complete normed algebra can be embedded in a complete normed algebra.

Proof:- suppose R is the completion of the normed space R. Now define multiplication in R suppose $\bar{x}, \bar{y} \epsilon R$ and $< x_n >, < y_n >$ be fundamental sequences in R. Which define $\bar{x}, \bar{y}$ respectively. It follows from inequality (2) with x_n, x_m in place of x, x_o and y_n, y_m in place of y, y_o that $< x_n, y_n >$ also is a fundamental sequence. The element in $\bar{R}$ which it define will be considered to be the product $\bar{x}\bar{y}$ of the elements $\bar{x}, \bar{y}$. Again applying inequality (2) it it can also be easily verified that $\bar{x}\bar{y}$, does not depend on the choice of the fundamental sequence.

$< x_n >, < y_n >$ which define $\bar{x}, \bar{y}$ if in particular $\bar{x} = x \epsilon R$, $\bar{y} \epsilon R$, then setting $x_n = x$, $y_n = y$, we conclude that in this case the product coincides with the product in R passing to the limit in the relations for the elements in the algebra R, it is easily shown that $\bar{R}$ is an algebra and that the inequality $|\bar{x}\bar{y}| \le |\bar{x}||\bar{y}|$ is satisfy for elements of the ring $\bar{R}$ consequently $\bar{R}$ is a complete normed which contains Ra sub algebra.

The algebra $\bar{R}$ is called the completion of the algebra R.

Example 1.1.59[1]: The algebra $C(T)$ suppose T is topological space. the set $C(T)$of all bounded continuous function $x(t)$ on T forms a Banach space recall that the norm $|x|$ in $C(T)$ is defined by the formula $|x| = sup_{t \epsilon T} |x(t)|$.

Multiplication in $C(T)$ can be define as the multiplication of function that mean $(xy)(t) = x(t).y(t)$

1) this easily seen that the condition $|xy| \leq |x||y|$ will be satisfied so that $C(T)$ becomes a Banach algebra. If T is compact then the boundedness condition on the functions $x(t)$ is redundant in virtue.

2) The algebra $B(x)$. Recall that $B(x)$ denotes the set of all bounded linear operations in the Banach space x. We saw above that $B(x)$ is also defined as the multiplication of operator with $|AB| \leq |A||B|$

According to that we proved consequently, $B(x)$ is a Banach algebra

3) The algebra W. We denote by W the set of all absolutely convergent series $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{int}$ with norm $|x|=\sum_{n=-\infty}^{\infty}|C_n|$

We obtain a Banach algebra by defining addition, scalar multiplication as the corresponding operations on

2. Adjunction of the identity. Suppose R a normed algebra without identity and let R be the algebra obtained from R upon adjunction of the identity we may introduce a norm in R by setting

$$|xe + x| = |x| + |x|$$

It is easily verified that R then becomes a normed algebra. If R is a complete algebra without identity, then R' is also a complete algebra. The proof is simple and so we shall omit it.

The Radical in a Normed Algebra

Theorem 1.1.60[1]: for every x of the normed algebra R,

$\lim_{n\to\infty} \sqrt{|x|^n} < \infty$ exists.

Theorem 1.1.61[1]: If the element x of the normed algebra R belongs to the radical of the algebra $\lim_{n\to\infty} \sqrt{|x|^n} = 0$

Banach Algebra with identity

Theorem 1.1.62[1]: Every Banach algebra with identity is an algebra with continuous inverse moreover, every element x_1 satisfying the inequality $|x - e| < 1$ is invertible.

Theorem 1.1.63[1]: In a Banach algebra R with identity:

1) The set of all elements x having a (left, right, two- sided) inverse is an open set.
2) The inverse x^{-1} is a continuous function of x at all points for which x^{-1} exists.
3) The closure of a (left, right, two- sided) ideal a (left, right)ideal.
4) Maximal (left, right, two- sided) ideal is closed.
5) The set R_x of all regular points of the element $x\epsilon R$ is open and the resolve $X_\lambda = (x - \lambda e)^{-1}$ is an analytic function of λ.
6) The spectrum of every element $x\epsilon R$ is a no avoid set.

Theorem 1.1.64[1]: (Gelfand [1] and Mazur [1]): Every complete normed division algebra is isomorphic to the field of complex number.

Theorem 1.1.65[1]: In a Banach algebra R the quotient algebra R/I modulo a closed two- sided ideal I is a Banach algebra.

Theorem 1.1.66[1]: If in the Banach algebra R with identity every element $x \neq 0$ has a left inverse. Then R is isomorphic to the field of complex number.

Theorem 1.1.67[1]: For $|\lambda| > \lim_{n\to\infty} \sqrt{|x|^n}$ the resolve X_λ can be expended in absolutely convergent Laurent series.

Theorem 1.1.68[1]: For an arbitrary $x\epsilon R$, $r(x) = \lim_{n\to\infty} \sqrt[n]{|x|^n}$.

Theorem 1.1.69[1]: The spectral radius possesses the following properties.

1) $r(x^k) = [r(x)]^k$
2) $r(xx) = |x| r(x)$
3) $r(x) \leq |x|$

continuous homomorphism of normed algebra

Theorem 1.1.70[1]: Every continuous homomorphism $x \to x'$ of the normed algebra R into the normed algebra R' satisfies in the inequality.

$$|x'| \leq C|x|$$

Theorem 1.1.71[1]: Every continuous homomorphism $x \to x'$ of a normed algebra R into a normed algebra R' is uniquely extendible to a continuous homomorphism of the completion R of the algebra R into the completion R' of the algebra R'.
Theorem 1.1.72[1]: Every continuous isomorphism of a Banach algebra R anto a Banach algebra R is a topological isomorphism.
Theorem 1.1.73[1]: Under a continuous homomorphism of the Banach algebra R'. The kernel I of the homomorphism is a closed two- sided ideal in R. And the algebra R' it self is topologically isomorphism to the quotient algebra R/I. Can certainly every closed two- sided ideal I of the Banach algebra R induces a continuous homomorphism (the so- called natural homomorphism) of the algebra R into the algebra R/I.

(regular representation of a normed algebra. Recall that the left and right regular representations $a \to A_a$ and $a \to B_a$of the algebra R are defined by means of the formulas.)
Theorem 1.1.74[1]: A left (right) regular representation of a normed algebra $R$is a continuous. Homomorphism of the algebra R into the algebra $R(a)$ of all bounded linear operations in the space R. In fact, the inequalities
$|A_a X| \le |a| \, |X|, |B_a X| \le |a| \, |X|$
$|A_a| \le |a|, |B_a| \le |a|$.
Theorem 1.1.75[1]: If R is a normed algebra with identity, then a left (right) regular representation of the algebra R is an isometric isomorphism (anti- isomorphism) of the algebra R into the algebra $R(R)$.
In fact, for $x = e$, inequalities (1) go over into equalities, and hence $|A_a| = |a|, |B_a| = |a|$
Theorem 1.1.76[1]: R is a minimal invariant sub space in R if and only if it is the annihilator of maximal right ideal in R.
Theorem 1.1.77[1]: If R is an algebra with identity, then every closed invariant sub space in R, with is distinct from (0) contains a minimal invariant sub space and consequently, in contains elementary functions.
Theorem 1.1.78[1]: Suppose $R$is an algebra with identity and let $F \ne 0$ be a function in R there exists an elementary functional which is a weak contact point of functional of the form $f_a(x) = f(xa)$. Regular

representations of an algebra can also be used in the proof of the following proposition.

Theorem 1.1.79[1]: Suppose R is a complete topological algebra with identity in which the topology is defined by the norm $|x|$. Then R is topologically isomorphic to a Banach algebra.

Definition 1.1.80[1]: R is called a symmetric algebra if:

1) R is an algebra
2) an operation is defined in R which assigns to each element x in R the element x^* in R in such a way that the following conditions are satisfies:-

a) $(\lambda x + \lambda y)^* = \lambda x^* + \mu y^*$

b) $x^{**} = x$

c) $(xy)^* = y^* x^*$

An element x is said to be Hermitian if $x^* = x$.

Theorem 1.1.81[1]: Every element x of a symmetric algebra can be uniquely represented in the form $x = x_1 + ix_2$, where x_1 , x_2 are Hermitian elements.

In fact, if such a representation holds, then $x^* = x_1 - ix_2$ consequently

$x_1 = \frac{x+x^*}{2}$, $x_2 = \frac{x-x^*}{2i}$

Thus, this representation is unique. Conversely, the elements x_1, x_2 defined by equalities (1) are Hermitian and $x = x_1 + ix_2$.

These elements x_1, x_2 will be called the Hermitian components of the element x an element x is called normal if $x^* x = x x^*$.

Theorem 1.1.82[1]: Every element of the form $x^* x$ is Hermitian

In fact, in virtue of c and b). $(x^* x)^* = x^* x^{**} = x^* x$

Theorem 1.1.83[1]: The identity e is a Hermitian element. In fact $e^* = e^* e$ is a Hermitian element. Consequently, $e^* = e$

If R is asymmetric algebra without identity and R' is the algebra obtained from R by adjunction of the identity, then setting $(\lambda e + x)^* = \lambda e + x^*$ for $x \epsilon R$.

Theorem 1.1.84[1]: If x^{-1} exists, then $(x^{*})^{-1}$ also exists and $(x^{*})^{-1} = (x^{-1})^{*}$

Theorem 1.1.85[1]: If R is a maximal commutative symmetric sub algebra containing a normal element x and if x^{-1} exists, then $x^{-1}\epsilon R$. In fact since x and x^{*} commute with all elements in R, x^{-1} and $x^{*} = (x^{-1})^{*}$

Definition 1.1.86[1]: The mapping $x \to x'$ of a symmetric algebra R into the symmetric algebra R' is called a symmetric homomorphism if

$\beta)$ $x \to x'$ is a homomorphism

$\alpha)$ $x \to x'$ implies that $x^{*} \to x'^{*}$.

Theorem 1.1.87[1]: The radical of a symmetric two- sided ideal.

Example 1.1.88[1]:

1) The algebra $C(T)$ is a symmetric algebra if we set $x^{*} = \overline{X(t)}$ for $X = X(t)$ (where the vinculum denotes conjugate complex number)
2) Suppose R is a Hilbert space. the algebra $R(R)$ that mean $R(x)$ with $X = R$ is a symmetric algebra if involution is under stood to be passage over to the adjoint operator
3) The algebra W is asymmetric algebra if we set

$x^{*} = \sum_{n=-\infty}^{\infty} \bar{C}_{-n} e^{int}$ for $x = \sum_{n=-\infty}^{\infty} C_n e^{int}$

Chapter one

Section two

Definition 1.2.1[1]: (Positive functional)

A linear functional f in the symmetric algebra R is said to be real- valued if f assumes real value on all Hemitian elements of the algebra R.

Theorem 1.2.2[1]: Every linear functional in a symmetric algebra can be represented in the form $f = f_1 + if_2$ where f_1, f_2 are real valued functional. Namely it suffices to set

$f_1(x) = [f(x) + \overline{f(x^*)}]$,$f_2(x) = \frac{1}{2i}\,[f(x) - \overline{f(x^*)}]$.

Then f_1, f_2 are real valued functional and $f(x) = f_1(x) + if_2(x)$ these functional f_1, f_2 are called the real components.

Theorem 1.2.3[1]: If f is a real- valued functional then $f(x^*) = \overline{f(x)}$ for an arbitrary $x \epsilon R$. In fact setting $x = x_1 + ix_2$ where x_1, x_2 are Hermitian we have $f(x^*) = f(x_1 - ix_2) = \overline{f(x_1) + \imath f(x_2)} = \overline{f(x)}$

Inasmuch as $f(x_1), f(x_2)$ are real- valued by assumption. A linear functional f is said to be positive if $f(x^*x) \geq 0$ for an arbitrary element x of the algebra R.

Theorem 1.2.4[1]: For every positive functional f in the symmetric algebra R.

1) $f(y^*x) = f(\overline{x^*y})$
2) $|f(y^*x)|^2 \leq f(y^*y)f(x^*x)$
3) $f((\lambda x + uy).(\lambda x + uy)) > 0$
4) $|\lambda|^2 f(x^*x) + \lambda\bar{u}f(y^*x) + \lambda u f(x^*y) + |u|^2 f(y^*y) \geq 0$

Theorem 1.2.5[1]: Every positive functional f in a symmetric algebra R with identity is real and $|f(x)^2| \leq f(e)f(x^*x)$.

Theorem 1.2.6[1]: Suppose R is a symmetric algebra without identity and that R' is the symmetric algebra obtained from R by adjunction of the identity. A positive functional f in R can be extended to a positive functional in R' if and only if f is real and satisfies inequality $|f(x)|^2 \leq cf(x^*x)$ for all $x\epsilon R$ where c is some constant

Theorem 1.2.7[1]: If in a symmetric normed algebra R

a) $|x^*| = |x|$

b) There exists a set $\{e_x\}$ approximating the identity, then every continuous positive functional in R can be extended to a positive functional in R'

Definition 1.2.8[1]: R is called a normed symmetric algebra if

a) R is a normed algebra

b) R is a symmetric algebra

c) $|x^*|$

CHAPTER TWO

Chapter two

Section one

Definition 2.1.1[2] Let U be a universe and E be a set of parameters. Let P (U) denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U). In other words, a soft set over U is a parametrized family of subsets of the universe U. For e ∈ A, F (e) may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.1.2[2] For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if

1. A ⊆ B and

2. for all e ∈ A, F(e) ⊆ G(e).

We write (F,A)⊂˜(G,B).

(F,A) is said to be a soft superset of (G,B), if (G,B) is a soft subset of (F,A). We denote it by (F,A)⊃˜(G,B).

Definition 2.1.3[2] Two soft sets (F,A) and (G,B) over a common universe U are said to be equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

$$H(e) = \begin{cases} f(e) & if e \in A\_B \\ G(e) & if e \in B - A \\ f(e) \cup G(e) & if e \in AB \end{cases}$$

Definition 2.1.4[2] The complement of a soft set (F,A) is denoted by (F,A)c = (Fc,A), where Fc : A → P(U) is a mapping given by Fc(α) = U − F(α), for all α ∈ A.

Definition 2.1.5[2] A soft set (F,E) over U is said to be an absolute soft set denoted by Uˇ if for all e ∈ E, F(e) = U.

Definition 2.1.6[2] A soft set (F,E) over U is said to be a null soft set denoted by Φ if for all e ∈ E, F(e) = ϕ.

Definition 2.1.7[2] The union of two soft sets (F,A) and (G,B) over the common ∪

universe U is the soft set (H,C) , where C = A B and for all e ∈ C,

We express it as (F,A)∪˜(G,B) = (H,C)

.

Definition 2.1.8[2] The intersection of two soft sets (F,A) and (G,B) over the common universe U is the soft set (H,C), where C = A B and for all e ∈ C, H(e) = F(e)∩G(e). We write (F,A)∩˜(G,B) = (H,C). ∩

Definition 2.1.9[2] Let X be an initial universal set and E be the non-empty set of parameters. The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by (F,E) − (G,E), is defined by H(e) = F(e) − G(e) for all e ∈ E.

Proposition 2.1.10[2] Let (F,E) and (G,E) be two soft sets over X. Then

•• (i) ((F,E)∪˜∩(G,E))c = (F,E)c∩˜∪(G,E)c

(ii) ((F,E)˜(G,E))c = (F,E)c ˜(G,E)c

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Definition 2.1.11[2] Let X be a non-empty set and E be a non-empty parameter set. Then a function $\epsilon : E \rightarrow X$ is said to be a soft element of X. A soft element ϵ of X is said to belong to a soft set A of X, which is denoted by $\epsilon\tilde{\in}A$, if $\epsilon(e) \in A(e)$, $\forall e \in E$. Thus for a soft set A of X with respect to the index set E, we have $A(e) = \{\epsilon(e); \epsilon\tilde{\in}A\}$, $e \in E$.

It is to be noted that every singleton soft set (a soft set (F,E) for which F(e) is a singleton set, $\forall e \in E$) can be identified with a soft element by simply identifying the singleton set with the element that it contains $\forall e \in E$.

Definition 2.1.12[2] Let R be the set of real numbers and B(R), the collection of all non-empty bounded subsets of R and A be taken as the set of parameters. Then a mapping $F : A \rightarrow B(R)$ is called a soft real set. It is denoted by (F,A). If specifically (F,A) is a singleton soft set, then after identifying (F,A) with the corresponding soft element, it will be called a soft real number. We use notations $\tilde{r}, \tilde{s}, \tilde{t}$ to denote soft real numbers whereas $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in A$ etc. For example $\bar{0}$ is the soft real number where $\bar{0}(\lambda) = 0$, for all $\lambda \in A$.

For two soft real numbers $\tilde{r}, \tilde{s}$ it is defined

$\tilde{r} \tilde{\leq} \tilde{s}$ if $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$, for all $\lambda \in A$.

$\tilde{r} \tilde{\geq} \tilde{s}$ if $\tilde{r}(\lambda) \geq \tilde{s}(\lambda)$, for all $\lambda \in A$.

$\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(\lambda) < \tilde{s}(\lambda)$, for all $\lambda \in A$.

$\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(\lambda) > \tilde{s}(\lambda)$, for all $\lambda \in A$.

Let X be an initial universal set and A be the non-empty set of parameters. Let us consider the collection of those soft sets (F,A) over X for which $F(\lambda) \neq \phi$, for all $\lambda \in A$, which is denoted by

S(X˘). For any soft set (F,A) ∈ S(X˘), the collection of all soft elements of (F,A) is denoted by SE (F,A) and let Y be any collection of soft elements of (F,A), then SS (Y) is the soft set generated byYsuch that (SS (Y))(λ)= {x˜(λ);x˜ ∈ Y }, ∀λ ∈ A.

Definition 2.1.13[2] (Sums and Scalar products of soft sets) Let F1,F2,......,Fn be n soft sets in (V,A). Then F = F1 + F2 + ... + Fn is a soft set over (V,A) and is defined as F(λ) = {x1 + x2 + ... + xn;xi ∈ Fi(λ),i = 1,2,...,n}, ∀λ ∈ A. Let α ∈ K(R or C) be a scalar and F be a soft set over (V,A), then αF is a soft set over (V,A) and is defined as follows: αF = G, G(λ) = {αx;x ∈ F(λ)}, λ ∈ A.

Definition 2.1.14[2] Let V be a vector space over a field K(R or C) and let A be a parameter set. Let G be a soft set over (V,A). Now G is said to be a soft vector space or soft linear space of V over K if G(λ) is a vector subspace of V , ∀λ ∈ A.

Proposition **2.1.15[2]** α(F +G) = αF +αG for all soft sets F,G over (V,A) and α ∈ K.

Definition **2.1.16[2]** (Soft Vector Sub spaces) Let F be a soft vector space of V over K. Let G : A →P(V) be a soft set over (V,A). Then G is said to be a soft vector subspace of F if(i) for each λ ∈ A, G(λ) is a vector subspace of V over K and (ii) F(λ) ⊇ G(λ), ∀λ ∈ A.

Theorem 2.1.17[2] A soft subset G of a soft vector space F is a soft vector sub-space of F if and only if for all scalars α,β ∈ K, αG + βG ⊂ G.

Definition 2.1.18[2] Let G be a soft vector space of V over K. Then a soft element of V is said to be a soft vector of G. In a similar manner a soft element of the soft set (K,A) is said to be a soft scalar, K being the scalar field.

Definition 2.1.19[2] A soft vector x˜ in a soft vector space G is said to be the null soft vector if $\tilde{x}(\lambda) = \theta$, $\forall\lambda \in A$, θ being the zero element of V . It will be denoted by Θ. A soft vector is said to be non-null if it is not a null soft vector

Definition 2.1.20[2] Let x,˜ y˜ be soft vectors of G and k˜ be a soft scalar. Then the addition x˜ + y˜ of x,˜ y˜ and scalar multiplication k.˜ x˜ of k˜ and x˜ are defined by $(\tilde{x} + \tilde{y})(\lambda) = \tilde{x}(\lambda) + \tilde{y}(\lambda)$, $\tilde{k}.\tilde{x}(\lambda) = \tilde{k}(\lambda).\tilde{x}(\lambda)$, $|\tilde{k}|(\lambda) = |\tilde{k}(\lambda)|, \forall\lambda \in A$. Obviously, x˜ + y,˜ k.˜ x˜ are soft vectors of G.

Theorem 2.1. 21[2] In a soft vector space G of V over K,

(i). $\bar{0}.\tilde{\alpha} = \Theta$, for all $\tilde{\alpha} \tilde{\in} G$;

(ii). $\tilde{k}.\Theta = \Theta$, for all soft scalar k˜.

(iii). $(-\bar{1})\tilde{\alpha} = -\tilde{\alpha}$, for all $\tilde{\alpha} \tilde{\in} G$.

Definition 2.1.22[2] Let $\check{X}$ be the absolute soft vector space i.e., $\check{X}(\lambda) = X$, $\forall\lambda \in A$. Then a mapping $\|.\| : SE(\check{X}) \to R(A)^*$ is said to be a soft norm on the soft vector space $\check{X}$ if $\|.\|$ satisfies the following conditions:

(N1).$\|\tilde{x}\| \tilde{\geq} \bar{0}$, for all $\tilde{x} \tilde{\in} \check{X}$;

(N2).$\|\tilde{x}\| = \bar{0}$ if and only if $\tilde{x} = \Theta$;

(N3).$\|\tilde{\alpha}.\tilde{x}\| = |\tilde{\alpha}|\|\tilde{x}\|$ for all $\tilde{x} \tilde{\in} \check{X}$ and for every soft scalar $\tilde{\alpha}$;

(N4). For all $\tilde{x}, \tilde{y} \tilde{\in} \check{X}$, $\|\tilde{x} + \tilde{y}\| \tilde{\leq} \|\tilde{x}\| + \|\tilde{y}\|$. The soft vector space $\check{X}$ with a soft norm $\|.\|$ on $\check{X}$ is said to be a soft normed linear space and is denoted by $(\check{X}, \|.\|, A)$ or $(\check{X}, \|.\|)$.

(N1),(N2),(N3)and (N4) are said to be soft norm axioms.

Example 2.1.23[2] Every parametrized family of crisp norms ||.||λ : λ ∈ A on a crisp vector space X can be considered as a soft norm on the soft vector space X˘

.

Definition 2.1.24[2] Let (X,˘ ||.||,A) be a soft normed linear space and r˜>˜¯0 be a soft real number. We define the followings;

B(x,˜ r˜) = {y˜∈˜X˘ : ||x˜ − y˜||<˜r˜}⊂ SE(X˘) B¯(x,˜ r˜) = {y˜∈˜X˘ : ||x˜-y˜||≤˜r˜}⊂SE(X˘)

S(x,˜ r˜) = {y˜∈˜X˘ : ||x˜ − y˜|| = r˜}⊂SE(X˘)

B(x,˜ r˜), B¯(x,˜ r˜) and S(x,˜ r˜) are respectively called an open ball, a closed ball and a sphere with centre at x˜ and radius r˜. SS(B(x,˜ r˜)), SS(B(x,˜ r˜)) and SS(S(x,˜ r˜)) are respectively called a soft open ball, a soft closed ball and a soft sphere with centre at x˜ and radius r˜.

Definition 2.1.25[2] A sequence of soft elements {x˜n} in a soft normed linear space (X,˘ ||.||,A) is said to be convergent and converges to a soft element x˜ if ||x˜n−x˜||→ ¯0 as n → ∞. This means for every ϵ˜>˜¯0, chosen arbitrarily, ∃ a natural number N = N(ϵ˜), such that ¯0≤||˜ x˜n −x˜||<˜ϵ˜ , whenever n > N. i.e., n > N ⇒ x˜n ∈ B(x,˜ ϵ˜). We denote this by x˜n → x˜ as n →∞ or by limn→∞x˜n = x.˜ x˜ is said to be the limit of the sequence x˜n asn →∞.

Definition 2.1.26[2] Let τ be the collection of soft sets over X, then τ is said to be soft topology on X if

ϕ, X belong to τ

the union of any number of soft sets in τ belongs to τ

the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

be a soft sets over (U,E).

Define τ(e) 2.1.27[2] = {F(e) : F ∈ τ} for e ∈ E. Then τ is said to be a topology of soft subsets over (U,E) if τ(e) is a crisp topology on U∀e ∈ E. In this case, ((U,E),τ) is said to be a topological space of soft subsets. If τ is a topology of soft subsets over 1(U,E), then the members of τ are called open soft sets and a soft set F over (U,E) is said to be closed soft set if Fc ∈ τ.

Chapter two

Section two

2[. Soft convergence and soft topology

In this section we discuss about a new type of convergence on a soft normed linear space and introduce soft topology generated by soft norm on a soft normed linear space and study some of its **basic properties.**

Lemma 2.2.1[2] In a soft normed linear space (X,˘ ||.||), for any x˜∈˜X˘ and λ∈ A, ||x˜||(λ) = 0 if and only if x˜(λ) =. θ

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Proof. Let us consider a soft scalar α˜ such that α˜(μ) = 1 if μ = λ, α˜(μ) = 0 if μ ≠ λ. Then (α˜x˜)(μ) = θ for μ ≠ λ, (α˜x˜)(μ) = x˜(λ) for μ = λ. From N

$$|\tilde{a}\,\tilde{x}\| = |\tilde{a}|\ \|\tilde{x}\|$$

$$\tilde{\theta} \text{ iff } \tilde{x}(\lambda) = \theta$$

(3) we have. This shows that ||x˜||(λ) = 0 iff |α˜|||x˜|| = 0¯ iff ||α˜x˜|| = ¯0 iff α˜x˜ =□

Lemma 2.2.2[2] In a soft normed linear space (X,˘ ||.||), for each ξ ∈ X and λ ∈ A, {||x˜||(λ) : x˜(λ) = ξ} is a singleton set.

Proof. In a soft normed linear space, by N (4), we have, for all ˜

$\tilde{x},\tilde{y}$ ∈̃ x||$\tilde{x}$ ||≅||$\tilde{x}$–||$\tilde{y}$ ||-||$\tilde{x}$ ||≦̃||$\tilde{x}$-$\tilde{y}$||.so

$\tilde{y}$+$\tilde{y}$||≦||$\tilde{x}$-$\tilde{y}$||+||$\tilde{y}$||→ ||$\check{x}$||-||$\tilde{y}$||≦̃||$\tilde{x}$-$\tilde{y}$||.similarly

Then ‖|$\tilde{x}$‖(λ) –now if ~x-y) (λ)=$\tilde{x}$(λ)−̃$\tilde{y}$(λ)= θ i.e.oby

Lemma 2.2.3[2] since(˜||x˜||(λ) = ||y˜||(λ), which proves the lemma. □

Proposition 2.2.([4])

(Decomposition Theorem) In a soft normed linear space

X,˘ ||.||), if∈ we define|| || for|| ||each λ ∈ A, ∈||.||λ : X → R+ be a mapping such that

˜ ˘ is such that x˜(λ) = ξ. Then for each for each ξ X, ξ λ = x˜ (λ), where x˜ X λ ∈ A, (X,||.||λ) is a normed linear space.

Proof. Since for λ ∈ A, {||x˜||(λ) : x˜(λ) = ξ} is a singleton set, the mapping ||.|o| λ : X → R+ is well defined. Hence from soft norm axioms, it follows that (X,||.||λ) is a normed linear space λ ∈ A. □

Definition 2.2.4[2] In a soft normed linear space a sequence x_n f soft elements is said to be soft convergent and soft converges to a soft element x,˜ if for any soft real number ϵ˜>˜¯0 there exists a soft natural number N˜ such that $||x_n$ − x˜||(λ) < ϵ˜(λ) ∀ n ≥ N˜ (λ), ∀ λ ∈ A and is denoted by limn→∞ x_n = x˜ or x_n ,→ x˜, where x˜ is called the soft limit of the sequence x_n

Proposition 2. 2.5[2]. Soft limit of a sequence of soft elements in a soft normed linear space is unique.

Proof. Let x_n is sequence of soft elements in a soft normed linear space (X,˘ ||.||) such

Exist that lim soft n→∞ natural x_n=˜x numbers andlimN˜n1→∞and x_nN˜=2 y˜such. Thenthatfor

$\|\widetilde{x_n}-\tilde{x}\|(\lambda)< \frac{\in(x)}{2} \forall n \geq n_1(\lambda)$

any soft real number ϵ˜>˜¯0 there ,

$\forall\lambda\in A\ and \|x_n-\tilde{y}\|(\lambda)\frac{\in(\lambda)}{2}\ \forall n \geq n_2(\lambda), \forall\lambda\in A$.i.e. $\|x_{n-}\tilde{x}\|(\lambda)< \frac{\in(\lambda)}{2}$ and

$\|x_n-\tilde{y}\|\,|(\lambda)< \frac{\in(\lambda)}{2} \forall n \geq \tilde{n}(\lambda)$,mam $\{\ n_1, n_2$

where maximum of these

soft natural numbers taken as component wise) ∀ λ ∈ A. Now for ∀ n ≥ N˘ (λ), ||x˜−y˜||(λ) <|| x_n −x˜||(λ)+|| −x_ny˜||(λ)< ϵ˜(λ)∀ λ ∈ A, which shows that x˘ = y.˘ □

Proposition 2.2.6[2] A sequence x_n of soft elements in a soft normed linear space (X,˘ ||.||||)||is soft convergent to x˘ iff x˜n (λ) is convergent to x˜(λ) in (X,||.||λ) ∀λ ∈ A, where . λ defined as in Proposition 2 .2.([6])

Proof. Let x_n be sequence→ soft converging to the soft element x˘ in (X,˘ ||.||). Take ϵ > 0, then since x_n, x˘, so there exists a soft natural number N˘ such that

$\|x_n-\tilde{x}\|(\lambda)< \bar{\in}(\lambda)=\in \forall n \geq \tilde{N}(\lambda), \forall \lambda\in A$ which shows that $x_n(\lambda)\rightarrow \tilde{x}(\lambda)\forall\lambda \in A$.

But $\|x_n$ −x˜‖(λ)=| x_n|(λ)−x˜(λ)‖|λ, Conversely, x_n(λ) → x˜(λ) ∀λ ∈ A. Take ϵ˜>˜¯0, since x_n (λ) → x˜(λ) ∀λ ∈ A, so for each λ ∈ A ∃Nλ, ‖ x_n−x˜‖(λ) = ‖x_n (λ)−x˜(λ)‖|λ < ϵ˜(λ) ∀n > Nλ. Now if we define N˜ (λ) = Nλ ∀λ ∈ A then ‖ x_n− x˜‖(λ) < ϵ˜(λ) ∀ n ≥ N˜ (λ), ∀ λ ∈ A. This proves the proposition. □

Definition 2.2.7[2]. Asequence x_n in a soft normed linear space is said to be soft Cauchy if for any soft real number ϵ˜>˜¯0 there exists a soft natural number N˜ such that $\|x_n - x_m\|$(λ) < ϵ˜(λ) ∀ n,m ≥ N˜ (λ) , ∀ λ ∈ A.

Propositionn 2.2.8[2] Asequence x_n a soft normed linear space (X,ˇ ‖.‖) is soft

Cauchy iff x˜ (λ) is Cauchy in (X, . ˇ ‖.‖) λ A, where . is defined as in Proposition 2" 2.

Proof. Proof is same as in Proposition 2.6 □

Proposition 2.2.9[2] Every soft convergent sequence of soft elements is soft Cauchy.

Proof. Let x_n ,→ x˜, then the relation ‖ x_n −x_m‖(λ) x_n≤‖ −x˜‖(λ)+‖ x_m −x˜‖(λ)∀ λ ∈ A gives the result. □

Definition 2.2.10[2] Asequence x_n of soft elements in a soft normed linear space

(x_n ‖.‖) is said to be bounded if there exists a soft real number M˜ such that ‖x˜ ‖≤ ˜M˜ , n N (The set of all natural numbers).

Proposition 2.2.11[2] Every soft Cauchy sequence $x_n$of soft elements in a soft normed linear space (nX,ˇ ‖.‖) is bounded. (‖‖‖)

Proof. Let x˜ be a soft Cauchy sequence in X,ˇ . . Then there exists a soft

Real number such that $\|x_n - x_n\|$(λ)< $\bar{1}$ ∀ n,m≥ $\tilde{N}$ (λ),∀ λ∈ $A\|$ $\|$λ ≤ $\|$ λ$\|$ $\|$λ$\|$ $\|$λ$\|$ $\|$λ∀≥ N λ ∀ λ A

N˜. x_n (x_n− x˜N˜(λ) ( ) + x˜N˜(λ) ( ) < 1 + x˜N˜(λ) ∈ .

Now if we take

M˜(λ)= maxn{||||x˜1||(λ),||x˜2||∀(λ)∈,||x˜∀3||(λ∈),......||x˜||N˜(nλ||) ≤1||(λ∀), ∈1 + ||x˜N˜(λ)||(λ)},

then clearly ||x˜ (λ) < M˜ (λ) n N λ A. i.e. x˜ ˜M˜ n N. □

Corollary 2.2.11[2] Every soft convergent sequence x_n of soft elements in a soft normed linear space (X,ˇ ||.||) is bounded. λ(||||) ((||||) (||||) ||||λ

Definition 2.2.12[2]. A soft normed linear space X,ˇ . is said to be soft complete if every soft Cauchy sequence in X,ˇ . is soft convergent in X,ˇ . .

Proposition 2.2.13[2]. Asoft normed linear space (X),ˇ ||.||) is soft complete iff (X, . ||.||) is complete ∀λ ∈ A, where defined as in Proposition 2.2

Proof. Let (X,ˇ ||.||) is soft complete and λ ∈ A. Consider (X,||.||λ). Let $\{x_n\}$\{be\}

aCauchy sequence in (X, .). Now if we construct a sequence of soft elements x_n such that

$$\widetilde{Xn}(\mu) = \begin{cases} Xn & if\,\mu = \lambda \\ \theta & \text{if}(\mu \neq \lambda \end{cases}.$$

Then clearly⇒{{x˜nn}} is soft Cauchy. So by soft λ completeness of (X,ˇ ||.||), $\{x_n\}$ is convergent x is convergent. i.e. (X,||.||) is complete.

Converse of the **proof** directly follows from Proposition 3.3, Proposition 3.6 and Proposition 3.8. □

Corollary 2.2.14[2] The soft set R(A) over R(set of real numbers) is soft complete.

Proposition 2.2.15[2] In a soft normed linear space (X,ˇ ||.||), SS(B(x,˜ r˜))(λ) =

S $(\tilde{x}(\lambda)),\tilde{r}(\lambda)\forall\lambda \in A, where$ $\boldsymbol{B}(\tilde{\boldsymbol{x}},\tilde{\boldsymbol{r}})$= $\{\tilde{y} \,\tilde{\in}\, \tilde{\boldsymbol{x}} : \|\, \tilde{\boldsymbol{x}} - \tilde{y} \,\| \,\tilde{<}\, \tilde{r}\}$CSE(x) and S($\tilde{\boldsymbol{x}}$(λ)(λ))= z λ : $\tilde{\boldsymbol{x}}$ (λ)z λ $\tilde{r}$ (λ)

r˜{ ∈||− || < }

Proof. Let λ ∈ A and z ∈ SS(B(x,˜ r˜))(λ). Then there exists a soft element y˜ such that

$\|\, \tilde{\boldsymbol{x}} - \tilde{y} \,\| \,\tilde{<}\, \tilde{r}$ and $\tilde{y}$(λ))= z,So $\|\, \tilde{\boldsymbol{x}} - \tilde{y} \,\|$ (λ)< $\tilde{r}$(λ)→|| $\tilde{\boldsymbol{x}}$(λ) ||

i. ||˜ (xλ) − z||$_\lambda$ < ˜r (λ)⇒ z ∈ S (˜x (λ) , r˜ (λ)).

Now let z ∈ S (˜x (λ) , r˜ (λ)).. Then if we take a soft element z˜ such that z˜(μ) = z when μ = λ, z˜(μ) = x˜(μ) when μ =/λ. Then clearly z˜∈˜B(x,˜ r˜). Hence z ∈ SS(B(x,˜ r˜))(λ).

Corollary2.2.16[2](. If S (xλ,rλ) are open balls in (X,||.||λ) ∀λ ∈ A. Then the soft set U such that U (λ) = S (xλ,rλ) ∀λ ∈ A is the soft∀ open∈ ball in (X,ˇ ||.||) with centre x˜ and radius r˜, where x˜(λ) = xλ and r˜(λ) = rλ, λ A.

Proof. Consider the soft element x˜ and soft real number r˜ such that x˜(λ) = xλ and r˜(λ) = rλ,∀λ ∈ A. Now if we consider the open ball B(x,˜ r˜). Then clearly, by the previous proposition, SS(B(x,˜ r˜)) = U. □

Proposition 2.2.17[2] Let in a soft normed linear space (X,ˇ ||.||), τ be the set of all soft sets in X such that(U˜||∈||τ) iff U˜ can be expressed as a union of finite intersections of soft open balls of X,ˇ . . Then τ forms a soft topology [15] on X.

Proof. The proof is straightforward. □

All the members of τ are(said to) be the soft open in (X,||ˇ||||.||). A soft set F˜ is said to be the soft closed in X,ˇ ||.|| if F˜c ∈ τ. The topology defined as in Proposition 2.16 is called the topology generated by the soft norm . on Xˇ.

Proposition 2.2.18[2]. For any α ∈ A the collection τα { U˜(α) : U˜∈τ} is a t opology

in X. i.e. τ is a topology of soft sets on X.

Proof. Proof directly follows from the Definition of soft topology τ and Proposition

2.16. □

Corollary 2.2..19[2] (|Let) U be soft set in a soft normed| linear space (X,ˇ ||.||). If/ U is soft open in X,ˇthen U (λ) is open in (X, . λ)∀ λ∈ A. Further, if X = θ , then the converse is also true.

Proof. Let U be soft open in (X,||.||). Then U = $U_{i\in\Delta} \cap_{i=1}$ Gi,j, where n is a positive integer and Gi,j is a soft open ball, i ∈ Δ. Now U (λ) = $U_{i\in\Delta} \bigcap_{n=1}$ Gi,j (λ)

and Gi,j (λ) is a open ball in (X,||.||λ), so U (λ) is an open ball in (X,||.||λ) ∀λ ∈ A. For the converse part consider the following Cases:

• **Case-1**: Let U ∈S (X). i.e. U (λ) ≠ ϕ ∀λ ∈ A. Now since for each λ ∈ A,

U (λ) is open in (X,||.||λ). So U (λ) can be expressed as aunion of open balls in (X,||.||λ), ∀λ ∈ A. Choose for each λ ∈ A one such open ball in((X,˘ ||.||)) and thereby construct a soft{set.∈ Then this soft} sets are soft open balls in X,˘and their union is the soft set U. Hence U is soft open in

(X,˘||.||)

• **Case-2**: If U ∈ S(X). Let A1 ={λ A;U (λ) = ϕ }. Since X ≠{θ ,}

∃ x(≠ θ) ∈ X. Take two disjoint balls B (x,r) and B (θ,r) in X. Now construct the soft sets U1 and U2 as in Case-1 by taking U1 (λ) = B (x,r) if λ ∈ A1, U1(λ) = U(λ) otherwise and U2 (λ) = B (θ,r) if λ ∈ A1, U2(λ) =

U(λ) otherwise. Then U1 and U2 are soft open by Case-1. Hence U = U1∩U2 is soft open in X,˘ . .

□

Definition2 .2.20[2] In a soft normed linear space (X,˘ ||.||), a soft element x˜ is said to be an interior point of a soft set U if there exists a open ball B(x,˜ r˜) containing x˜ such that SS(B(x,˜ r˜))⊆˜U.

Proposition 2.2.21[2]. In a soft normed linear space||) (X,˘ ||.||)||a||soft set U ∈S (X) is soft open in X,˘ . iff any soft element x˜˜U is an interior point of U.

Proof. Let U((X)) be soft open in X,˘ . and x˜˜U. Then U (λ) is open in (X, . λ) λ A(by first part of Corollary 3.20). Now (X, . λ) is a normed linear space so U (λ) can be expressed as a union of open balls in (X,||.||λ)

∀λ ∈ A, where at least one of the balls contains the point x˜(λ) ∀λ ∈ A, since x˜∈˜U. If we take the soft set whose λ components are these open balls containing x˜(λ) in (X,||.||λ) ∀λ ∈ A, by Proposition 2.16, this soft set will be soft open ball in (X,˜ ||.||) containing x˜ and contained in U, which proves that x˜ is an interior point of U.

Conversely, let any soft element x˜∈˜U be an interior point of U. Then for each x˜∈˜U there exists a open ball B(x,˜ r˜) such that SS(B(x,˜ r˜))⊆˜U. Now if we take all soft elements of U then U = SS (∪x˜∈˜U {x˜})⊆∪˜ x˜∈˜U { SS(B(x,˜ r˜))}⊆˜U. i.e. U is a soft open set. □

Proposition 2.2.21[2] Let (X,ˇ ||.||) bea soft normed∈ linear space and $x_n$be any sequence in a soft closed set F. If x_n x then x˜˜F.

Proof. Let x˜∈˜/F, then x˜(λ) ∈/ F (λ) for some λ ∈ A ⇒x˜(λ) ∈ X − F (λ), where X −F (λ) is open in (X,||.||λ). Now since x_n,→ x˜ ⇒ x_n (λ) → x˜(λ), so the sequence x_n (λ) is eventually in X − F (λ), which contradicts that the sequence x˜n is in F.

Hence x˜∈˜F

Chapter tow

Section three

3. Soft Banach algebra and its properties

Definition 2.3.1[2] Let V be an algebra over a field C of complex numbers and let A be the parameter set and (G,A) be a soft set over V . Now (G,A) is said to be a soft algebra of V over C if G(λ) is a sub algebra of V ∀λ∈A.

It is very easy to see that in a soft algebra the soft elements satisfy the properties:

(x˜y˜)z˜ = x˜(y˜z˜)

x˜(y˜+ z˜) = x˜y˜+ x˜z,˜ (x˜ + y˜)z˜ = x˜z˜+ y˜z˜

α˜(x˜y˜) = (α˜x˜)y˜ = x˜(α˜y˜)

where for all x,˜ y,˜ z˜∈˜G˘ and for any soft scalar α˜, x˜y˜(λ) = x˜(λ)y˜(λ) and α˜x˜(λ) = α˜(λ)x˜(λ). If (G,A˘) is||also||≤||a soft||||Banach|| space(with) respect to a soft norm that satisfies the inequality x˜y˜ ˜ x˜ y˜ and if G,A˘ contains an identity e¯ such that x˜e¯=¯ex˜=x˜ with ||e¯|| = ¯1, then (G,A) is called a soft Banach algebra. In addition, if in a soft Banach algebra (G,A), x˜y˜ = y˜x˜, ∀x,˜ y˜∈˜G˘ then (G,A) is called a commutative soft Banach algebra.

Proposition 2.3.2[2]. (G,A) is a soft Banach algebra iff G(λ) is a Banach algebra ∀λ ∈ A.

Proof. Proof follows from the definition of soft algebra and Proposition 2.14. □

Proposition 2.3.3[2]. In a soft Banach algebra if x_n ,→ x˜ and y_n ,→ y˜ then x_n y_n ,→ xy˜. i.e. multiplication in a soft Banach algebra is continuous.

Proof. Sincex_n,→ x˜ and y_n,→ y in (G,A). So x_n (λ) → x˜(λ) and y_n (λ) → y˜(λ) ∀λ ∈ A in (G(λ),||.||λ). Now since G(λ) is Banach algebra ∀λ ∈ A (by Proposition 4.2) and in Banach algebra multiplication is continuous so, x_n (λ) y_n (λ) → x˜(λ)y˜(λ)∀λ ∈ A, which proves that y_n,→ x_n x˜y˜ (by Proposition 3.6). □

Proposition 2.3.4[2] Every parametrized family of crisp Banach algebras on a crisp vector space V can be considered as a soft Banach algebra on the soft vector space

Vˇ.

Proof. Let ||.||λ : λ ∈ A be a family of crisp norms on the vector space V such that (V,||.||λ) are Banach algebra ∀λ ∈ A. Now let us define ||.|| : Vˇ → R(A)∗ by ||x˜||(λ) = ||x˜(λ)||λ, ∀λ ∈ A,∀x˜∈˜Vˇ. Then by Example 2.23 (Vˇ,||.||) is a soft normed linear space. Now to show that (Vˇ,||.||) is a soft Banach algebra we have to show that ||x˜y˜||≤||˜ x˜||||y˜|| ∀ x,˜ y˜∈˜Vˇ and (Vˇ,||.||) is complete.

Now ||x˜y˜||(λ) = ||x˜(λ)y˜(λ)||λ ≤ ||x˜(λ)||λ||y˜(λ)||λ ≤||x˜||(λ)||y˜||(λ) ∀λ ∈ A, which shows that ||x˜y˜||≤||˜ x˜||||y˜||.

Now let x_n be a Cauchy sequence in Vˇ. Then for any ϵ˜>˜0¯ there exists a soft natural number N˜ such that

||˜x_n+p − x_n||(λ)< $\frac{\epsilon}{2}$(λ) ∀n ≥ N (λ)∀λ ∈ A⇒||x_n+p(λ) − (λ)||x_n(λ)< $\frac{\epsilon}{2}$ (λ) ∀n ≥ N (λ), ∀λ ∈ A. i.e x_n (λ) is a Cauchy sequence in (V, ||.||λ)
∀λ ∈ A . Since (V,||.||λ) are Banach algebra ∀λ ∈ A, so there exist xλ such that x˜n(λ) converge to xλ, ∀λ ∈ A. Hence there must exist some Nλ(> N (λ)) such that
||x_n(λ)−xλ||λ<($\frac{\epsilon}{2}$λ) ∀n ≥Nλ, ∀λ ∈A .Now || x_n−x||(λ) =||(λx_n)−x λ||λ<||(λx_n)− λ x_N (λ) ||λ+||˜xN$_\lambda$ −xλ (λ) ||λ<˜(λ) ∀n > N (λ), ∀λ ∈ A, where x(λ) = xλ. This

shows that (V,||.||) is a soft Banach space. Hence (V,||.||) is a soft Banach algebra.

□

Definition 2.3.5[2] Asoft element x˜∈˜Gˇ is said to be invertible if it has an inverse in Gˇ i.e. if there exists a soft element y˜∈˜Gˇ such that x˜y˜ =

y˜x˜ = e¯ and then y˜ is called the inverse of x˜, denoted by x˜−1. Otherwise x˜ is said to be non-invertible soft element of Gˇ.

Remark 2.3.6[2] Clearly e¯ is invertible. If x˜ is invertible, then we can verify that the inverse is unique. because if y˜x˜ = e¯ = x.˜z˜ Then y˜ = y˜e¯ = y˜(x˜z˜) = (y˜x˜)z˜ = e¯z˜ = z.˜ Further, if x˜ and y˜ are both invertible then x˜y˜ is invertible and (x˜y˜)−1 =(y^{-1} x^{-1})(. For (x˜y˜ (y^{-1} x^{-1}) = x˜(y y^{-1})x˜x^{-1}) = x˜e¯ x^{-1}) = e¯ and similarly ((y^{-1} x^{-1})(x˜y˜)= e¯ .

Definition 2.3.7[2] Let (G,∗) be a group and (F,A) be a soft set over G. Then (F,A) is said to be a soft group over G if and only if F(λ) is a subgroup of (G,∗) for all λ ∈ A.

Proposition 2.3([8]). Let (G,∗) be a group and (F,A) be a soft set over G. If for any

x˜, y˜ ∈˜(F,A)

x˜˜∗y˜∈˜(F,A)

x˜−1∈˜(F,A), where x˜˜∗y˜(λ) = x˜(λ) ∗ y˜(λ) and x˜−1 (λ) = (x˜(λ))−1. Then (F,A) is a soft group over G.

Proof. Proof is obvious. □

Note 3.9. This shows that in a soft algebra, the soft set generated by the all invertible elements is a soft group with respect to the composition defined as in Proposition ??.

Definition 2.3.10.[2] A series $\sum_{n=1}^{\infty} x_n$of soft elements is said to be soft convergent if the

partial sum of the series sk= $\sum_{n=1}^{k} x_n$˜ is soft convergent.

Proposition 2.3.11[2]. Let (G,A) be a soft Banach algebra. If x˜∈˜Gˇ satisfies ||x˜||<˜¯1,

then (¯e − x˜) is is invertible and $(\tilde{e}\text{-}x)^{-1}$=e+$\sum_{n=1}^{\infty} x_n$

Proof. Since (G,A) is soft algebra, so we have ||x˜j||≤||˜ x˜||j for any positive integer j, so that the infinite series ∑∞ $\|x\|^n$ is soft convergent because ||x˜||<˜1.¯ So the sequence sk= $\sum_{n=1}^{k} x_n$

of partial sum ˜ is a soft Cauchy sequence since ||∑x˜n|| <˜ ∑||x˜||n.

n=k $\sum_{n=1}^{\infty} x_n$

Since (G,A) is soft complete sois soft convergent.

Now let s˜ = e¯+ ∑ x˜ .

n=1 Now it is only we have to show that s˜ = (¯e − x˜)−1.

We have

(3.1) (¯e − x˜)(¯e + x˜ + x_2 + ... x_n) = (¯e + x˜ + +x_2 ..)(e˜− xx_n˜) = e¯−x_n +1

Now again since ||x˜||<˜¯1 so x_{n+1} ,→ θ¯ as n→ ∞. Therefore letting n→ ∞ in and remembering that multiplication in G is continuous we get,

(¯e − x˜)s˜ = s˜(¯e − x˜) = e˜

. So that s˜ = (¯e −$x)^{-1}$ This proves the proposition. □

Corollary 2.3.12[2] Let G be a soft Banach algebra. If x˜ ∈ G and ||e¯− x˜|| < ¯1, then x^{-1} =$\bar{e}$+$\sum_{j=1}(\bar{e}\text{-}x)^j$

x^{-1} exists and .

Corollary 2.3.13[2] Let G be a soft Banach algebra. Let x˜ ∈ G and µ˜ be a soft scalar such that |µ˜|>˜||x˜||. Then (µ˜e¯−$x)^{-1}$ exists and

$(\tilde{\mu}\ \bar{e}$-¯−$x)^{-1}$=$\sum_{n=1} \mu^{-n} x^{n-1}\ (x_0 = \widetilde{e})$

Proof. y˜ ∈ G be such that y˜−1exists in G and α˜ be a soft scalar such that α˜(λ) ≠0 ,

∀λ ∈ A. Then it is clear that

(α˜y˜)−1 = α˜−1y˜−1.

Having noted this we can write

µ˜e¯− x˜ = µ˜(¯e − µ˜−1x˜)

and now we show that (¯e− µ˜−1x˜)−1 exists. We have ||e¯− (¯e −µ˜−1x˜)|| = ||µ˜−1x˜|| = |µ˜|−1||x˜|| <˜¯1 by hypothesis. So, By Corollary 3.12 (¯e − μ^{-1}x˜)−1 exists and hence (µ˜e¯− x˜)−1 exists. For the infinite series representation, using the Proposition 3.11 we have

(µ˜e¯− x˜)−1 = µ˜−1(¯e − µ−1x˜)−1

μ^{-1} $(\bar{e}+\sum_{n=1}^{\infty}[e-(e-]\mu^{-1}[\![x)]^n\ [\mu^{-1}[\![x)]^n$

μ^{-1} $(\bar{e}+\sum_{n=1}^{\infty}[\ \mu^{-1}[\![x)]^n$

$=\sum_{n=1}^{\infty} \mu^{-n} x^{n-1}$

This proves the corollary.

Proposition..2.3.14[2] Let G be a soft Banach algebra. The soft set S generated by the set of all invertible soft elements of G is a soft open subset in G.

Proof. x˜0∈˜S. We have to show that x˜0 is a soft interior point of G. Consider the open sphere) $s(x_0 \frac{1}{\|x_0{}^{-1}\|}$ with centre at x˜0 and radius . $\frac{1}{\|x_0{}^{-1}\|}$ Every soft element x˜

of this sphere satisfies the inequality $\|\ x_{0-}\ \mathrm{x}\| < \frac{1}{\|x_0{}^{-1}\|}$

Let y˜ = x_0 1x˜ and z˜ = e¯ − y˜ then we have ||z˜|| = ||y˜ − e¯|| = || 1x˜ x_0 − x_0 − 1x˜0|| ≤||˜ 1||||x˜ −x_0 - x_0 ||<˜¯1 . So by Proposition 3.11, e¯− z˜ is invertible i.e. y˜ is invertible.

Hence y˜∈˜S. Now x˜0∈˜S, y˜∈˜S and so by Remark3.6, x˜0y˜ ∈ S. But

x_0 y˜ = x_0 x˜−0 1x˜ = x˜

So any x˜ satisfying the inequality (3.2) belongs to S. This shows that S is a soft open subset of G. □

Corollary 2.3.15[2] The soft set P(= Sc) of G is soft closed subset of G.

Definition 2.3.16[2] Amapping T from a soft normed linear space G onto G is said to be continuous if for any sequence x_n,→ x˜ implies T(x_n) ,→ T(x˜).

Proposition 2.317[2]. In a soft Banach algebra G, the mapping x˜ ,→ x˜−1 of S onto S is continuous.

Proof. Let x_0 ∈ S and let $\{x_n\}$ be a sequence of soft elements in S such that x_n ,→ x_0 as n →∞. To prove x˜ ,→ x˜−1 is continuous, it is enough to show that ${x_n}^{-1}$,→ ${x_0}^{-1}$. Now

|| ${x_n}^{-1} - {x_0}^{-1}$|| = |${x_n}^{-1}$| (x_0 −x_n) ${x_0}^{-1}$||

(2.3) ≤||˜ |${x_n}^{-1}$||||| x_0 −x_n ||${x_0}^{-1}$||||.

Sincex_n ,→ x_0 , for any given ϵ˜>˜0¯, there exists N˜ such that for all n ≥ N˜(λ) ,

$$\| x_{n-} x_0 \|(\lambda) < \frac{1}{2\|{x_0}^{-1}\|}(\lambda \in = \frac{1}{2\|x^{-1}\|}^{-}$$

4))where we have taken Now

(3.5) ||e¯− ${x_0}^{-1} x_0$|| = ||x_0− 1(${x_0}^{-1}$ −x_n)||≤||˜ ${x_0}^{-1}$ 1||||(x˜ − x_n||

Using (3.4) and (3.5) we get

$\|\tilde{e}\text{-}{x_0}^{-1}x_n\|(\lambda) < \frac{1}{2}(\lambda)=\frac{1}{2}\,\forall n \geq \tilde{N}(\lambda$

${x_0}^{-1}x_0$

So by Corollary is invertible and its inverse is given by $={x_n}^{-1}x_0$

$({x_0}^{-1}x_n)^{-1} = \bar{e}+\sum(\bar{e}\text{-}{x_0}^{-1}x_n)^n$

$\left\|{x_0}^{-1}x_0\right\| \leq 1 + \sum_{n=1}\left\|e - {x_0}^{-1}x_n\right\|^n \leq \frac{1}{1-\|e-{x_0}^{-1}x_n\|}$. Thus

≤˜¯2 by (3.6). This gives ||x˜−n1x˜0||<˜¯2 so that we have

(3.7) $||{x_n}^{-1}|| = |{x_n}^{-1}|{x_0}^{-1}||\leq||\,||{x_n}^{-1}|||{x_0}^{-1}x_0||\leq\tilde{\ }\bar{\ }2||\,|{x_0}^{-1}|$

From (3.3))and (3.7) we get

$||\,{x_n}^{-1}- {x_0}^{-1}||(\lambda)\leq\tilde{\ }2||{x_0}^{-1}||(\lambda)||\,x_{n-}\,x_0||(\lambda)|\,{x_0}^{-1}|||(\lambda) \to 0$asn $\to\infty$.

This proves that ${x_n}^{-1}\,{x_0}^{-1}$,→ as n → ∞. So the mapping x˜ ,→ $x^{-1}$of S onto S is continuous. □

Corollary 2.3.18.[2] In a soft Banach algebra G, the mapping x˜−1 ,→ x˜ of S onto S is continuous.

Definition 2.3.19[2] Let G be a soft Banach algebra. A soft element z˜ ∈ G is called a soft topological divisor of zero if there exists a sequence $\{z_n\}$, z_n ∈ G, $|z_n|||$ = ¯1 for n = 1,2,3..... and such that either z˜z_n ,→ Θ orz_n z˜ ,→ Θ.

Proposition 2.3.20.[2] The soft set Z is a soft subset of P, where Z denotes the set of all soft topological divisors of zero.

Proof. Let z˜ ∈ Z. The there exists a sequence $\{z_n\}$ such that $||z_n||$ = ¯1 for n = 1,2,3..... and either z˜z_n ,→ Θ or z_nz˜ → Θ as n → ∞. Suppose that z˜z_n ,→ Θ.

If possible, let z˜/∈ P. Then z˜(λ)−1 exists for some λ. Now as multiplication is continuous operation, we should have

z_n (λ) = z˜(λ)−1 (z˜z_n)(λ) → z˜(λ)−1 Θ(λ) = θ asn →∞.

This contradicts the fact that ||z_n || = ¯1 for n = 1,2,3..... . Hence Z is a soft subset of P. □

Definition 2.3.21[2] Let (X,ˇ ||.||) be a soft normed linear space and Y ∈ S (X). A soft element α˜ Xˇ is called a soft boundary elements of Y if there exist two sequence x_n and y_n of soft elements in Y and Y c respectively such that x_n ,→ α˜ and y_n ,→ α˜. Proposition 3.22. The boundary of P is a soft subset of Z.

Proof. Let z˜ be a boundary point of P. So there exist two sequences of soft elements r_n in S and s_n in P such that

(3.8) r_n ,→ z˜and s_n ,→ z.˜

Since P is soft closed so z˜∈˜P. Now let us write r˜n−1z˜ − e¯ = r˜n−1 (z˜− r_n). The sequence∈ { ${r_n}^{-1}$ (λ)} given above|| is unbounded− || ∀λ ∈ A.If not, then∀ ∈there exists some λ A and n(λ) such that ${r_n}^{-1}$ z˜ e¯ (λ) < 1 n n(λ), λ A. So that by

Corollary 3.12, ${r_n}^{-1}$z˜(λ) is regular| and hence z˜(λ)∀ = r_n (λ)(${r_n}^{-1}$z˜)(λ) is regular, contradicting ∈ z˜˜P. Hence { ${r_n}^{-1}$ (λ)} is regular unbounded λ A so that

(3.9) $\| {r_n}^{-1} \| \to \infty,^{-}$ as $n \to \infty$.

Now let us define $z_n = \frac{{r_n}^{-1}}{\|{r_n}^{-1}\|}$˜. From the definition of z_n, we have(3.10.$\| z_n \| = -1$Further

$zz_n = \frac{{r_n}^{-1}}{\|{r_n}^{-1}\|} = \frac{e+z{r_n}^{-1}-e}{\|{r_n}^{-1}\|} = \frac{e+(z-r_n){r_n}^{-1}}{\|{r_n}^{-1}\|}$(3.11).But

$\frac{e+(z-r_n)r_n^{-1}}{r_n^{-1}} = \frac{\bar{e}}{\|r_n^{-1}\|} + (\widetilde{z}\text{-}r_n)z_n(3.12)$.From (3.11) and (3.12), we get

$$zz_n = \frac{\bar{e}}{\|r_n^{-1}\|} + (\widetilde{z}\text{-}r_n)z_n(3.13) \quad .$$

Using (3.8), (3.9) and (3.10) in (3.13) we see that z˜ z_n,→ Θ as n → ∞. Hence z˜ □is a topological divisor of zero.

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