

**Republic of Iraq  
Ministry of Higher Education  
& Scientific Research  
University of Qadisiyah  
Faculty of Education  
Mathematics Department**



**"On Jordan Generalized"  
Higher tri-derivations on prime Gamma  
Rings**

*Are search submitted to the department of mathematics college  
of education as partial fulfillment of the requirement's for the  
degree of Bachelor of science in mathematics*

By

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Supervision

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﴿ بل هو آياتٌ بِيناتٍ فِي صُدُورِ الَّذِينَ أَوْتُوا الْعِلْمَ وَمَا يَجِدُ بِأَيْمَانِهِنَّ ﴾

﴿ لَا الظَّالِمُونَ ﴾

صدق الله العلي العظيم

# الاهداء

الى من علم البشرية مبادئ العلم والثقافة الى منارة العلم الى سيد الخلق  
رسولنا وحبيبنا الاكرم محمد (ص)

الى من علمني كيف أرتقي سلم النجام بصره وحكمته والدي العزيز ...

الى من كان دعائهما سر نجاهي ودعائهم منطلق أفكارني الى حبيبتي الاهدي  
روحوي وكل طموحي والدتي العزيزة ...

الى عيني ومهجتي الى من أعتمد عليهم وقت شدتني أختي وإخوانني الأعزاء ...

الى من شاطرني أفكاره وابداعاته الى من سار معه في طريق النجام زملائي  
وزميلاتي الأعزاء ...

# الشكر والتقدير

إلى استاذي الفاضل مازن عمران كريم

للنجاح اناس يقدرون معناه . . . ولابداع اناس يحصدونه . . . فمنك تعلمنا ان للنجاح قيمة

... ومعنى

ومنك تعلمنا **كيف يكون التفاني والاخلاص في العمل . . .**

ومعك امنا ان لا مستحيل في سبيل الابداع والرقي . . .

فأنت اهل للشكرا والتقدير . . .

فوجب علينا تقديرك . . .

فلك منا كل الثناء والتقدير والاحترام .

## ***Introduction:-***

In this researcher consists of two chapters in chapter one some basic definitions of fringe , $\tau$ -subsing , $\tau$ -ideal commentator and give some proper lies of definition and the definition of derivation , tri-derivation , Jordon derivation .

In chapter two give the definition of generalized higher tri-derivation Jordan generalized higher tri-derivation higher tri-derivation , Jordan higher tri-derivation and the relationships below these conepts.

## ***Abstract***

In this study , we define the concepts of a generalized higher tri-derivation Jordan generalized higher tri-derivation and Jordan triple generalized higher tri-derivation on  $\Gamma$ - rings and show that a Jordan generalized higher tri-derivation on 2-torsion free prime  $\Gamma$ -ring is triple a generalized higher tri-derivation

# *Chapter One*

## Definition :1-1 "Γ-ring"

Let  $M$  and  $\Gamma$  be two additive abelian groups if there exists a mapping  $(a, \alpha, b) \rightarrow a\alpha b$  of  $M \times \Gamma \times M \rightarrow M$  satisfying the following for all  $a, b, c \in M$  and  $\alpha, \beta \in \Gamma$ .

$$1- (a + b)\alpha c = a\alpha c + b\alpha c$$

$$a(\alpha + \beta)b = a\alpha b + a\beta b$$

$$a\alpha(b + c) = a\alpha b + a\alpha c$$

$$2- (a\alpha b)\beta c = a\alpha(b\beta c)$$

Then  $M$  is called a  $\Gamma$ -ring.

## Definition :- 1-2 $\Gamma$ -subring

Let  $(M, +, \cdot)$  be a  $\Gamma$ -ring and  $S$  nonempty subset of  $M$  then  $(S, +, \cdot)$  is called a subring of  $(M, +, \cdot)$  if  $(S, +, \cdot)$  itself  $\Gamma$ -ring.

Definition :- 1-3 " ideal"

Let  $M$  be  $\Gamma$ -ring and  $U$  be subset of  $M$  then  $U$  is called left (ring) ideal of  $M$  if  $a\alpha x \in U(x\alpha aU)$  for all  $a \in M, x \in U$  and  $\alpha \in \Gamma$  and  $U$  is called ideal if it aleft and right ideal.

Definition :- 1-4 "commutitive  $\Gamma$ -ring "

A  $\Gamma$ -ring  $(M, +, \cdot)$  is called commutative  $\Gamma$ -ring if  $a\alpha b = b\alpha a$  for all  $a, b \in M, \alpha \in \Gamma$ .

Definition:- 1-5 "commutator"

Let  $(M, +, \cdot)$  be a  $\Gamma$ -ring then  $a\alpha b - b\alpha a$  is called commutater of  $a, b \in M, \alpha \in \Gamma$  and denoted by  $[a, b]_\alpha$ .

Definition :- 1-6 "2-torsion"

A  $\Gamma$ -ring  $(M, +, \cdot)$  is called 2-torsion free if  $2a = 0$  imphies  $a = 0$  for all  $a \in M$  where 2 is a positive integer number.

Definition 1-7 "prime  $\Gamma$ -ring"

A  $\Gamma$ -ring  $M$  is called prime if  $a\Gamma M\Gamma b = 0$  implies that  $a = 0$  or  $b = 0$  where  $a, b \in M$ .

## Definition:- 1-8 "Semiprime"

A  $\Gamma$ -ring  $M$  is called semiprime if  $a\Gamma M\Gamma b = 0$  implies that  $a = 0$  where  $a \in M$ .

Remark:

It is clear that every prime  $\Gamma$ -ring is semiprime  $\Gamma$ -ring .But the converse is not true in general .The following example shows that a semi prime  $\Gamma$ -ring may be not prime  $\Gamma$ -ring.

Example: 1-9

Let  $M$  be a prime  $\Gamma$ -ring, we put  $M = M_1 \oplus M \oplus M_1$  and  $\Gamma = \Gamma_1 \oplus \Gamma_1 \oplus \Gamma_1$  then  $M$  is a  $\Gamma$ -ring. It is obvious that  $M$  is semiprime  $\Gamma$ -ring but not prime  $\Gamma$ -ring since if  $a \neq 0$  and  $b \neq 0$  where  $a, b \in M$ , then for all  $(\alpha, \beta, M), (w, \tau, s) \in \Gamma$  and  $(x, y, z) \in M$  we have

$$(a, 0, 0)(\alpha, \beta, M)(x, y, z)(\sigma, \tau, s)(0, 0, b) = (a\alpha x\sigma 0, \beta y\tau 0, \gamma z s b)$$
$$= (0, 0, 0)$$

But  $(a, 0, 0) \neq (0, 0, 0)$  and

$$(0, 0, b) \neq (0, 0, 0)$$

Definition :- 1-10 " Permuting"

To be permuting if  $D(a, b, c) = D(a, c, b) = D(b, a, c) = D(b, c, a) = D(c, a, b) = D(c, b, a)$ .

Definition :-1-11 "Derivation"

An additive mapping  $D: M \rightarrow M$  is called a derivation of  $M$  if  
 $d(a\alpha b) = d(a)\alpha b + a\alpha d(b)$  for all  $a, b \in m$  and  $\alpha \in \Gamma$ .

Definition:- 1-12

A mapping  $D: M \times M \times M \rightarrow M$  is called tri-additive mapping if :

$$1- D(a + b, c, d) = D(a, c, d) + D(b, c, d)$$

$$2- D(a, b + c, d) = D(a, b, d) + D(a, c, d)$$

$$3- D(a, b, c + d) = D(a, b, c) + D(a, c, d)$$

### Definition :- 1-13 "tri-Derivation"

A tri additive mapping  $D: M \times M \times M \rightarrow M$  is called tri-Derivation if :

$$1- D(x_1 \alpha x_2, y, z) = x_1 \alpha D(x_2, y, z) + D(x_1, y, z) \alpha x_2$$

$$2- D(x, y_1 \alpha y_2, z) = y_1 \alpha (x, y_2, z) + D(x, y_1, z) \alpha y_2$$

$$3- D(x, y, z_1 \alpha z_2) = z_1 \alpha D(x, y, z_2) + D(x, y, z_1) \alpha z_2$$

### Definition :-1-14 "Jordan derivation"

An addivation mapping  $D: M \rightarrow M$  ia called Jordan derivation of  $M$  if :

$$d(a\alpha a) = d(a)\alpha a + a\alpha d(a) \text{ for all } a, b \in m \text{ and } \alpha \in \Gamma.$$

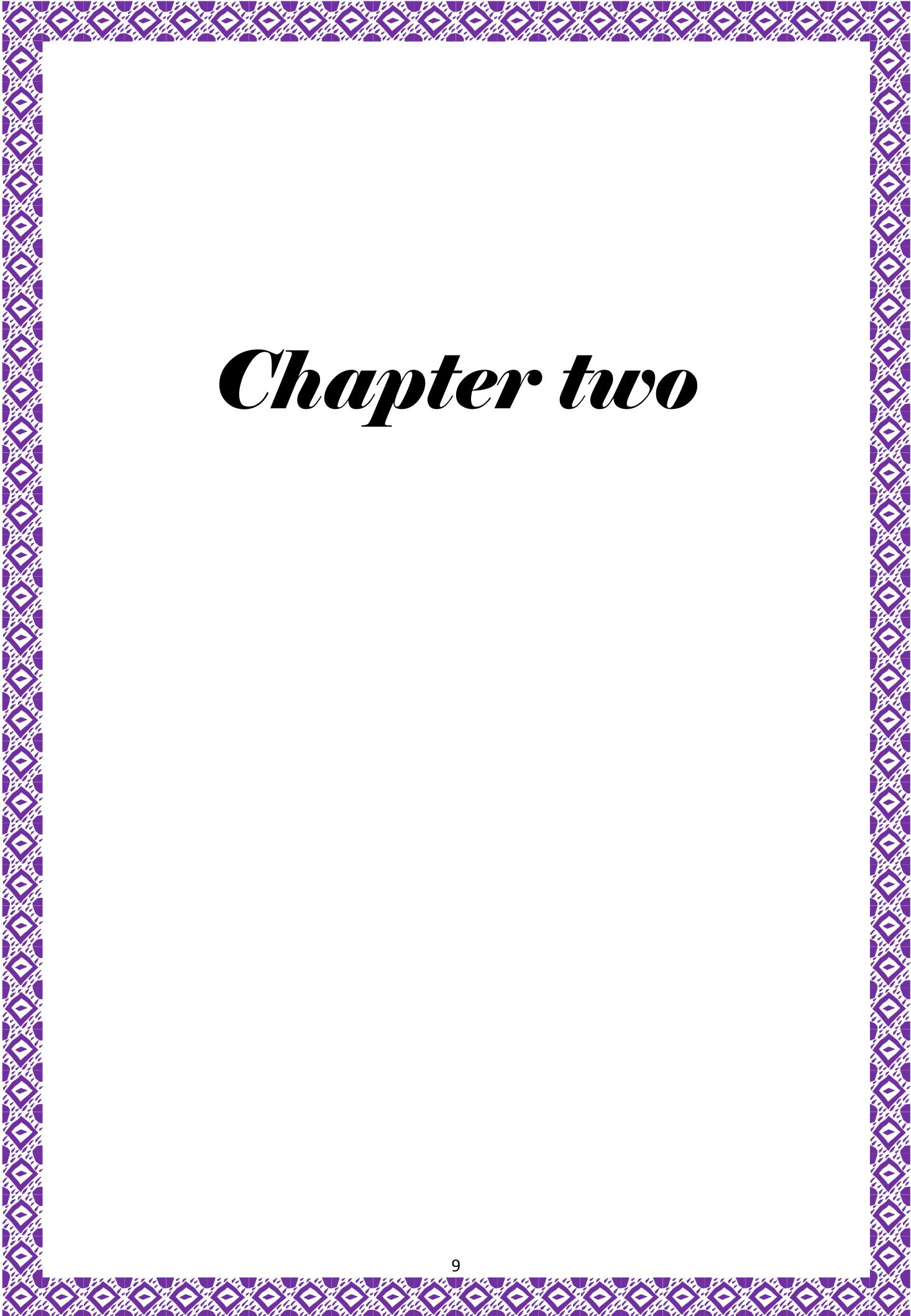
### Definition : 1-15 "Jordan tri-derivation"

A tri additive mapping  $D: M \times M \times M \rightarrow M$  is called Jordan tri-derivation if

$$1- D(a\alpha a, b, c) = a\alpha D(a, b, c) + D(a, b, c)\alpha a$$

$$2- D(a, b\alpha b, c) = b\alpha D(a, b, c) + D(a, b, c)\alpha b$$

$$3- D(a, b, c\alpha c) = c\alpha D(a, b, c) + D(a, b, c)\alpha c$$



# *Chapter two*

## Introduction

In this chapter give the definition of generalized tri-derivation , Jordan generalized tri-derivation , higher tri-derivation , Jordan higher tri-derivation , triple higher tri-derivation generalized higher tri-derivation and relationships between these concepts.

Definition :- 2-1 "generalized tri-derivation"

A tri -additive mapping  $F: M \times M \times M \rightarrow M$  is called generalized tri-derivation on  $M \times M \times M$  in to  $M$  if there exists tri-derivation

$D: M \times M \times M \rightarrow M$  such that.

$$1- F(a\alpha b, c, d) = F(a, c, d)\alpha b + a\alpha D(b, c, d)$$

$$2- F(a, b\alpha c, d) = F(a, b, d)\alpha c + b\alpha D(a, c, d)$$

$$3- F(a, b, c\alpha d) = F(a, b, c)\alpha d + c\alpha D(a, b, d)$$

Definition: 2-2

A tri -additive mapping  $F: M \times M \times M \rightarrow M$  is called Jordan generalized tri-derivation on  $m \times m \times m$  in to  $m$  suchthat

$$1- F(a\alpha a, b, c) = F(a, b, c)\alpha a + a\alpha D(a, b, c)$$

$$2- F(a, b\alpha b, c) = F(a, b, c)\alpha b + b\alpha D(a, b, c)$$

$$3- F(a, b, c\alpha c) = F(a, b, c)\alpha c + c\alpha D(a, b, c)$$

### Definition 2-3 "higher tri-dorivation"

Let  $M$  be a  $\Gamma$ -ring and  $D = (d_i)_{i \in N}$  be a family of tri additive mapping from  $M \times M \times M$  into  $M$  such that  $d_0(a, b, c) = a$  for all  $a, b, c \in M$  then  $D$  is called a higher tri-derivation on  $M \times M \times M$  into  $M$  if for all  $a, b, c, d, e, f \in M, \alpha \in \Gamma$  and  $n \in N$  such that

$$d_n(a\alpha b, c\alpha d, e\alpha f) = \sum_{i+j=n} d_i(a, c, e)\alpha d_j(b, d, f)$$

### Definition :- 2-4 "Jordan higher tri-derivation"

Let  $M$  be a  $\Gamma$ -ring and  $D = (d_i)_{i \in N}$  be a family of tri- additive mapping from  $M \times M \times M$  into  $M$  then  $D$  is called Jordan higher tri-derivation if

$$d_n(a\alpha a, b\alpha b, c\alpha c) = \sum_{i+j=n} d_i(a, b, c)\alpha d_j(a, b, c)$$

Definition :- 2-5 "Jordan triple higher tri-derivation"

Let  $M$  be a  $\Gamma$ -ring and  $D = (d_i)_{i \in N}$  be a family of tri-additive mapping from  $M \times M \times M$  in to  $M$  then  $D$  is called Jordan triple higher tri-derivation if:

$$d_n(a\alpha b\beta a, c\alpha d\beta c, e\alpha f\beta e) = \sum_{i+j+k=n} d_i(a, c, e)\alpha d_j(b, d, f)d_k(a, c, e)$$

Definition :- 2-6 "generazied higher tri-derivation"

Let  $M$  be a  $\Gamma$ -ring and  $F = (f_i)_{i \in N}$  be a family tri-additive mapping from  $M \times M \times M$  in to  $M$  such that  $f_0(a, b, c) = a$  for all  $a, b, c \in R$  then  $F$  is called a generalized higher tri-derivation from  $M \times M \times M$  into  $M$  if there exists a higher tri-derivation  $D = (d_i)_{i \in N}$  from  $M \times M \times M$  into  $M$  such that for all  $n \in N$  we have

$$f_n(a\alpha b, c\alpha d, e\alpha f) = \sum_{i+j=n} f_i(a, c, e)\alpha d_j(b, d, f)$$

For all  $a, b, c, d, e, f \in M$  and  $\alpha \in \Gamma$

## Definition :- 2-7 "Jordan generalized higher tri-derivation"

Let  $M$  be a  $\Gamma$ -ring and  $f = (f_i)_{i \in N}$  be a family of tri-additive mapping from  $M \times M \times M$  into  $M$  such that  $f_0(a, b, c) = 0$  for all  $a, b, c \in M$  then  $F$  is called Jordan generalized higher tri-derivation from  $M \times M \times M$  into  $M$  if there exists Jordan higher tri-derivation  $D = (d_i)_{i \in N}$  from  $M \times M \times M$  into  $M$  such that for all  $n \in N$  we have

$$f_n(a\alpha a, b\alpha b, c\alpha c) = \sum_{i+j=n} f_i(a, b, c)\alpha d_j(a, b, c)$$

### Example:- 2-8

Let  $M = \left\{ \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} : x, y \in R \right\}$   $R$  is real number  $M$  be a  $\Gamma$ -ring of  $2 \times 2$  matric and  $F = \left\{ \begin{pmatrix} r & 0 \\ 0 & 0 \end{pmatrix} : r \in R \right\}$  we use the usual addition and multiplication on matric of  $M \times \Gamma \times M \times \Gamma \times M$  we defined

$F_i: M \times \Gamma \times M \times \Gamma \times M \rightarrow M$ ,  $i \in N$  by

$$F_i \left( \left( \begin{matrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \right) \left( \begin{matrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \right) \right) = \left( \begin{matrix} ka_1 & (1+i)b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \right)$$

$$k = \frac{(i^2 - in + 1) + |i^2 - in + 1|}{2} = \begin{cases} 1 & \text{if } i \in \{0, n\} \\ 0 & \text{if } i \notin \{0, n\} \end{cases}$$

$$n \in N, 0 \leq j \leq n$$

Then  $f$  is generalized higher tri-derivation on  $\Gamma$ -ring because there exists a higher tri-derivation on  $\Gamma$ -ring  $d_i: M \times \Gamma \times M \times \Gamma \times M \rightarrow M$ ,  $i \in N$

**Defined by**

$$d_i \left( \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} ma & (m+i)b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{Such that } m = \frac{(1-i)+|1-i|}{2} = \begin{cases} 1 & \text{if } i = 0 \\ 0 & \text{if } i \neq 0 \end{cases}$$

$$\text{For all } \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in M$$

Sol:-

$$M \times \Gamma \times M \times \Gamma \times M \rightarrow M$$

$$f_2 \left( \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} e_1 & f_1 & d_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_2 & f_2 & d_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) =$$

$$\varepsilon f_0 \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_1 & f_1 & d_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} d_2 \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} e_2 & f_2 & d_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + f_1 \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_1 & f_1 & d_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$d_1 \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_2 & f_2 & d_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ f_2 \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_1 & f_1 & d_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$d_0 \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_2 & f_2 & d_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & b_1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} o & 2b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} a_1 & 2b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} a_1 & 3b_1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} a_1 r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} b & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} a_1 r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} a_1 r a_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 r a_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$f_2 \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} a_1 r a_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$d_n \left( \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right),$$

$$\left( \begin{pmatrix} e_1 & f_1 & d_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_2 & f_2 & d_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)$$

$$= \sum f_i \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_1 & f_1 & d_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} d_j \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_2 & f_2 & d_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 r a_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

### Lemma : 2-9

Let  $M$  be a  $\Gamma$ -ring and  $f = (f_i)_{i \in N}$  be a Jordan generalized higher tri-derivation on  $M \times M \times M$  into  $M$  associated with Jordan higher tri-derivation  $D = (d_i)_{i \in N}$  of  $M \times M \times M$  into  $M$

Then for an  $a, b, c, d, e, f \in M$ ,  $\alpha \in \Gamma$  and  $n \in N$  then following statements hold

$$fn(a\alpha b + b\alpha c, c\alpha d + d\alpha c, e\alpha f + f\alpha e)$$

$$= \sum_{i+j} f_i(a, c, e)\alpha d_i(b, d, f) + f_i(a, c, e)\alpha d_i(b, d, f)$$

Proof :- Since  $f = (f_i)_{i \in N}$  be Jordan generalized higher tri-derivation on  $M \times M \times M$  into  $M$  then

$$fn[(a\alpha a + b\alpha b), (c\alpha c + d\alpha d), (e\alpha e + f\alpha f)]$$

$$\sum_{i+j=n} f_i(a+b, c+d, e+f)\alpha d_i(a+b, c+d, e+f)]$$

$$= \sum_{i+j=n} [f_i(a, c, e) + f_i(b, d, f)][d_i(a, c, e) + d_i(b, d, f)]$$

$$\sum_{i+j=n} f_i(a, c, e)\alpha d_i(a, c, e) + f_i(a, c, e)\alpha d_i(b, d, f)$$

$$+ f_i(b, d, f)\alpha d_i(a, c, e)$$

$$+ f_i(b, d, f)\alpha d_i(b, d, f) \dots \dots \dots (1)$$

## On the other hand

$$= f_n(a\alpha a + a\alpha b + b\alpha a, c\alpha c + c\alpha d + d\alpha c + d\alpha d, e\alpha e + e\alpha f + f\alpha e)$$

$$\begin{aligned} & -f_n(a\alpha a, c\alpha c, e\alpha e) + f_n(a\alpha b + b\alpha a, c\alpha d + d\alpha c, e\alpha f + f\alpha e) \\ & \quad + f_n(b\alpha b, d\alpha d, f\alpha f) \end{aligned}$$

$$\begin{aligned} & = \sum_{i+j=n} f_i(a, c, e) \text{adi}(a, c, e) \\ & \quad + f_n(a\alpha b + b\alpha a, c\alpha d + d\alpha c, e\alpha f + f\alpha e) \\ & \quad + \sum_{i+j=n} (b, d, f) \text{adi}(b, d, f) \dots \dots (2) \end{aligned}$$

Compare between (1) and (2) we get

$$\begin{aligned} & f_n(a\alpha b + b\alpha a, c\alpha d + d\alpha c, e\alpha f + f\alpha e) \\ & = \sum_{i+j=n} f_i(a, c, e) \text{adi}(b, d, f) + f_i(b, d, f) \text{adi}(a, c, e) \end{aligned}$$

Lemma : 2-10

Let  $M$  be a  $\Gamma$ -ring and  $\mathbf{f} = (f_i)_{i \in N}$  be a Jordan triple generalized higher tri-derivation on  $M \times M \times M$  into  $M$  associated with Jordan higher tri-derivation  $D = (d_i)_{i \in N}$  of  $M \times M \times M$  into  $M$

Then for an  $a, b, c, d, e, f \in M, \alpha \in \Gamma$  and  $n \in N$

$$fn(a\alpha b\alpha c + c\alpha b\alpha a, s\alpha d\alpha t + t\alpha d\alpha c, e\alpha f\alpha r + r\alpha f\alpha e)$$

$$= \sum fi(a, s, e) \alpha di(b, d, f) \alpha dk(s, t, r) \\ + fi(c, t, r) \alpha di(b, d, f) \alpha dk(a, s, e)$$

Proof :-

Replacing  $a+c$  for  $a$  and  $s+t$  for  $c$  and  $e+r$  for  $e$

### **Proposition (2-9)**

$$fn(a + c) \alpha b \alpha (a + c), (s + t) \alpha d \alpha (s + t), (e + r) \alpha f \alpha (e + r)$$

$$\sum fi(a + c, s + t, e + r) \alpha di(b, d, f) \alpha dk(a + c, s + t, e + r) \\ = \sum [fi(a, s, e) + fi(c, t, r) \alpha di(b, d, f) \alpha [dk(a, s, e) + dk(c, t, r)]] \\ = \sum fi(a, s, e) \alpha di(b, d, f) \alpha (a, s, e) \\ + fi(a, s, e) \alpha di(b, d, f) \alpha dk(e, t, r) \\ + fi(c, t, r) \alpha di(b, d, f) \alpha dk(a, s, e) \\ + fi(c, t, r) \alpha di(b, d, f) \alpha dk(c, t, r) \dots \dots (1)$$

## On the other hand

$$fn(a + c) \alpha b \alpha (a + c), (s + t) \alpha d \alpha (s + t), (e + r) \alpha f \alpha (e + r)$$

$$fn(a \alpha b \alpha a + a \alpha b \alpha c + c \alpha b \alpha a + c \alpha b \alpha c, s \alpha d \alpha s$$

$$+ s \alpha d \alpha t + t \alpha d \alpha s + t \alpha d \alpha t, e \alpha f \alpha e + e \alpha f \alpha r$$

$$+ r \alpha f \alpha e + r \alpha f \alpha r)$$

$$= fn(a \alpha b \alpha a, s \alpha d \alpha s, e \alpha f \alpha e)$$

$$+ fn(a \alpha b \alpha c + c \alpha b \alpha a, s \alpha d \alpha t$$

$$+ t \alpha d \alpha s, e \alpha f \alpha r + r \alpha f \alpha e)$$

$$+ fn(c \alpha b \alpha c, t \alpha d \alpha t, r \alpha f \alpha r)$$

$$= \sum fi(a, s, e) \alpha dj(b, d, f) \alpha d_k(a, s, e) + fn(a \alpha b \alpha c)$$

$$+ c \alpha b \alpha a, s \alpha d \alpha t + t \alpha d \alpha s, e \alpha f \alpha r + r \alpha f \alpha e)$$

$$+ \sum fi(c, t, r) \alpha dj(b, d, f) \alpha d_k(c, t, r) \dots \dots (2)$$

## Compare between (1) and (2)

$$fn(a \alpha b \alpha c + c \alpha b \alpha a, s \alpha d \alpha t + t \alpha d \alpha s, e \alpha f \alpha r  
+ r \alpha f \alpha e$$

$$= \sum fi(a, s, e) \alpha dj(b, d, f) \alpha d_k(s, t, r)$$

$$+ fi(c, t, r) \alpha dj(b, d, f) \alpha d_k(a, s, e)$$

## Proposition :2-11

Let  $M$  be 2-torsion free ring then every Jordan generalized higher tri derivation on  $M \times M \times M$  into  $M$  such that  $a\alpha b\beta c = a\beta b\alpha c$  for all  $a, b, c \in M$  and  $\alpha, \beta \in \Gamma$  is a Jordan triple generalized higher tri-derivation on  $M \times M \times M$  into  $M$ .

Proof :-

Replace  $a\beta b + b\beta a$  for  $b$  and  $c\beta d + d\beta c$  for  $d$  and  $e\beta f + f\beta e$  for  $f$  in **lemma(2-9)**

$$\begin{aligned}
 & f_n [ a \alpha (a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a, c \alpha (c\beta d + d\beta c) \\
 & \quad + (c\beta d + d\beta c)\alpha c, e \alpha (e\beta f + f\beta e) + (e\beta f + f\beta e)\alpha e ] \\
 &= \sum_{i+j=n} f_i(a, c, e) \alpha d_i(a\beta b + b\beta a, c\beta d + d\beta c, e\beta f + f\beta e) \\
 & \quad + f_i(a\beta b + b\beta a, c\beta d + d\beta c, e\beta f + f\beta e) \alpha d_i(a, c, e) \\
 &= \sum_{i+j=n} f_i(a, c, e) \alpha d_j(a\beta b, c\beta d, e\beta f) + d_j(b\beta a, d\beta c, f\beta e) ] \\
 & \quad + [ f_i(a\beta b + c\beta d + e\beta f) + f_i(b\beta a + d\beta c + f\beta e) ] \\
 & \quad + \alpha d_j(a, c, e) \\
 &= \sum_{i+j=n} f_i(a, c, e)(a\beta b, c\beta d, e\beta f) + f_i(a, c, e) \alpha d_j(b\beta a, d\beta c, f\beta e)
 \end{aligned}$$

$$\begin{aligned}
& \sum_{k+i+j=n} f_i(a, c, e) \alpha [d_j(a, c, e) \beta d_k(b, d, f)] \\
& + f_i(a, c, e) \alpha [d_j(b, d, f) \beta d_k(a, c, e)] \\
& + f_i(a, c, e) \alpha d_j(b, d, f) \beta d_k(a, c, e) \\
& + f_i(b, d, f) \alpha d_j(a, c, e) \beta d_k(a, c, e) \\
& = \sum_{i+j+k=n} f_i(a, c, e) \alpha d_j(a, c, e) \beta d_k(b, d, f) \\
& + 2 \sum_{i+j+k=n} f_i(a, c, e) \alpha d_j(b, d, f) \beta d_k(a, c, e) \\
& + f_i(b, d, f) \alpha d_j(a, c, e) \beta d_k(a, c, e) \dots \dots (1)
\end{aligned}$$

**On the other hand**

$$\begin{aligned}
& f_n(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a, c\alpha(c\beta d + d\beta c) \\
& + (c\beta d + d\beta c)\alpha c, e\alpha(e\beta f + f\beta e) \\
& + (e\beta f + f\beta e)\alpha e)
\end{aligned}$$

$$\begin{aligned}
& \text{fn}(a\alpha a\beta b + a\alpha b\beta a + a\alpha b\beta a + a\alpha b\beta a + b\alpha a\beta a, c\alpha c\beta d \\
& \quad + c\alpha d\beta c + d\alpha c\beta c, e\alpha e\beta f + e\alpha f\beta e + e\alpha f\beta e \\
& \quad + f\alpha e\beta e) \\
& = \text{fn}(a\alpha a\beta b + b\alpha a\beta a, c\alpha c\beta d + d\alpha c\beta c, e\alpha e\beta f + f\alpha e\beta e) \\
& \quad + \text{fn}(a\alpha b\beta a, c\alpha d\beta c, e\alpha f\beta e) \\
& \quad + \text{fn}(a\alpha b\beta a, c\alpha d\beta c, e\alpha f\beta e) \\
& = \sum_{i+j=n} f_i(a\alpha a, c\alpha c, e\alpha e)\beta dj(b, d, f) \\
& \quad + f_i(b, d, f) dj(a\alpha a, c\alpha c, e\alpha e) \\
& \quad + 2 \text{fn}(a\alpha b\beta a, c\alpha d\beta c, e\alpha f\beta e) \\
& = \sum f_i(a, c, e) \alpha di(a, c, e) \beta dk(b, d, f) \\
& \quad + i(b, d, f) f\alpha dj(a, c, e) \beta dk(a, c, e) \\
& \quad + 2 \text{fn}(a\alpha b\beta a, c\alpha d\beta c, e\alpha f\beta e) \dots \dots (2)
\end{aligned}$$

**Compare between (1) and (2) we get**

$$\begin{aligned}
& 2 \sum f_i(a, c, e) \alpha dj(b, d, f) \beta dk(a, c, e) \\
& = 2 \text{fn}(a\alpha b\beta a, c\alpha d\beta c, e\alpha f\beta e)
\end{aligned}$$

Since  $R$  is 2-torsion free then

$$\begin{aligned} f_n(a \alpha b \beta a, c \alpha d \beta c, e \alpha f \beta e) \\ = \sum_{i+j+k=n} f_i(a, c, e) \alpha d j(b, d, f) \beta d k(a, c, e) \end{aligned}$$

Definition 2-12 :-

Let  $M$  be a  $\Gamma$ -ring and  $F = (f_i)_{i \in N}$  be a Jordan generalized higher tri-derivation on  $M \times M$  into  $M$  associated with Jordan higher bi-derivation  $D = (d_i)_{i \in N}$  on  $M \times M$  into  $M$ . Then for all  $a, b, c, d, s, t \in M$ ,  $\alpha, \beta \in \Gamma$  and  $n \in N$  we define

$$\phi_n(a, b, c, d) = F_n(a \alpha b, c \alpha d) -$$

$$\sum_{i+j=n} f_i(a, c) \alpha d j(b, d)$$

Lemma 2-13 :-

Let  $M$  be a  $\Gamma$ -ring and  $F = (f_i)_{i \in N}$  be a Jordan generalized higher tri-derivation on  $M \times M \times M$  into  $M$  associated with Jordan higher tri-derivation  $D = (d_i)_{i \in N}$  on  $M \times M \times M$  into  $M$ . Then for all  $a, b, c, d, s, e, f \in M$ .

$$i. \Phi_n(a, b, c, d, e, f)_\alpha = -\Phi_n(b, a, d, c, f, e)_\alpha$$

$$\text{ii. } \Phi_n(a + s, b, c, d, e, f)_\alpha = \Phi_n(a, b, c, d, e, f)_\alpha +$$

$$\Phi_n(s, b, c, d, e, f)$$

$$\text{iii. } \emptyset_n(a, b + s, c, d, e, f)_\alpha = \emptyset_n(a, b, c, d, e, f)_\alpha +$$

$$\emptyset_n(a, s, c, d, e, f)_\alpha$$

$$\text{iv. } \emptyset_n(a, b, c + s, d, e, f)_\alpha = \emptyset_n(a, b, c, d, e, f)_\alpha +$$

$$\emptyset_n(a, b, s, d, e, f)_\alpha$$

$$\text{v. } \emptyset_n(a, b, c, d + s, e, f)_\alpha = \emptyset_n(a, b, c, d, e, f)_\alpha +$$

$$\emptyset_n(a, b, c, s, e, f)_\alpha$$

$$\text{vi. } \emptyset_n(a, b, c, d, e + s, f)_\alpha = \emptyset_n(a, b, c, d, e, f)_\alpha +$$

$$\emptyset_n(a, b, c, d, s, f)_\alpha$$

$$\text{vii. } \emptyset_n(a, b, c, d, e, f + s)_\alpha = \emptyset_n(a, b, c, d, e, f)_\alpha +$$

$$\emptyset_n(a, b, c, d, e, s)_\alpha$$

proof:- by Lemma 2-9 and the definition of  $\emptyset_n(a, b, c, d)_\alpha$

$$F_n(a\alpha b + b\alpha a, c\alpha d + d\alpha c, e\alpha f + f\alpha e)$$

$$= \sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d, f) + f_i(b, d, f) \alpha d_j(a, c, e)$$

$$F_n(a\alpha b, c\alpha d, e\alpha f) + f_n(b\alpha a, d\alpha c, f\alpha e) =$$

$$\sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d, f) + \sum_{i+j=n} f_i(b, d, f) \alpha d_j(a, c, e)$$

$$\text{iii- } (a, b + s, c, d, e, f) =$$

$$F_n(a\alpha(b+s), c\alpha d, e\alpha f) - \sum_{i+j=n} f_i(a, c, e)\alpha d_j(b+s, d, f)$$

$$f_n(a\alpha b, +a\alpha s, c\alpha d, e\alpha f)$$

$$- \sum_{i+j=n} f_i(a, c, e)\alpha d_j(b, d, f) + d_j(s, d, f)$$

$$= f_n(a\alpha b, c\alpha d, e\alpha f) + F_n(a\alpha s, c\alpha d, e\alpha f)$$

$$- \sum_{i+j=n} f_i(a, c, e)\alpha d_j(b, d, f)$$

$$= \emptyset_n(a, b, c, d, e, f)_\alpha + \emptyset_n(a, s, c, d, e, f)_\alpha$$

$$\text{iv- } \emptyset_n(a, b, c + s, d, e, f) = F_n(a\alpha b, (c + s)\alpha d, e\alpha f) -$$

$$\sum_{i+j=n} f_i(a, c + s, e)\alpha d_j(b, d, f)$$

$$\begin{aligned}
&= f_n(a\alpha b, c\alpha d + s\alpha d, e\alpha f) \\
&\quad - \sum_{i+j=n} [f_i(a, c, e) + f_i(a, s, e)]\alpha d_j(b, d, f) \\
&= f_n(a\alpha b, c\alpha d, e\alpha f) + f_n(a\alpha b, s\alpha d, e\alpha f) \\
&\quad - \sum_{i+j=n} f_i(a, c, e)\alpha d_j(b, d, f) + f_i(a, s, e)\alpha d_j(b, d, f) \\
&= f_n(a\alpha b, c\alpha d, e\alpha f) \\
&\quad - \sum_{i+j=n} f_i(a, c, e)\alpha d_i(b, d, f) + F_n(a\alpha b, s\alpha d, e\alpha f) \\
&\quad - \sum_{i+j=n} f_i(a, s, e)\alpha d_j(b, d, f) \\
&= \emptyset_n(a, b, c, d, e, f)_\alpha + \emptyset_n(a, b, s, d, e, f)_\alpha
\end{aligned}$$

$$v - \emptyset_n(a, b, c, d + s, e, f)_\alpha = f_n(a\alpha b, c\alpha(d + s), e\alpha f)$$

$$\begin{aligned}
& - \sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d + s, f) \\
& = f_n(a\alpha b, c\alpha d + c\alpha s, e\alpha f) \\
& - \sum_{i+j=n} f_i(a, c, e) \alpha [d_j(b, d, f) + d_j(b, s, f)] \\
& = F_n(a\alpha b, c\alpha d, e\alpha f) + f_n(a\alpha b, c\alpha s, e\alpha f) \\
& - \sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d, f) \\
& - \sum_{i+j=n} (a, c, e) \alpha d_j(b, s, f) = f_n(a\alpha b, c\alpha d, e\alpha f) \\
& - \sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d, f) + f_n(a\alpha b, c\alpha s, e\alpha f) \\
& - \sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, s, f) \\
& = \emptyset_n(a, b, c, d, e, f)_\alpha + \emptyset_n(a, b, c, s, e, f)_\alpha
\end{aligned}$$

$$vi- f_n(a\alpha b, c\alpha d, e\alpha f + s\alpha f) - \sum_{i+j=n} [f_i(a, c, e) +$$

$$f_i(a, c, s)] \alpha d_j(b, d, f) = F_n(a\alpha b, c\alpha d, e\alpha f) +$$

$$f_n(a\alpha b, c\alpha d, s\alpha f) - \sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d, f) -$$

$$\sum_{i+j=n} f_i(a, c, s) \alpha d_j(b, d, f) = f_n(a\alpha b, c\alpha d, e\alpha f) -$$

$$\sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d, f) + f_n(a\alpha b, c\alpha d, s\alpha f) -$$

$$\sum f_i(a, c, s) \alpha d_j(b, d, f) = \emptyset_n(a, b, c, d, e, f)_\alpha +$$

$$\emptyset_n(a, b, c, d, f)_\alpha$$

$$\text{vii. } \quad \emptyset_n(a, b, c, d, f + s)_\alpha = f_n(a\alpha b, c\alpha d, e\alpha(f + s))$$

$$- \sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d, f + s)$$

$$= f_n(a\alpha b, c\alpha d, e\alpha f + e\alpha s)$$

$$- \sum_{i+j=n} f_i(a, c, e) \alpha [f_i(b, d, f) + d_j(b, d, s)]$$

$$= f_n(a\alpha b, c\alpha d, e\alpha f) + f_n(a\alpha b, c\alpha d, e\alpha s)$$

$$- \sum_{i+j=n} f_i(a, c, e) \alpha d_j(b, d, f)$$

$$- \sum f_i(a, c, e) \alpha d_j(b, d, s)$$

$$f_n(a\alpha b, c\alpha d, e\alpha s) - f_i(a, c, e) \alpha d_i(b, d, s)$$

$$= \phi_n(a, b, c, d, e, f)\alpha + \phi(a, b, c, d, e, s)\alpha$$

Lemma 2-14

Let  $M$  be a 2-torsion free and  $F=(f_i)_{i \in N}$  be a Jordan generalized higher tri-derivation on  $M \times M \times M$  into  $M$  associated

with Jordan higher tri-derivation  $D=(d_i)_{i \in N}$  of  $M \times M \times M$  into  $M$  then for all  $a, b, c, d, s, t, e, f \in M$ ,  $\alpha, \beta \in \Gamma$  and new if  $\phi_t(a, b, c, d, e, f)\alpha = 0$  for every  $t > 0$  then :-

$$\begin{aligned}\phi_n(a, b, c, d, e, f)\alpha\beta m\beta [a, b]\alpha + [a, b]\alpha\beta m\beta \\ \phi_n(a, b, c, d, e, f)\end{aligned}$$

Proof :-

Let  $S.P \in R$

Since  $f_n$  is tri-additive mapping then lemma(2. ) iv we abtain

$$\begin{aligned}f_n(a\alpha b)\beta m\beta (b\alpha a) \\ + (b\alpha a)\beta m\beta (a\alpha b), (c\alpha d)\beta s\beta (d\alpha c) \\ + (d\alpha c)\beta s\beta (c\alpha d), (e\alpha f)\beta p\beta (f\alpha e) \\ + (f\alpha e)\beta p\beta (e\alpha f) \\ = \sum_{i+j+k=n} f_i(a\alpha b, c\alpha d, e\alpha f)\beta d_j(m, s, p)d_k(b\alpha a, d\alpha c, (f\alpha e)) \\ + f_i(b\alpha a, d\alpha c, f\alpha e)\beta d_j(m, s, p)d_k(a\alpha b, c\alpha d, e\alpha f) \\ = f_n(a\alpha b, c\alpha d, e\alpha f)\beta m\beta b\alpha a + f_n(b\alpha a, d\alpha c, f\alpha e)\beta m\beta a \\ + a\alpha b\beta m\beta (b\alpha a, d\alpha c, f\alpha e) + b\alpha a\beta m\beta (a\alpha b, c\alpha d, e\alpha f) \\ + \sum_{i+j+k=n} f_i(a\alpha b, c\alpha d, e\alpha f)\beta d_j(m, s, p)\beta d_k(b\alpha a, d\alpha c, f\alpha e) \\ + F_i(b\alpha a, d\alpha c, f\alpha e)\beta d_j(m, s, p)\beta d_k(a\alpha b, c\alpha d, e\alpha f)\end{aligned}$$

$$\begin{aligned}
&= f_n(a\alpha b, c\alpha d, e\alpha f)\beta m\beta b\alpha a + F_n(b\alpha a, d\alpha c, F\alpha e)\beta m\beta a\alpha b \\
&+ a\alpha b\beta m\beta d_n(b\alpha a, d\alpha c, f\alpha e) + b\alpha a\beta m\beta d_h(a\alpha b, c\alpha d, e\alpha f) \\
&+ \sum_{q+i+j} f_q(a, c, e)\alpha d_j(b, d, f)\beta d_j(m, s, p)\beta d_h(b, d, f)\alpha d_q(* a, c, e) \\
&+ F_q(b, d, f)\alpha d_t(a, c, e)\beta d_j(m, s, p)\beta d_h(a, c, e)\alpha d_q(b, d, f)
\end{aligned}$$

On the other hand lemma (2. )iii

$$\begin{aligned}
&F_n(a\alpha b, \beta m\beta b\alpha a + b\alpha a\beta m\beta a\alpha b, c\alpha(d\beta s\beta d)\alpha d \\
&+ d\alpha(c\beta s\beta c)\alpha d, e\alpha(f\beta p\beta f)\alpha e + f\alpha(e\beta p\beta e)\alpha f) \\
&= f_n(a\alpha(b\beta m\beta b)\alpha a + b\alpha(a\beta m\beta a)\alpha b, c\alpha(d\beta s\beta d)\alpha c \\
&+ d\alpha(c\beta s\beta c)\alpha d, e\alpha(f\beta p\beta f)\alpha e + f\alpha(e\beta p\beta e)\alpha f)
\end{aligned}$$

$$\begin{aligned}
fn &= fn(a \alpha(b\beta m\beta b) \alpha a, c \alpha(d\beta s\beta d) \alpha c, e \alpha(f\beta p\beta f) \alpha e \\
&+ fn(b \alpha(a\beta m\beta a)\alpha b, d\alpha(c\beta s\beta c)\alpha d, f\alpha(e\beta p\beta e)\alpha f) \\
&= \sum_{q+g+k=n} f_q(a, c, e)\alpha d_k(b\beta m\beta b, d\beta s\beta d, f\beta p\beta f)d_g(a, c, e) \\
&+ f_q(b, d, f)\alpha d_k(a\beta m\beta a, c\beta s\beta c, e\beta p\beta e)\alpha d_g(b, d, f)
\end{aligned}$$

$$\begin{aligned}
&= \sum_{h+t+g+q=n} f_q(a, c, e) \alpha dt(b, d, f) \beta di(m, s, p) \beta dn(b, d, f) \alpha dg(a, c, e) \\
&\quad + Fg(b, d, f) \alpha dt(a, c, e) \beta di(m, s, p) \beta d_h(a, c, e) \alpha dg(b, d, f) \\
&= \sum_{g+t=n} f_g(a, c, e) \alpha dt(b, d, f) \beta m \beta b \alpha a \\
&\quad + \sum_{g+t=n} fg(b, d, f) \alpha dt(a, c, e) \beta m \beta a \alpha b \\
&\quad + a \alpha b \beta m \beta \sum_{h+g=n} dh(a, c, e) \alpha dg(b, d, f) \\
&\quad + \sum_{q+t+h=n} f_q(a, c, e) \alpha dt(b, d, f) \beta di(m, s, p) \beta dn(b, d, f) \alpha dh(a, c, e) \\
&\quad + fg(b, d, f) \alpha dt(a, c, e) \beta d_j(m, s, p) \beta dh(a, c, e) \\
&\quad \alpha dg(b, d, f) \dots \dots (2)
\end{aligned}$$

**Compare between (1) and (2)**

$$\begin{aligned}
& \text{fn}(a\alpha b, c\alpha d, e\alpha f)\beta m\beta b\alpha a \\
& - \sum_{g+t=n} fg(a, c, e)\alpha dt(b, d, f)\beta m\beta b\alpha a \\
& + \text{fn}(b\alpha a, d\alpha c, f\alpha e)\beta m\beta a\alpha b \\
& - \sum_{g+t+h=n} fg(b, d, f)\alpha dt(a, c, e)\beta m\beta a\alpha b \\
& + b\alpha a\beta m\beta d_h(a\alpha b, c\alpha d, e\alpha f) \\
& - b\alpha a\beta m\beta \sum_{h+g=n} d_h(a, c, e)\alpha dg(b, d, f) = 0 \\
& [\text{fn}(a\alpha b, c\alpha d, e\alpha f) - \sum_{h+g+t=n} fg(a, c, e)\alpha dt(b, d, f)]\beta m\beta b\alpha a \\
& + a\alpha b\beta m\beta [d_h(b\alpha, d\alpha c, f\alpha e)] \\
& - \sum_{h+g=n} d_h(b, d, f)\alpha dg(a, c, e) + [\text{fn}(b\alpha a, d\alpha c, f\alpha e) \\
& - \sum_{h+g=t=n} fg(b, d, f)\alpha dt(a, c, e)\beta m\beta a\alpha b \\
& + b\alpha a\beta m\beta [d_h(a\alpha b, c\alpha d, e\alpha f)] \\
& - \sum_{g+h=n} d_h(a, c, e)\alpha dg(b, d, f) = 0 \\
& \phi n(a, b, c, d, e, f)\alpha \beta m\beta b\alpha a + a\alpha b\beta m\beta + \Psi n(b, a, d, c, e, f)\alpha \\
& + \phi n(b, a, d, c, e, f)\alpha \beta m\beta b\alpha a + a\alpha b\beta m\beta \\
& \Psi n(a, b, c, d, e, f)\alpha = 0
\end{aligned}$$

$$\begin{aligned}
& \phi_n(a, b, c, d, e, f) \beta m \beta b \alpha a + \phi_n(a, b, c, d, e, f) \beta m \beta a \alpha b \\
& \quad + b \alpha a \beta m \beta + \phi_n(a, b, c, d, e, f) \alpha \\
& - a \alpha b \beta m \beta \Psi_n(a, b, c, d, e, f) \alpha = 0 \\
& = \phi_n[(a, b, c, d, e, f) \beta m \beta] (b \alpha a - a \alpha b) \\
& \quad + (b \alpha a - a \alpha b) [\beta m \beta \Psi_n(a, b, c, d, e, f) \alpha] \\
& = \phi_n(a, b, c, d, e, f)_\alpha \beta m \beta [b, a] \alpha + [b, a] \alpha \beta m \beta \Psi_n(a, b, c, d, e, f)_\alpha = 0 \\
& \phi_n(a, b, c, d, e, f)_\alpha \beta m \beta [a, b] \alpha + [a, b] \alpha \beta m \beta \Psi_n(a, b, c, d, e, f)_\alpha = 0
\end{aligned}$$

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