

**Ministry of Higher Education  
& Scientific Research  
Qadisiyah University  
College of Education  
Department of Mathematics**



# **On Jordan Generalized Higher TRI – Derivations on Prime Rings**

Research presented by the student

**Walaa Aziz Salmaan**

Submitted The Council of the College of Education – Department of Mathematics In partial fulfillment of requirements for the Bachelor degree in Science of Mathematics

**Under the Supervision of**

**Dr. Mazen Omran**

**2018**

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

اللَّهُ لَا إِلَهَ إِلَّا هُوَ الْحَيُّ الْقَيُومُ لَا تَأْخُذُهُ سِنَةٌ وَلَا نُوْمٌ  
لَهُ مَا فِي السَّمَاوَاتِ وَمَا فِي الْأَرْضِ مَنْ ذَا أَلَّذِي يَشْفَعُ عِنْدَهُ  
إِلَّا بِإِذْنِهِ يَعْلَمُ مَا بَيْنَ أَيْدِيهِمْ وَمَا خَلْفَهُمْ وَلَا يُحِيطُونَ  
بِشَئٍ مِنْ عِلْمِهِ إِلَّا بِمَا شَاءَ وَسَعَ كُرْسِيُهُ السَّمَاوَاتِ  
وَالْأَرْضَ وَلَا يَعُودُهُ حِفْظُهُمَا وَهُوَ الْعَلِيُّ الْعَظِيمُ

٢٥٥

صدق الله العلي العظيم

سورة البقرة - الآية (255)

# Acknowledgment

Praise be to Allah the lord of the worlds and prayers and peace be upon Ashraf created by Muhammad and The God of the good and virtuous .

It gives me great :

Pleasure has finished the requirements of this research to extend my sincere thanks and Altkadir walamthan to my professor virtuous please Omran six Mazen for its sincere efforts in the direction and supervision and follow – up and moral support throughout the duration of the study.

Kmatkdm thanks to the gentlemen chairman and members of the discussion to your favorite agreeing to discuss research .

# **Gifting**

**To the Prophet of Mercy and the savior of mankind Muhammad (Peace be upon him) .**

**Love and Respect**

**Who said I won the lord of the Kaaba**

**Love and Respect**

**To the martyrs and the liberal Abe mingled with the blood of pure soil of Karbala for Islam symbol .**

**Love and Respect**

# **Abstract**

**In this study, we define the concepts of a generalized higher tri – derivation , Jordan generalized higher tri – derivation and Jordan triple generalized higher tri – derivation on rings and show that a Jordan generalized higher tri – derivation on 2 – torsion free ring is a Jordan triple generalized higher tri derivation .**

## **Introduction**

**In this research consists of two chapters in chapter one some basic definitions of ring , sub ring , ideal , commutator and give some properties of commutator and the definitions of derivation , tri – derivation , Jordan tri – derivation Jordan generalized tri – derivation .**

**In chapter two give the definition of higher tri – derivation , Jordan higher tri – derivation , Jordan triple higher tri – derivation , generalized higher tri – derivation , Jordan generalized higher tri – derivation and study the relationships between these concepts .**

# **Chapter One**

## **Introduction**

In this chapter some basic definitions of ring , subring , ideal , commutator and give some properties of commutator and the definitions of derivation , tri – derivation , Jordan tri – derivation and Jordan generalized tri – derivation and some relationship between them .

### **Definition : 1 – 1 “Ring”**

Let  $R$  be a nonempty set and let  $+$ ,  $\cdot$  be two binary operation defined on  $R$  then  $(R, +, \cdot)$  is a ring if :

1-  $(R, +)$  commutative group .

2-  $(R, \cdot)$  semi group .

$$3- a \cdot (b + c) = a \cdot b + a \cdot c$$

$$(b + c) \cdot a = b \cdot a + c \cdot a$$

For all  $a, b, c \in R$  .

### **Definition 1 – 2 “Subring”**

Let  $(R, +, \cdot)$  be a ring and  $S$  be a nonempty subset of  $R$  then  $(S, +, \cdot)$  is called a subring of  $(R, +, \cdot)$  if  $(S, +, \cdot)$  itself ring .

### **Definition 1 – 3 “Ideal”**

Let  $(R, +, \cdot)$  be a ring and  $I$  nonempty sub set of  $R$  then  $(I, +, \cdot)$  is called an ideal of  $(R, +, \cdot)$  if the following conditions are satisfying.

$$1. a - b \in I .$$

$$2. r \cdot a \in I \text{ and } a \cdot r \in I \quad \text{for all } a, b \in I \text{ and } r \in R .$$

### **Definition 1 – 4 “2 – torsion”**

A ring  $(R, +, \cdot)$  is called 2 – torsion free if  $2 a = 0$  implies  $a = 0$  for all  $a \in R$  where 2 is a positive integer number .

### **Definition 1 – 5 “commutative ring”**

A ring  $(R, +, \cdot)$  is called commutative ring if  $ab = ba$  for all  $a, b \in R$  .

### **Definition 1 – 6 “ Commutator”**

Let  $(R, +, \cdot)$  be a ring define  $[a, b]$  as  $[a, b] = ab - ba$  is called commutator of  $a, b$  .

Lemma :

If  $R$  is a ring then for all  $x, y, z \in R$  :

$$1- [x, y] + [y, x] = 0$$

$$2- [x + y, z] = [x, z] + [y, z]$$

$$3- [x, y + z] = [x, y] + [x, z]$$

### **Definition 1 – 7 “Permmuting”**

A mapping  $D : R \times R \times R \rightarrow R$  is said

To be permmutting if

$$\begin{aligned} D(a, b, c) &= D(a, c, b) = D(b, a, c) \\ &= D(b, c, a) = D(c, a, b) = D(c, b, a) \end{aligned}$$

### **Definition “Prime ring”**

A ring  $R$  is called prime if for any  $a, b \in R$ ,  $aRb = \{0\}$  implies  $a = 0$  or  $b = 0$ .

### **Definition 1 – 9 “Semi prime”**

A ring  $R$  is called semi prime if for any  $a \in R$ ,  $aRa = \{0\}$  implies  $a = 0$ .

### **Remark :**

Every prime ring is semi prime ring but the convers is not true in general.

The following example show this fact:

**Example:**

A ring  $(Z_{35}, +_{35}, \cdot_{35})$  is semi prime ring since for all  $a \in Z_{35}$  if  $a \in Z_{35} \setminus \{0\}$  implies  $a = 0$  but we note that  $(Z_{35}, +_{35}, \cdot_{35})$  is not prime ring since  $5 \cdot_{35} 7 = \{0\}$  but  $5 \neq 0$  and  $7 \neq 0$ .

**Definition 1 – 10 “tri-additive mapping”**

A mapping  $D : R \times R \times R \rightarrow R$  is called tri – additive mapping if:

$$(1) D(a + b, c, d) = D(a, c, d) + D(b, c, d)$$

$$(2) D(a, b + c, d) = D(a, b, d) + D(a, c, d)$$

$$(3) D(a, b, c + d) = D(a, b, c) + D(a, c, d)$$

**Definition 1 – 11 “Derivation”**

An additive mapping  $D : R \rightarrow R$  is called a derivation of  $R$  if :

$$D(ab) = D(a)b + aD(b) \text{ for all } a, b \in R.$$

**Definition 1 – 12 “tri – Derivation”**

A tri additive mapping  $D : R \times R \times R \rightarrow R$  is called tri – Derivation

if :

$$(1) D(x_1 x_2, y, z) = x_1 D(x_2, y, z) + D(x_1, y, z))$$

$$(2) D(x, y_1 y_2, z) = y_1 D(x, y_2, z) + D(x, y, z) y_3$$

$$(3) D(x, y_1, z_1 z_2) = z_1 D(x, y_1, z_2) + D(x, y, z_1) z$$

### **Definition 1 – 13 “Jordan tri – derivation”**

A tri additive mapping  $D : R \times R \times R \rightarrow R$  is called Jordan tri derivation if :

$$(1) D(a^2, b, c) = a D(a, b, c) + D(a, b, c) a$$

$$(2) D(a, b^2, c) = b D(a, b, c) + D(a, b, c) b$$

$$(3) D(a, b, c^2) = c D(a, b, c) + D(a, b, c) c$$

### **Definition 1 – 14 “generalized tri – derivation”**

A tri – additive mapping  $F : R \times R \times R \rightarrow R$  is called generalized tri – derivation on  $R \times R \times R$  into  $R$  if there exists tri – Derivation  $D : R \times R \times R \rightarrow R$  such that :

$$(1) F(ab, c, d) = F(a, c, b) b + a D(b, c, d)$$

$$(2) F(a, bc, d) = F(a, b, d) c + b D(a, c, d)$$

$$(3) F(a, b, cd) = F(a, b, c) d + c D(a, b, d)$$

## **Definitions 1 – 15 “Jordan generalized tri – derivation”**

A tri – additive mapping  $F : R \times R \times R \rightarrow R$  is called Jordan generalized tri – derivation on  $R \times R \times R$  into  $R$  if there exists Jordan tri – derivation on  $R \times R \times R$  into  $R$  such that :

$$(1) F(a^2, b, c) = F(a, b, c)a + aD(a, b, c)$$

$$(2) F(a, b^2, c) = F(a, b, c)b + bD(a, b, c)$$

$$(3) F(a, b, c^2) = F(a, b, c)c + cD(a, b, c)$$

# Chapter Two

# **On Jordan Generalized Higher TRI – Derivations on Prime Rings**

## **Introduction**

In this chapter give the definition of higher tri – derivation , Jordan higher tri – derivation , Jordan triple higher tri – derivation , generalized higher tri – derivation , Jordan generalized higher tri – derivation and the relationships between these concepts .

### **Definition 2 – 1 “higher tri – derivation”**

Let R be a ring and  $D = (d_i)_{i \in N}$  be a family of tri additive mapping on  $R \times R \times R$  into R such that  $d(a, b, c) = a$  for all  $a, b, c \in R$  then D is called a higher tri – derivation on  $R \times R \times R$  into R if for all  $a, b, c, d, e, f \in R$  and  $n \in N$  such that :

$$dn(ab, cd, ef) = \sum_{i+j=n} d_i(a, c, e)d_j(b, d, f)$$

### **Definition 2 – 2 “Jordan higher tri – derivation”**

Let R be a ring and  $D = (d_i)_{i \in N}$  be a family of tri – additive mapping on  $R \times R \times R$  into R then D is called Jordan higher tri – derivation if  $dn(a^2, b^2, c^2) = \sum_{i+j=n} d_i(a, b, c)d_j(a, b, c)$  .

**Definition 2 – 3 “Jordan triple higher tri – derivation”**

Let R be a ring and  $D = (d_i)_{i \in N}$  be a family of tri – additive mapping on  $R \times R \times R$  into R then D is called Jordan triple higher tri – derivations if :

$$dn(aba, cdc, efe) = \sum_{i+j+k=n} d_i(a, c, e) d_j(b, d, f) d_k(a, c, e)$$

**Definition 2 – 4 “Generalized higher tri – derivation”**

Let R be a ring and  $F = (f_i)_{i \in N}$  be family of tri additive mapping on  $R \times R \times R$  in to R such that  $f_o(a, b, c) = a$  for all  $a, b, c \in R$  then F is called a generalized higher tri derivation on  $R \times R \times R$  into R if there exist a higher tri derivation  $D = (d_i)_{i \in N}$  on  $R \times R \times R$  into R such that for all  $n \in N$  we have :  $fn(ab, cd, ef) = \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f)$  for all  $a, b, c, d, e, f \in R$ .

**Definition 2 – 5 “Jordan generalized higher tri – derivation”**

Let R be a ring and  $F = (f_i)_{i \in N}$  be a family of tri – additive mapping on  $R \times R \times R$  into R such that  $f_o(a, b, c) = a$  for all  $a, b, c \in R$  then F is called Jordan generalized higher tri – derivation on  $R \times R \times R$  into R

if there exist Jordan higher tri – derivation  $D = (d_i)_{i \in \mathbb{N}}$  on  $R \times R \times R$  into  $R$  such that for all  $n \in \mathbb{N}$  we have  $f_n(a^2, b^2, c^2) = \sum_{i+j=n} f_i(a, b, c) d_j(a, b, c)$ .

**Example 2 – 6 :**

Let  $R = \left\{ \begin{pmatrix} 0 & 0 & a \\ 0 & b & c \\ 0 & 0 & 0 \end{pmatrix}, a, b, c \in Z \text{ be a ring} \right.$

Under matrix addition and matrix multiplication .

Define  $f_n : R \times R \times R \rightarrow R, n \in N$  by

$$f_n \left( \begin{bmatrix} 0 & 0 & a_1 \\ 0 & b_1 & c_1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_2 \\ 0 & b_2 & c_2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_3 \\ 0 & b_3 & c_3 \\ 0 & 0 & 0 \end{bmatrix} \right)$$

$$= \begin{cases} \begin{bmatrix} 0 & 0 & a_1 \\ 0 & b_1 & c_1 \\ 0 & 0 & 0 \end{bmatrix} & \text{if } n = 0 \\ \begin{bmatrix} 0 & 0 & a_1 + a_2 + a_3 \\ 0 & 0 & nc_1 \\ 0 & 0 & 0 \end{bmatrix} & \text{if } n \geq 1 \end{cases}$$

for all  $\begin{bmatrix} 0 & 0 & a_1 \\ 0 & b_1 & c_1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_2 \\ 0 & b_2 & c_2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_3 \\ 0 & b_3 & c_3 \\ 0 & 0 & 0 \end{bmatrix} \in R$

Then  $F$  is generalized higher tri – derivation because there exists a higher tri – derivation  $d_n : R \times R \times R \rightarrow R, n \in N$  defined by :

$$\begin{aligned}
& d_n \left( \begin{bmatrix} 0 & 0 & a_1 \\ 0 & b_1 & c_1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_2 \\ 0 & b_2 & c_2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_3 \\ 0 & b_3 & c_3 \\ 0 & 0 & 0 \end{bmatrix} \right) \\
&= \begin{cases} \begin{bmatrix} 0 & 0 & a_1 \\ 0 & b_1 & c_1 \\ 0 & 0 & 0 \end{bmatrix} & \text{if } n = 0 \\ \begin{bmatrix} 0 & 0 & a_1 + a_2 + a_3 \\ 0 & 0 & nc_1 \\ 0 & 0 & 0 \end{bmatrix} & \text{if } n \geq 1 \end{cases} \\
&\text{for all } \begin{bmatrix} 0 & 0 & a_1 \\ 0 & b_1 & c_1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_2 \\ 0 & b_2 & c_2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_3 \\ 0 & b_3 & c_3 \\ 0 & 0 & 0 \end{bmatrix} \in R
\end{aligned}$$

Such that  $f_n(AB, CD, EF)$

$$= \sum_{i+j=n} f_i(A, C, E) f_j(B, D, F)$$

For all  $A, B, C, D, E, F \in R$

### Lemma 2- 7

Let  $R$  be a ring and  $F = (f_i)_{i \in N}$  be a Jordan generalized higher tri – derivation on  $R \times R \times R$  into  $R$  associated with Jordan higher tri – derivation  $D = (d_i)_{i \in N}$  on  $R \times R \times R$  into  $R$  then for all  $a, b, c, d, e, f \in R$  and  $n \in N$ .

$$f_n(ab + ba, cd + dc, ef + fe)$$

$$= \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f) + f_i(b, d, f) d_j(a, c, e)$$

Since  $F = (f_i)_{i \in N}$  be Jordan generalized higher tri – derivation on  $R \times R$

$\times R$  into  $R$  then :

$$\begin{aligned} & f_n[(a+b)^2, (c+d)^2, (e+f)^2] \\ &= \sum_{i+j=n} f_i(a+b, c+d, e+f) d_j(a+b, c+d, e+f) \\ &= \sum_{i+j=n} [f_i(a, c, e) + f_i(b, d, f)] [d_j(a, c, e) + d_j(b, d, f)] \\ &= \sum_{i+j=n} f_i(a, c, e) d_j(a, c, e) + f_i(a, c, e) d_j(b, d, f) \\ &\quad + f_i(b, d, f) d_j(a, c, e) + f_i(b, d, f) d_j(b, d, f) \dots \dots \dots (1) \end{aligned}$$

On the other hand :

$$\begin{aligned} & f_n((a+b)^2, (c+d)^2, (e+f)^2) \\ &= f_n(a^2 + ab + ba + b^2, c^2 + cd + dc + d^2, e^2 + ef + fe + f^2) \\ &\quad f_n(a^2, c^2, e^2) + f_n(ab + ba, cd + dc, ef + fe) \\ &\quad + f_n(b^2, d^2, f^2) \\ &= \sum_{i+j=n} f_i(a, c, e) d_j(a, c, e) + f_n(ab + ba, cd + dc, ef + fe) \end{aligned}$$

$$+ \sum_{i+j=n} f_i(b, d, f) d_j(b, d, f) \dots \dots \dots \quad (2)$$

Compare between (1) and (2) we get :

$$F_n(ab + ba, cd + dc, ef + fe)$$

$$= \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f) + f_i(b, d, f) d_j(a, c, e)$$

## Proposition 2 – 8

Let  $R$  be  $2 -$  torsion free ring then every Jordan generalized higher tri – derivation on  $R \times R \times R$  in to  $R$  is a Jordan triple generalized higher tri – derivation on  $R \times R \times R$  into  $R$  .

### Proof :

Replace  $ab + ba$  for  $b$  and  $cd + dc$  for  $d$  and  $ef + fe$  for  $f$  in Lemma 2.7

$$\begin{aligned}
& f_n [a(ab + ba) + (ab + ba)a, c(cd + dc) + cd + dc)c, e(ef \\
& \quad + fe) + (ef + fe)e \\
= & \sum_{i+j=n} f_i(a, c, e)d_j(ab + ba, cd + dc, ef + fe) \\
& + f_i(ab + ba, cd + dc, ef + fe)d_j(a, c, e) \\
= & \sum_{i+j=n} f_i(a, c, e)[d_j(ab, cd, ef) + d_j(ba, dc, fe)] \\
& + [f_i(ab, cd, ef) + f_i(ba, dc, fe)]d_j(a, c, e) \\
= & \sum_{i+j=n} f_i(a, c, e)d_j(ab, cd, ef) + f_i(a, c, e)d_j(ba, dc, fe) \\
& + f_i(ab, cd, ef)d_j(a, c, e) + f_i(ba, dc, fe)d_j(a, c, e) \\
= & \sum_{i+j=n} f_i(a, c, e)[d_j(a, c, e)d_k(b, d, f)]
\end{aligned}$$

$$\begin{aligned}
& + f_i(a, c, e) [d_j(b, d, f) d_k(a, c, e)] \\
& + [f_i(a, c, e) d_j(b, d, f)] d_k(a, c, e) \\
& + [f_i(b, d, f) d_j(a, c, e)] d_k(a, c, e) \\
= & \sum_{i+j+k=n} f_i(a, c, e) d_j(a, c, e) d_k(b, d, f) \\
& + 2 \sum_{i+j+k=n} f_i(a, c, e) d_j(b, d, f) d_k(a, c, e) \\
& + f_i(b, d, f) d_j(a, c, e) d_k(a, c, e) \dots \dots \dots (1)
\end{aligned}$$

On the other hand :

$$\begin{aligned}
& f_n(a(ab + ba) + (ab + ba)a, c(cd + dc) \\
& \quad + (cd + dc)c, e(ef + fe) + (ef + fe)e) \\
= & f_n(a^2b + aba + aba + ba^2, c^2d + cdc + dc^2, e^2f + efe + efe \\
& \quad + fe^2) \\
& f_n(a^2b + ba^2, c^2d + dc^2, e^2f + fe^2) \\
& + f_n(aba, cdc, efe) + f_n(aba, cdc, efe) \\
= & \sum_{i+j=n} f_i(a^2, c^2, e^2) d_j(b, d, f) + f_i(b, d, f) f_j(a^2, c^2, e^2) \\
& + 2 f_n(aba, cdc, efe)
\end{aligned}$$

Compare between (1) and (2) we get :

$$2 \sum fi(a,c,e) d_j(b,d,f) d_k(a,c,e)$$

Since  $R$  is 2 – torsion free then :

$$fn(aba,cdc,efe) =$$

$$\sum_{i+j+k=n} f_i(a,c,e) d_j(b,d,f) d_k(a,c,e)$$

**Lemma 2 – 9 :**

Let  $R$  be a ring and  $F = (f_i)_{i \in N}$  be a Jordan triple generalized higher tri-derivation on  $R \times R \times R$  into  $R$  associated with Jordan higher tri-derivation  $D = (d_i)$   $i \in N$  on  $R \times R \times R$  in to  $R$  then for all  $a, b, c, d, e, f, r, s, t \in R$  and  $n \in N$ .

$$fn(abc, cba, sdt + tds, efr + rfe)$$

$$\begin{aligned} &= \sum_{i+j+k=n} f_i(a, s, e) d_j(b, d, f) d_k(c, t, r) \\ &\quad + f_i(c, t, r) d_j(b, d, f) d_k(a, s, e) \end{aligned}$$

**Proof :** replacing  $a + c$  for  $a$  and  $s + t$  for  $c$  and  $e + r$  for  $f$  in proposition 2 – 8 :

$$fn((a + c)b(a + c), (s + t)d(s + t), (e + r)f(e + r))$$

$$\begin{aligned} &= \sum_{i+j+k=n} f_i(a + c, s + t, e + r) d_j(b, d, f) d_k(a + c, s + t, e + r) \\ &= \sum [f_i(a, s, e) + f_i(c, t, r)] d_j(b, d, f) \end{aligned}$$

On the other hand :

$$\begin{aligned}
& f_n((a+c)b(a+c), (s+t)d(s+t), (e+r)f(e+r)) \\
&= f_n(aba + abc + cba + cbc, sds + sdt + tds + tdt, efe + efr \\
&\quad + rfe + rfr) \\
&= f_n(aba, sds, efe) \\
&\quad + f_n(abc + cba, sdt + tds, efr + rfe) \\
&\quad + f_n(cbc, tdt, rfr) \\
&= \sum_{i+j+k=n} f_i(a, s, e) d_j(b, d, f) d_k(a, s, e)
\end{aligned}$$

$$+ f_n(abc + cba, sdt + tds, efr + rfe)$$

$$+ \sum_{i+j+k=n} f_i(c, t, r) d_j(b, d, f) d_k(c, t, r) \dots \dots \dots (2)$$

Compare between (1) and (2) we get :

$$fn(abc + cba, sdt + tds, efr + rfe)$$

$$= \sum f_i(a,s,e) d_j(b,d,f) d_k(c,t,r)$$

$$+ f i \, (c \, , t \, , r) \, d_j \, (b \, , d \, , f) \, d_k \, (a \, , s \, , e)$$

## Definition 2 – 10 :

Let  $R$  be a ring and  $F = (f_i)_{i \in N}$  be a Jordan generalized higher tri – derivation on  $R \times R \times R$  into  $R$  associated with Jordan higher tri – derivation  $D = (d_i)_{i \in N}$  on  $R \times R \times R$  into  $R$ . Then for all  $a, b, c, d, e, f \in R$  and  $n \in N$  we denote by :

$$\emptyset_n(a,b,c,d,e,f)$$

$$= f_n(ab, cd, ef) - \sum f_i(a, c, e) d_j(b, d, f)$$

**Lemma 2 – 11 :**

Let  $R$  be a ring and  $F = (f_i)$   $i \in N$  be a Jordan generalized higher tri – derivation on  $R \times R \times R$  into  $R$  associated with Jordan higher tri – derivation  $D = (d_i)_{i \in N}$  on  $R \times R \times R$  into  $R$  then for all  $a, b, c, d, s, e, f$ .

$$(i) \emptyset_n(a, b, c, d, e, f) = \emptyset_n(b, a, d, c, f, e)$$

$$(ii) \emptyset_n(a + s, b, c, d, e, f) = \emptyset_n(a, b, c, d, e, f)$$

$$+ \emptyset_n(s, b, c, d, e, f)$$

$$(iii) \emptyset_n(a, b + s, c, d, e, f) = \emptyset_n(a, b, c, d, e, f)$$

$$+ \emptyset_n(a, s, c, d, e, f)$$

$$(v) \emptyset_n(a, b, c + s, d, e, f) = \emptyset_n(a, b, c, d, e, f)$$

$$+ \emptyset_n(a, b, s, d, e, f)$$

$$(iv) \emptyset_n(a, b, c, d + s, e, f)$$

$$= \emptyset_n(a, b, c, d, e, f) + \emptyset_n(a, b, c, s, e, f)$$

$$(ii)v\emptyset_n(a,b,c,d,e+s,f)$$

$$= \emptyset_n(a,b,c,d,e,f) + \emptyset_n(a,b,c,d,s,f)$$

$$(iiiv)\emptyset_n(a,b,c,d,e,f+s)$$

$$= \emptyset_n(a,b,c,d,e,f) + \emptyset_n(a,b,c,d,e,s)$$

**Proof :**

**(i)** by lemma 2 – 7 :

$$f_n(ab + ba, cd + dc, ef + fe)$$

$$= \sum_{i+j=n} f_i(a,c,e)d_j(b,d,f) + f_i(b,d,f)d_j(a,c,e)$$

$$f_n(ab, cd, ef) + f_n(ba, dc, fe)$$

$$= \sum_{i+j=n} f_i(a,c,e)d_j(b,d,f) + \sum_{i+j=n} f_i(b,d,f)d_j(a,c,e)$$

$$f_n(ab, cd, ef) - \sum f_i(a,c,e)d_j(b,d,f)$$

$$+ f_n(ba, dc, fe) - \sum f_i(b,d,f)d_j(a,c,e) = 0$$

$$\emptyset_n(a, b, c, d, e, f) + \emptyset_n(b, a, d, c, f, e) = 0$$

$$\emptyset_n(a, b, c, d, e, f) = -\emptyset_n(b, a, d, c, f, e)$$

**(ii)**

$$\emptyset_n(a+s, b, c, d, e, f) = f_n[(a+s)b, cd, ef]$$

$$-\sum_{i+j=n} f_i(a+s, c, e) d_j(b, d, f)$$

$$= f_n[ab + sb, cd, ef] - [\sum f_i(a, c, e) + f_i(s, c, e)]$$

$$(d_j(b, d, f))$$

$$= f_n(ab, cd, ef) + f_n(sb, cd, ef)$$

$$-\sum_{i+j=n} f_i(a, c, e) d_j(b, d, f) - \sum_{i+j=n} f_i(s, c, e) d_j(b, d, f)$$

$$= f_n(ab, cd, ef) - \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f)$$

$$+ f_n(sb, sd, ef) - \sum_{i+j=n} f_i(s, c, e) d_j(b, d, f)$$

$$= \emptyset_n(a, b, c, d, e, f) + \emptyset_n(s, b, c, d, e, f)$$

**(iii)**

$$\emptyset_n(a, b+s, c, d, e, f) = f_n(a(b+s), cd, ef)$$

$$- \sum_{i+j=n} f_i(a, c, e) d_j(b+s, d, f)$$

$$= f_n(ab, as, cd, ef)$$

$$- \sum_{i+j=n} f_i(a, c, e) [d_j(b, d, f) + d_j(s, d, f)]$$

$$= f_n(ab, cd, ef) + f_n(as, cd, ef)$$

$$- \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f) - \sum_{i+j=n} f_i(a, c, e) d_j(s, d, f)$$

$$= f_n(ab, cd, ef) - \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f)$$

$$+ f_n(as, cd, ef) - \sum_{i+j=n} f_i(a, c, e) d_j(s, d, f)$$

$$= \emptyset_n(a, b, c, d, e, f) + \emptyset_n(a, s, c, d, e, f)$$

(v)

$$\emptyset_n(a, b, c + s, d, e, f) = f_n(ab, (c + s)d, ef)$$

$$- \sum_{i+j=n} f_i(a, c + s, e) d_j(b, d, f)$$

$$= f_n(ab, cd + sd, ef)$$

$$- \sum_{i+j=n} [f_i(a, c, e) + f_i(a, s, e)] d_j(b, d, f)$$

$$= f_n(ab, cd, ef) + f_n(ab, sd, ef)$$

$$- \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f) + f_i(a, s, e) d_j(b, d, f)$$

$$= f_n(ab, cd, ef) - \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f)$$

$$+ f_n(ab, sd, ef) - \sum_{i+j=n} f_i(a, s, e) d_j(b, d, f)$$

$$= \emptyset_n(a, b, c, d, ef) + \emptyset_n(a, b, s, d, e, f)$$

(vi)

$$\emptyset_n(a, b, c, d + s, e, f) = f_n(ab, c(d + s), ef)$$

$$- \sum_{i+j=n} f_i(a, c, e) d_j(b, d + s, f)$$

$$= f_n(ab, cd + cd, ef)$$

$$- \sum_{i+j=n} f_i(a, c, e)[d_j(b, d, f) + d_j(b, s, f)]$$

$$= f_n(ab, cd, ef) + f_n(ab, cs, ef)$$

$$- \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f)$$

$$- \sum f_i(a, c, e) d_j(b, s, f)$$

$$= f_n(ab, cd, ef) - \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f)$$

$$+ f_n(ab, cs, ef) - \sum_{i+j=n} f_i(a, c, e) d_j(b, s, f)$$

$$= \emptyset_n(a, b, c, d, e, f) + \emptyset_n(a, b, c, s, e, f)$$

(vii)

$$\emptyset_n(a, b, c, d, e + s, f) = f_n(ab, cd, (e + s)f)$$

$$- \sum_{i+j=n} f_i(a, c, e + s) d_j(b, d, f)$$

$$= f_n(ab, cd, ef + sf)$$

$$- \sum_{i+j=n} [f_i(a, c, e) + f_i(a, c, s)] d_j(b, d, f)$$

$$= f_n(ab, cd, ef) + f_n(ab, cd, sf)$$

$$- \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f) - \sum_{i+j=n} f_i(a, c, s) d_j(b, d, f)$$

$$= f_n(ab, cd, ef) - \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f)$$

$$+ f_n(ab, cd, sf) - \sum_{i+j=n} f_i(a, c, s) d_j(b, d, f)$$

$$= \emptyset_n(a, b, c, d, e, f) + \emptyset_n(a, b, c, d, s, f)$$

(viii)

$$\emptyset_n(a, b, c, d, e, f + s) = f_n(ab, cd, e(f + s))$$

$$- \sum f_i(a, c, e) d_j(b, d, f + s)$$

$$= f_n(ab, cd, ef + es)$$

$$- \sum_{i+j=n} f_i(a, c, e) [d_j(b, d, f) + d_j(b, d, s)]$$

$$= f_n(ab, cd, ef) + f_n(ab, cd, es)$$

$$- \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f) - \sum_{i+j=n} f_i(a, c, e) d_j(b, d, s)$$

$$= f_n(ab, cd, ef) - \sum_{i+j=n} f_i(a, c, e) d_j(b, d, f)$$

$$+ f_n(ab, cd, es) - \sum_{i+j=n} f_i(a, c, e) d_j(b, d, s)$$

$$= \emptyset_n(a, b, c, d, e, f) + \emptyset_n(a, b, c, d, e, s)$$

## Lemma 2 – 12

Let  $R$  a 2 – torsion free and  $F = (f_i)_{i \in N}$  be a Jordan generalized higher tri – derivation on  $R \times R \times R$  into  $R$  associated with Jordan higher tri derivation  $D = (d_i)_{i \in N}$  on  $R \times R \times R$  into  $R$  then for all  $a, b, c, d, e, f \in R$  and  $n \in N$  if  $\emptyset_t(a, b, c, d, e, f) = 0$  and  $\Psi_t(a, b, c, d, e, f) = 0$  for every  $t < n$  then  $\emptyset_n(a, b, c, d, e, f) r [a, b] + [a, b] r \Psi_n(a, b, c, d) = 0$

**Proof :** Let  $S, P \in R$

$$\begin{aligned}
 & f_n(ab r ba + ba r ab, cdscdc + dcscd, efpfe + fepef) \\
 &= f_n((ab)r(ba) + (ba)r(ab), (cd)s(dc) + (dc)s(cd), (ef)p(fe) + (fe) \\
 &\quad p(ef))
 \end{aligned}$$

By lemma 2 . 9

$$\begin{aligned}
 &= \sum_{i+j+k=n} f_i(ab, cd, ef) d_j(r, s, p) d_k(ba, dc, fe) \\
 &\quad + f_i(ba, dc, fe) d_j(r, s, p) d_k(ab, cd, ef) \\
 &= f_n(ab, cd, ef) rba + f_n(ba, dc, fe) r ab
 \end{aligned}$$

$$+ ab r dn (ba, dc, fe) + ba r dn (ab, cd, ef)$$

$$+ \sum_{i+j=n}^{i,k < n} f_i (ab, cd, ef) d_j (r, s, p) d_k (ba, dc, fe)$$

$$+ f_i (ba, dc, fe) d_j (r, s, p) d_k (ab, cd, ef)$$

$$= f_n (ab, cd, ef) r ba + f_n (ba, dc, fe) rab$$

$$+ ab r dn (ba, dc, fe) + ba r dn (ab, cd, ef)$$

$$+ \sum_{q+t+j+h+g=n}^{q+t,h+g < n} f_q (a, c, e) d_t (b, d, f) d_j (r, s, p) d_h (b, d, f) d_g (a, c, e)$$

..... (1)

On the other hand :

$$f_n (ab r ba + ba ra b, cd s dc + dcs cd, ef p fe + fe p ef)$$

$$= f_n (a (brb) a + b (ara)b, c (dsd)c + d (csc)d, e (fpf)e)$$

$$+ f (epe)f)$$

$$= f_n (a (brb)a, c (dsd)c, e (fpf)e )$$

$$+ f_n (b(ara)b, d (csc)d, f (epe)f)$$

$$\begin{aligned}
&= \sum_{q+k+g=n} f_q(a, c, e) d_k(b r b, d s d, f p f) d g(a, c, e) \\
&\quad + f_q(b, d, f) d_k(a r a, \csc, e p e) d_g(b, d, f) \\
&= \sum f_q(a, c, e) d_t(b, d, f) d_j(r, s, p) d_h(b, d, f) d_g(a, c, e) \\
&\quad + f_q(b, d, f) d_t(a, c, e) d_j(r, s, p) d_h(a, c, e) d_g(b, d, f) \\
&= \sum_{q+t=n} f_q(a, c, e) d_t(b, d, f) r b a \\
&\quad + \sum_{q+t=n} f_q(b, d, f) d_t(a, c, e) r a b \\
&\quad + a b r \sum_{h+g=n} d_h(b, d, f) d_g(a, c, e) \\
&\quad + b a r \sum_{h+g=n} d_h(a, c, e) d_g(b, d, f) \\
&\quad + \sum_{q+t, h+g < n} f_q(a, c, e) d_t(b, d, f) d_j(r, s, p) d_h(b, d, f) d_g(a, c, e) \\
&\quad + f_q(b, d, f) d_t(a, c, e) d_j(r, s, p) d_h(a, c, e) d_g(b, d, f) \dots \dots \dots \quad (2)
\end{aligned}$$

Compare between (1) and (2) :

$$\begin{aligned}
& f_n(ab, cd, ef)r ba - \sum_{q+t=n} f_q(a, c, e) d_t(b, d, f) r ba \\
& + abr dn(ba, dc, fe) - abr \sum_{h+g=n} d_h(b, d, f) d_g(a, c, e) \\
& + f_n(ba, dc, fe)r ab - \sum_{q+t=n} f_q(b, d, f) d_t(a, c, e) r ab \\
& + ba v dn(ab, cd, ef) - b a r \sum d_h(a, c, e) d_g(b, d, f) = 0 \\
& [f_n(ab, cd, ef) - \sum_{q+t=n} f_q(a, c, e) d_t(b, d, f)] r ba \\
& + abr [dn(ba, dc, fe) - \sum_{h+g=n} d_h(b, d, f) d_g(a, c, e)] \\
& + [f_n(ba, dc, fe) - \sum f_q(b, d, f) d_t(a, c, e)] r ab \\
& + bar [dn(ab, cd, ef) - \sum d_h(a, c, e) d_g(b, d, f)] = 0 \\
& \emptyset_n(a, b, c, d, e, f)r ba + a br \Psi_n(b, a, d, c, f, e)
\end{aligned}$$

$$+\emptyset_n(b,a,d,c,f,e)r\,a\,b+b\,a\,r\,\Psi_n(a,b,c,d,e,f)=0$$

By :

$$\emptyset_n(a,b,c,d,e,f)r\,ba-\emptyset_n(a,b,c,d,e,f)r\,ab$$

$$+bar\,\Psi_n(a,b,c,d,e,f)-a\,b\,r\,\Psi_n(a,b,c,d,e,f)=0$$

$$[\emptyset_n(a,b,c,d,e,f)r](ba-ab)$$

$$+(ba-ab)[r\,\Psi_n(a,b,c,d,e,f)]=0$$

$$\emptyset_n(a,b,c,d,e,f)r\,[b,a]$$

$$+[b,a]r\,\Psi_n(a,b,c,d,e,f)=0$$

## **References:**

1. E.C.Ponser, "Derivation in prime ring", Proc.Amer.Math.Soc., 8(1957), P:1093-1100.
2. M.Bresar, "Jordan Derivations on Semiprime Rings", Proc.Amer.Math.Soc.104, No.4 (1988), P: 1003-1006.
3. N.Argac, "On Prime and semiprime rings with Derivations", Algebra col-loq.13 (3) (2006), p: 371-380.
4. S.M Salih, A.M. Mahirm "On Jordan generalized higher biderivations on Prime rings", International Journal of Advanced Scientific and Technical Research, Vol.5m No.6, 20016, P: 414-428.
5. K.K. Dey and A.C. Paul, "Permuting tri-derivation on semi Prime Gamma ring", J. Sci.Res., Vol.5, No.1, 2016, P: 55-66.