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***On Soft Topological Linear Spaces***

**A thesis**

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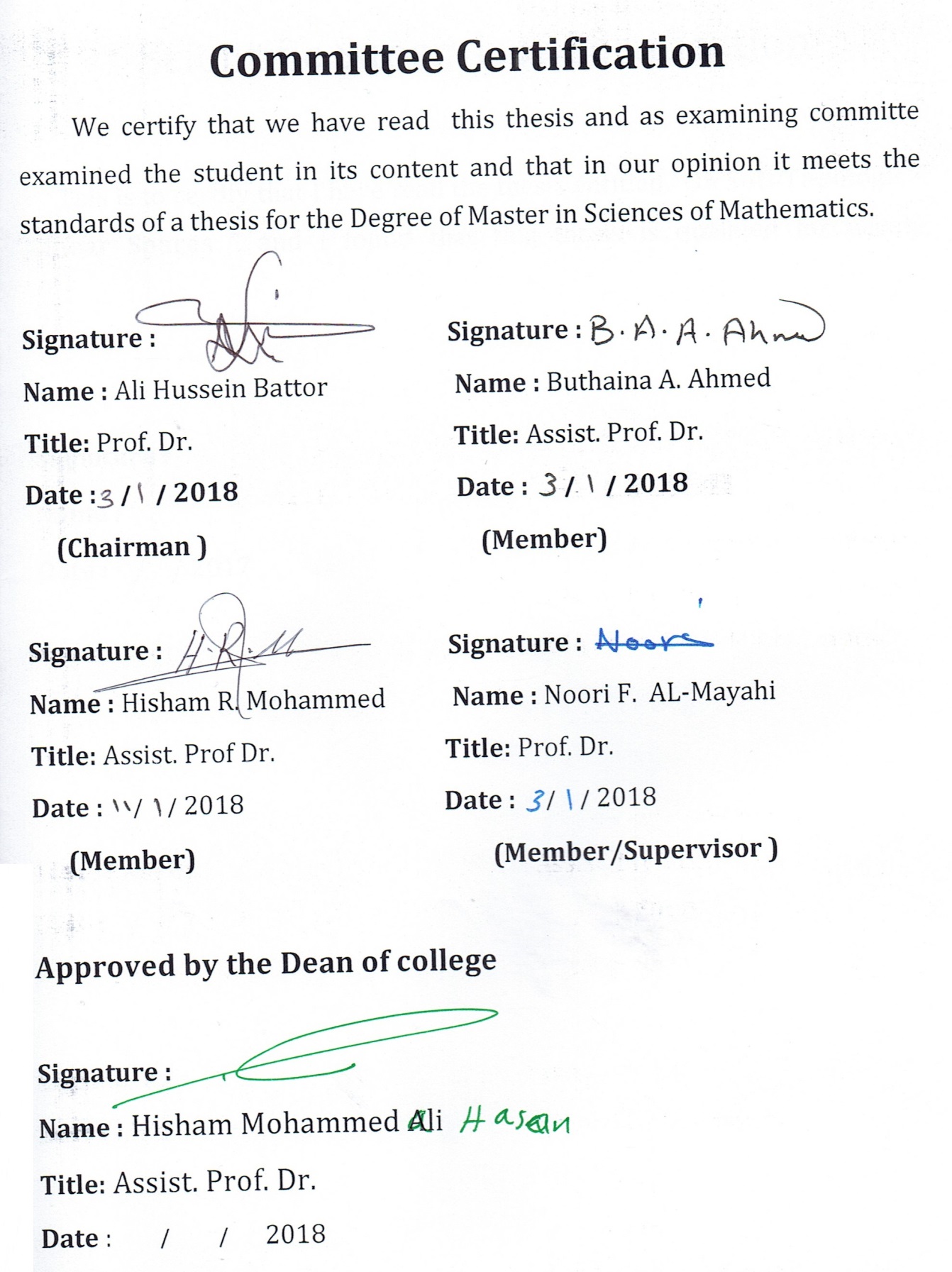
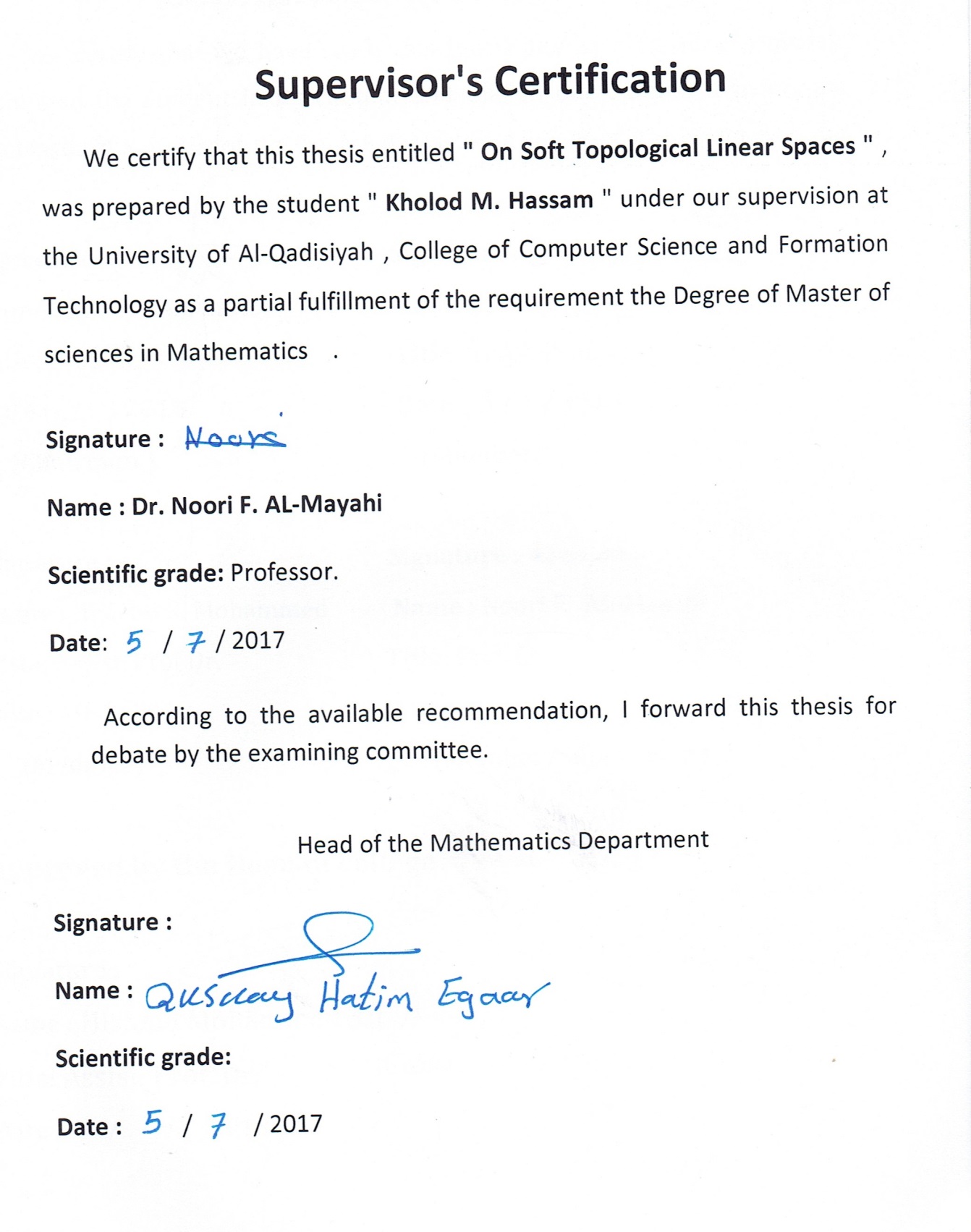
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**Abstract**

This thesis provides results on soft topological linear spaces and study of it is properties.

The main objective of this work is to create a new type of a soft topological linear space , namely soft topological linear space (which is soft locally convex space) and considered as the basis of our main definitions. Throughout this work, some important and new concepts have been illustrated including a soft topological linear space induced by a family of soft seminorms on a soft linear spaces over a soft scalar field , soft separated family, soft bounded set, soft bounded mapping , soft absolutely convex set and soft barrel set and their properties. Finally, we study another types of soft locally convex soft topological linear spaces called soft barrelled and soft bornological spaces. Moreover, we introduce some properties of this concepts.

Also, this study proves that:

Every soft nbhd of of a over contains a soft balanced nbhd of .

Every soft nbhd of of a  over is soft absorbing.

Every proper soft linear subspace of a over has a null soft interior.

Every soft closed and soft balanced nbhd of of a over , forms a soft local base at .

Every a over is soft Hausdorff and soft separated.

A which is determined by a family of soft seminorms on a over is which is soft separated.

Every soft normed space is , but need not to be a soft locally convex space.

A soft convex hull of a soft bounded set of a over (which is soft locally convex space ) is soft bounded.

Every soft nbhd of of a over is contained in a soft nbhd of which is soft barrel.

A over (which is soft locally convex and soft second category space or soft Baire space) is soft barreled space.

A over (which is soft locally convex and soft first countable space or soft metrizable space) is soft bornological space.

**Introduction**

The concept of soft set, coined by Molodtsov [15], in his seminar paper

"Soft set theory-first results " in 1999 has emerged as a fundamental and fresh idea exploring softness mathematically for first time. Soft is generalization of " Soft Topology " in classical mathematics, but it also has its own differentially marked characteristics.

It deepens the understanding of basic structures of classical new methods and significant results in this area. The art of defining soft norm one of the fundamental problems in soft mathematics. The application of the soft sets provided a natural framework for generalizing many concepts of topology which is called the soft topological space as initiated by [1], [4], [8], [11], [12], [16], [18] , [19] , [21] , [23] , [24], [30] , [31] and [32].

A lot of activity has been shown in soft set theory (see [5] , [14], [15], [17] and [25] .

The objective of this thesis is to analyze certain results in "soft topological linear space", where the soft point chosen that was previously defined in [26]. Some of his proofs were thoroughly revised. In addition, explain or more detailed descriptions and explanations were added to them. In 2014 [29], an idea of soft linear spaces and soft norm on a soft linear spaces are given and some of their properties are studied. Soft linears in soft linear spaces are introduced and their properties are studied.

Finally, we examine the properties of this soft normed space and present some investigations about soft continuous functional in the soft linear space, see [29].

In 2015, Chiney M. and Samanta S. [6], introduced a notion of a vector soft topology and studied some of its basic properties depending on the crisp point in soft set. After that, in 2016 [7], they also introduced a generalization of the concept of seminorm called soft seminorm on a soft linear space over a soft scalar field depending on soft element. But in our thesis terms are defined above depending on the concept of soft point. The thesis entitled **“ On Soft Topological Linear Spaces ”** consists of three Chapters. In this work, the main concentration is focused on defining different types of soft topological linear spaces and some soft topological linear spaces which are induced by the family of soft seminorms on a soft linear space over a soft scalar filed and which was defined by Intisar R. [9], it is a special type of soft topological linear spaces.

In the first chapter of the thesis preliminary definitions and results are briefly delineated. This chapter is divided into two sections, section one includes some basic algebraic operations on soft sets. We introduce equality of two soft sets, soft subsets, complement of soft sets, null soft set, absolute soft set, soft point and singleton soft set.

There are several basic properties which are not hold true in general and proved results with already defined operations on union, intersection and different soft sets, furthermore we introduced the notion of soft real sets, soft real numbers and we discuss their properties, see [25] and [28]. In section two some basic properties of soft topological spaces are studied. Chapter two, divided into two sections. In section one we introduced an important notions called soft convex, soft balanced, soft absorbing and soft symmetric set in a soft linear space over a soft scalar field and study some properties of them. Also, we introduce a notion of a linear soft topologies and studied some of its basic properties. In this section, some facts of the system of soft neighborhoods of the soft zero linear of a soft topological linear space are established. The concept of soft topological linear space is introduced in this section along with some basic properties of such spaces. Section two aims at studying some basic theorems which are needed in this chapter and the next one.

Chapter three contains three sections, in section one we study a soft linear spaces equipped with a soft topology generated by a family of soft seminorms on a soft scalar field are called soft topological linear spaces. It is possible to consider this soft topological linear space as the central theme in this thesis and some basic properties of this soft topological linear spaces were studied. In section two, we define a soft locally convex spaces in two equivalent ways, one by means of a soft local base at soft zero liear that consist of soft convex sets and one that requires soft seminorms. A soft locally convex space is a major topic and worthy of its own section. There is a soft topological linear spaces that are not soft locally convex space. It is difficult to find somewhat pathological examples in the soft analysis. Finally, in section three, we study a soft barreled and soft bornological spaces which considered the features of the soft locally convex space. Also, there is an equivalence to the definition of soft barrelled and soft bornological spaces. On the other hand, there is an example leads to is not soft barrelled space and soft bornological space.

**§ (1.1) Soft sets and its properties:**

In this section, we give some basic definitions of soft sets and their necessary operations.

**Definition (1.1.1) [15]:**

Let be a universe set and be a set of parameters , the power set of and . A pairis called soft set over with recpect to and is a mapping given by , .

**Remark (1.1.2**) **[15] , [18]:**

, may be arbitrary set , may be empty set.

The soft set can be represented by two ways:

• .

• By ordered pairs :.

denote to collection of all soft sets over a universe .

**Remark (1.1.3) [21]:**

It is clear that every set is soft set.

In sense , when , a soft set behaves similar to a set. In this case the soft set is same as the set , where .

**Definition (1.1.4) [26] , [25]:**

A soft over is called soft point and its denoted by:

, if exactly one , for some and for all .

A soft set for which is a singleton set , for all , is called singleton soft set.

**Remark (1.1.5) [26]:**

The fact that is a soft point of and will be denoted by , if .

For two soft points and , then if and only if or .

The collection of all soft points over a universe denotes by .

**Definition (1.1.6) [14]:**

A soft set over is called null soft set , denoted by , if for all , we have .

A soft set over is called absolute soft set and its denoted by , if for all , we have .

**Definition (1.1.7) [14] , [26]:**

The soft complement of a soft set over a universe is denoted by and it is defined by , where is a mapping given by , , for all .

i.e. .

It is clear that:

;.

If , then i.e. .

**Definition (1.1.8) [14] , [5]:**

Let and be two soft sets over , we say that is a soft subset of and denoted by , if:

• .

• , .

Also, we say that and are soft equal is denoted by , if and .

It is clear that:

is a soft subset of any soft set .

Any soft set is a soft subset of .

**Difinition (1.1.9) [14] , [16]:**

The intersection of two soft sets and over a universe is the soft set, where and for all , write such that .

The union of two soft sets and over is the soft set , where and we write such that .

It is clear that every soft set can be expressed as a union of all soft points belong to it. i.e. [26].

The difference of two soft sets and over , denoted by is defined as , for all .

**Definition (1.1.10) [25] , [7]:**

Let be the set of all real numbers , be the collection of all non-empty bounded subset of and be a set of parameters.

Then a mapping is called a soft real set .

If a soft real set is a singleton soft set it will be called a soft real number . We denote soft real numbers by , , where as will denote particular type of soft real numbers such that for all . The set of all soft real numbers it is denoted by and the set of all non-negative soft real numbers by

**Remark (1.1.11) [28]:**

Let . Then the soft addition of and soft scaler multiplication of and are defined by:

, for all .

, for all .

, for all .

= , and for all .

**Definition (1.1.12) [28]:**

The inverse of any soft real number , denoted by and defined as , for all .

**Theorem (1.1.13) [28]:**

The set of all soft real numbers forms a field and called soft real field.

**Definition (1.1.14) [4] , [23]:**

For two soft sets and over and respectively , the cartisian product of and is defined as:

• , such that and , , for all .

• , denotes the absolute soft set over with the parameter set.

A relation from to is a soft subset of as follows () .

In other words a relation from to is of the form , where and for all .

**Remark (1.1.15) [26]:**

For two soft real numbers , , then:

if ; if , .

if ;if , .

**Remark (1.1.16) :**

For two soft real numbers , , then:

If , then ; for all .

If , then ; for all .

**Definition (1.1.17) [18]:**

Let be a soft set in . Then is called soft bounded above , if there is a soft number so that any is soft less than or soft equal to . The soft number is called a soft upper bound for the soft set .

**Definition (1.1.18) [18]:**

Let be a soft set in the soft number is called the soft least upper bound (or soft supremum) of the soft set if:

is a soft upper bound if any satisfies .

is the smallest soft upper bound i.e. If is any other soft upper bound of the soft set then .

**Definition (1.1.19) [18]:**

Let be a soft set in the soft set is called soft bounded below, if there is a soft number so that any is soft bigger than or soft equal to , the soft number is called a soft lower bound for the soft set .

**Definition (1.1.20) [18]:**

Let be a soft set in the soft number is called the soft greatest lower bound (or soft inﬁmum) of the soft set if :

is a soft lower bound , if any satisfies .

is the greatest soft lower bound i.e. If is any other soft lower bound of the soft set then .

**Remark (1.1.21) [18].**

When the soft supremum of a soft set is a soft number that soft belong to then it is also called soft maximum .

When the soft infimum of a soft set is a soft number that soft belong to then it is also called soft minimum .

**Definition (1.1.22) [25]:**

For any be a soft real set and for any positive integer , we define by , for each . Obviously is soft real set.

For any soft real set of , we define the sequere root of by and denoted by is defined by , for each .

For any soft real set defined By:

.

Then for any soft real set , implies that .

**Definition (1.1.23) [17] , [25]:**

Let be a family of all bounded intervals of real numbers, then a mapping is known as a soft open interval. Each soft open interval may be expressed as an ordered pair of soft real numbers. That is if is defined by , , then the soft open interval may be expressed as an ordered pair of soft real numbers , where , , .

Similarty the mapping is called soft closed interval , if it is defined by , .

**Definition (1.1.24) [18]:**

A soft real set is called soft bounded , if there are two soft real numbers and such that for any . A soft set which is not soft bounded is called soft unbounded.

**§ (1.2) Soft topological space and some results:**

In this section some notations are introduced in soft topological space such as soft Hausdorff space , soft compact spaces , soft continuous mapping. Also, we introduce some results about soft convergence of soft net and investigate the relations between these concepts.

**Definition (1.2.1) [16]:**

Let be the collection of soft sets over , then is said to be a soft topology on , if :

.

If for all , then .

If , then .

The triple is called soft topological space (for short ) , the members of is called soft open sets . The complement of the members of is called soft closed set .

Indeed, the difference between and arise from this facts in remark (1.1.3.i) and (1.1.3.ii), [**21**].

**Definition (1.2.2) [19]:**

Let be a and be a soft set over , then:

The soft closure of denoted by is the intersection of all soft closed sets which is containing .

A soft point is called a soft interior point of , if there is a soft open set such that . The soft set which contains all soft interior points is called soft interior set and it is denoted by .

A soft set is called a soft neighborhood (for short soft nbhd ) of a soft point , if there is a soft open set such that . The soft nbhd system of , denoted by is the collection of all soft nbhd of .

**Proposition (1.2.3) [19] , [16]:**

Let be a and , be two soft sets over a universe . Then:

.

If , .

.

**.**

A soft set is soft open if and only if .

**Proposition (1.2.4) [19]:**

Let be a , and be two soft sets over a universe . Then:

.

If , .

.

.

A soft set is soft closed if and only if .

**Proposition (1.2.5):**

Let be a and , be two soft sets over a universe and . Then:

If and , then.

If , then .

If , then .

If , , then .

**Theorem (1.2.6) [8] , [19] , [30]:**

Let be a and , be two soft set over and . Then:

if and only if , for every soft open nbhd of .

A soft set is soft open if and only if is soft nbhd for all its soft points.

A soft set is soft open if and only if for every , there is a soft open set in such that .

**Definition (1.2.7) [24] , [11]:**

Let be a . Then a soft set is called:

Soft dense if and only if .

Soft nowhere dense if and only if .

**Definition (1.2.8) [16] , [12] :**

Let be a soft set over universe and be a non-empty subset of . Then:

The soft set over is denoted by and defined as follows , .

Let be a and be a non-empty subset of . Then the collection is called a soft relative topology on. Hence , is called a soft topological subspace of .

Let be a soft topological subspace of a , if is a soft set over , then .

**Definition (1.2.9) [1] , [31]:**

Let be a . Then:

A sub-collection of is called a soft base , if every member of can be expressed as a union of some members of .

A sub-collection of is called soft nbhd base at , if for each soft nbhd of , there is such that:

.

Let and be two s .

Let be a basise of soft nbhds and be a collection of all arbitrary union of elements of . Then is a soft topology on and is called soft product topological space. The form of any soft point in is .

**Definition (1.2.10) [18]:**

Let be a and , be two soft sets over . Then:

, are said to be soft disjoint , if .

A non-null soft ( which is soft disjoint )sets , are said to be soft separated of , if , .

**Theorem (1.2.11) [18]:**

Two soft closed (or soft open) subsets and of a are soft separated if and only if they are soft disjoint.

**Definition (1.2.12) [32]:**

A is soft compact if for each soft open cover of has finite subcover.

Also , the soft set of a is soft compact set with respect to the soft relative topology on , if for each soft open cover of , there is a finite subcover.

**Definition (1.2.13) [26]:**

A soft set in a is called finite (countable) soft set , if finite (countable) for all .

**Theorem (1.2.14) [18]:**

Every soft closed set of a soft compact space is soft compact.

The intersection of soft closed with soft compact is soft compact. Every finite soft set is soft compact.

(**v**) In , if the sets and are finite , then is soft compact.

If the soft sets in a are finite ( are finite mapping), then is soft compact space.

**Definition (1.2.15) [22] , [10]:**

A is called :

Soft -space , if for all , with , there are two soft open sets , in such that:

and .

Soft Hausdorff space (Soft -space) , if for all , with , there are two soft disjoint (soft open sets) and in such that and .

**Theorem (1.2.16) [22]:**

A soft Hausdorff space is soft -space.

**Theorem (1.2.17) [22] , [30]:**

Let be a. Then:

is soft if and only if soft closed for all .

Every soft compact of a soft Hausdorff space is soft closed.

**Definition (1.2.18) [31] , [11]:**

A is called:

soft first countable space , if each soft point over has a countable soft nbhd base.

Soft Baire space , if the intersection of each countable family of a soft open which is soft dense sets is soft dense.

Soft second category , if for any soft set over cannot be represented as a countable union of soft nowhere dense.

**Theorem (1.2.19) [31] , [11]:**

Let be a . Then:

If is a soft Baire space and be any countable soft closed family of soft covering of , then at least one of must contains a soft open set , that is for at least one . is soft first countable space at if and only if there is a countable soft open nbhd base at such that :

for all .

**Example (1.2.20) :**

is soft first countable space.

**Definition (1.2.21) [4] , [30]:**

Let and be soft sets over a universe sets respectively. A soft relation

is a soft set over, where such that: , for all , is called a soft mapping from to and denoted by , if for all , there is only one such that.

**Definition (1.2.22) [30]:**

Let be a soft mapping . Then :

The soft image of under is the soft set denoted by of the form:

.

• The image of any null soft set under the soft mapping is null soft set.

The soft inverse image of under is the soft set denoted by of the form:

.

The inverse image of any null soft set under the soft mapping is null soft set.

**Proposition (1.2.23) [30]:**

Let be two soft sets over with , and let be a soft mapping. Then the following hold :

If , then ;

also if , then .

; and

and .

and

.

**Definition (1.2.24) [18]:**

A soft mapping is called:

Soft injective , if for all , with , implies that .

Soft surjective , if for all , there is such that .

Soft bijective , if it is soft injective and soft surjective.

**Propositions** **(1.2.25) [18].**

Let be two soft sets over with , , let be a soft mapping. Then:

, if is soft injective.

, if is soft injective.

, if is soft surjective.

**Deﬁnition (1.2.26) :**

Let be a soft mapping , then is called soft continuous at if for each soft nbhd of , in , there is a soft nbhd of in such that . If is soft continuous mapping at , then it is called soft continuous.

**Theorem (1.2.27) [18]:**

Let be a soft mapping. Then the following

statements are equivalent:

is soft continuous .

is soft open set over for all soft open set over . is soft closed over for all soft closed set over . For all soft set over , .

**Definition (1.2.28) [18]:**

Let be a soft mapping , then is called:

Soft open (Soft closed), if is soft open (soft closed) set over for all soft open (soft closed) set over .

Soft homeomorphism , if is soft bijective, soft continuous and are soft continuous mapping.

**Theorem (1.2.29) [18]:**

Let be a soft bijective and soft continuous mapping . Then the following statements are equivalent:

is soft continuous.

is soft open.

is soft closed.

**Theorem (1.2.30):**

Let be a soft mapping . Then:

is soft open if and only if for each soft set of , then .

is soft closed if and only if for each soft set of , then .

**Theorem (1.2.31):**

Let be a soft bijective mapping . Then is soft continuous if and only if for every soft set of .

**Theorem (1.2.32) [30]:**

A soft continuous image of a soft compact set is soft compact.

**Definition (1.2.33) [10]:**

A soft sequence in is a soft mapping , where is the set of all natural numbers . The soft point denoted by , where such that be a sequence in and a sequence in a parameter set .

A soft sequence is called soft converge to in a and write , if for every soft nbhd of , there is such that for all .

**Definition (1.2.34) [23]:**

A soft net in is a soft mapping , where is directed set. A soft point denoted byand , where is a net in and be a net in a parameter set . Every soft sequence is soft net.

**Definition (1.2.35) [23]:**

Let be a soft net in a and be a soft set over a universe and . Then is called:

Eventually in , if with .

Frequently in , if , , then .

Soft convergence to if is eventually in each soft nbhd of (written ) . The soft point is called a limit soft point of .

Have as a soft cluster soft point if is frequently in each soft nbhd of (written ) .

**Remark (1.2.36) [23]:**

Let be a soft mapping. Then:

If is a soft net in , then is a soft net in .

If is soft surjective and be a soft net in , there is a soft net insuch that ,

Let be a soft point over . Then is soft continuous at if and only if , whenever a soft net in and , then in.

**Theorem (1.2.37) [23]:**

Let be a and be a soft set over a universe. Then if and only if there is a soft net in with .

Let be a , then is soft Hausdorff space if and only if every soft net in has a unique soft limite point.

**Theorem (1.2.38) [23]:**

Let and be two s . Then:

be a soft net in , with be a soft net in and be a soft net in . A soft net in is soft converge to a soft point if and only if and in and respectively.

**Definition (1.2.39) [26]:**

A soft mapping is said to be a soft meric on a soft set , if satisfaies the following conditions:

.

if and only if i.e. .

, .

, .

**•** A soft set with the soft metric is called a soft metric space and denoted by .

**•** defined by for all, . , then is soft metric space .

**Remark (1.2.40) [26]:**

A soft sequence in called soft converge to in , if as .

This means that for every , there is a natural number , such that , whenever .

**Theorem (1.2.41):**

Let be a soft mapping and be a soft first countable. Then is soft continuous at if and only if , whenever a soft sequence in and , then in.

**Definition (1.2.42) [26]:**

A soft set is called a soft open ball with center and a radius .

**:**

**Definition (1.2.43):**

A is called a soft metrizable , if generated by , where .

**Theorem (1.2.44):**

A soft metric space is soft first countable.

**Proof**:

forms countable a soft nbhd base at

**Theorem (1.2.45) [26]:**

Every soft metric space is soft Hausdorff.

**Definition (1.2.46) [27] , [26]:**

Let be a soft metric space and be a non-null soft subset of . Then is called soft bounded if there is and such that .

A soft sequence in called soft bounded in , if there is such that for all .

**Remark (1.2.47):**

The soft compact set of a soft metric space is soft closed and soft bounded.

In( the soft closed and soft bounded is soft compact.

A soft sequence in is soft converge to in, if , whenever .

**§ (2.1) Linear soft topology:**

In this section, by using the concept of soft point we introduced the soft linear space in a new point of view and investigate the properties of the soft topological linear space.

Let be a linear space over a scalar field () and the parameter set be the real number set .

**Definition (2.1.1) [29]:**

Let be a soft set over a universe rameter set.. The soft set is called the soft linear and denoted by a universe, if there is exactly one such that for some and, for all . The set of all soft linears of will be denoted by .

**Proposition (2.1.2) [29]:**

The set is called soft linear space (for short ) over according to the following operations:

; for all .

; for all and for all .

It is clear that:

• If be a zero linear and , then is a soft zero linear in .

• be the inverse of a soft linear .

**Remark (2.1.3):**

Let and be two soft sets of a over . Then:

If , then .

, for all .

**Definition (2.1.4) [29]:**

Let be a overand be a soft subset . If is a soft linear space , then is called soft linear subspace of and denoted by .

**Theorem (2.1.5):**

Let be a soft subset of a over . Then is soft linear subspace of if and only if:

and for all .

The soft intersection of any soft linears subspaces of is soft linear subspace of .

**Proof:**

If is a soft subspace of , the condition is holds.

Conversely, if and for all . Now, let and .

Then and .

This completes the proof.

It is clear.

**Example (2.1.6):**

Let

, with . Then is soft linear space over under the soft addition and soft scalar multiplication on :

, for all .

, for all and .

**Definition (2.1.7) [9] , [7]:**

A soft set of over is called:

Soft convex if , .

Soft balanced if , with.

Soft absorbing if for all there is such that . i.e. and is soft absorb In other words, is soft absorbs if there is such that for all .

Soft symmetric if

**Proposition (2.1.8) [9] , [7]:**

Letand are two soft sets of a over. Then:

If and is two soft convex sets , then , and are soft convex set ; .

If and is a soft balanced set , then is soft balanced set.

If is a soft balanced set and , and either or , then .

is a soft convex (soft balanced) set if and only if for all , is convex (balanced) set of .

**Remark (2.1.9):**

Let and are two 's over and is a soft linear mapping. Then:

If is a soft convex ( soft balanced and soft linear subspace ) of , then is a soft convex (soft balanced and soft linear subspace ) of .

If is a soft surjective , then the condition is valid , when be a soft absorbing set.

If is a soft convex (soft balanced and soft linear subspace and soft absorbing ) of, then is a soft convex (soft balanced and soft linear subspace and soft absorbing ) of .

**Definition (2.1.10):**

Let be a linear space over, be a parameter set . Then with the soft topology is said to be a linear soft topology (for short). A triple is said to be a soft tological linear space (for short) over, if provided with having the following properties:

For , then is soft closed set.

The soft mappings:

; (soft addition).

;

(soft scalar multiplication).

are soft continuous mappings , for all and.

In the sense that , for any soft nbhd of , there are soft nbhds and of and respectively such that :

.

And for any soft nbhd of , there are soft nbhds of of and of of such that:

.

By definition of , there is such that is a soft open ball centred at .

Let be a soft set contains of all soft real numbers which is satifay the condition:for all .

Then .

Thus is soft continuous if and only if there is such that , for all with and for all .

**Example (2.1.11):**

The with the soft indiscrete topology is not ( since its not satifaying from definition (2.1.10)).

The with the soft discrete topology is not , as follows:

If is a , where is a soft discrete topology unless .

Assume by a contradiction that it is a , there is in . A soft sequence  in , since the soft scaler multiplication is soft continuous , from definition (2.1.10.ii) , thus for every soft nbhd of of .

Then there is such that for all . If we take is a soft open nbhd of of . Hence , implies that , contradiction.

**Theorem (2.1.12):**

Let be a over. For and . Then:

The soft translation ; , ;

The soft multiplication ; , ; are soft homeomrphism for all .

**Proof:**

It is clearl that, and are soft bijective mapping on , thus inverse exists. In fact and . Consider the soft mapping take a soft nbhd of . Then by soft continuouty of soft addition at , there are soft nbhds and of and respectively such that:

.

Thus in particular, , i.e. , . So, is soft continuous .

Similarly , is soft contiuous . Hence is soft homeomorphism.

By the soft continuty of soft scalar multiplication at , then for any soft nbhd of , there is and soft nbhd of such that for all with and for all .

In particular, i.e. , thus is soft contionuous. Since, it follows that is also soft continuous. So is soft homeomorphism.

**Corollary (2.1.13):**

Let be a soft set of a over . For , we have:

.

.

**Proof:**

By using theorems (2.1.12.ii)) , (1.2.30.i) and (1.2.31).

By using theorems (2.1.12.ii)) , (1.2.27) and (1.2.30.ii).

**Theorem (2.1.14):**

Let be a over. For and . Then:

is a soft open if and only if is a soft open.

.

is a soft open if and only if is a soft open for any .

A soft set is soft openif and only if for all , there a soft open nbhd such that:.

**Proof:**

Proof and we get them directly from theorem (2.1.12.i) and (2.1.12.ii) respectively .

Suppose that be a soft open set of . Then from theorem (1.2.6.iii) , and , this completes the proof.

Conversely, directly deduce from a theorem (1.2.6.ii).

**Definition (2.1.15) [31]:**

A collection of soft nbhds of of a over is called a soft nbhd base (or soft local base) at , if for any soft nbhd of , there is such that .

**Theorem (2.1.16) :**

In a over , every soft nbhd of contains a soft balanced nbhd of .

**Proof:**

Let be a soft nbhd of of . By using soft continuity of soft scalar multiplication at , for every soft nbhd of , there is a soft nbhd of and such that for all and for all . Then , for all .

Let . Then , is soft nbhd of .

Let . Then . Thus. So , is soft balanced nbhd of such that .

**Remark (2.1.17):**

If , then .

In a over, every soft nbhd of contains a soft open , soft balanced nbhd of .

**Theorem (2.1.18) :**

In a over , every soft convex nbhd of contains soft convex , soft balanced nbhd of .

**Proof:**

Let be a soft convex nbhd of and let .

From proposition (2.1.8.i), we have is soft convex , this implies that is soft convex.

It is clear that is a soft nbhd of and . To show that is soft balanced . Let . If , then.

For with , then can be expressed as , where and .

Since , . Now , let . Since be a soft convex set containing , then there is such that:

.

Then . Thus . So is soft balanced set.

**Theorem (2.1.19) :**

In a over every soft nbhd of is soft absorbing.

**Proof:**

Let and be a soft nbhd of and by soft continuity of soft scalar multiplication, there is and soft nbhd of such that for all with and for all . i.e. .

Put , then . This implies that and . So , hence with .

i.e.. Therefore is soft absorbing set.

**Theorem (2.1.20):**

In a over every soft nbhd of , there is a soft symmetric soft nbhd of such that .

**Proof:**

Let be a soft nbhd of of . Since by using a soft continuity of soft addition of , it follows that there are soft nbhds , of such that . Let . Then is soft nbhd of , also is soft nbhd of .

Put , since , then is soft symmetric nbhd of .

So.

**Theorem (2.1.21):**

In a over , every non-null soft compact and soft closed sets , respectively with , there is a soft open nbhd of such that:

.

**Proof:**

Since , the there is . Since be a soft closed and there is a soft open nbhd of of such that …(\*) . Then from theorem (2.1.20) , there a soft symmetric nbhd of such that . Using theorem (2.1.20) again, there is a soft symmetric nbhd of such that . Since , then:

. Now, from (\*) above, we have.

By soft translation, we have . Since be a soft compact, then there are such that , where be a soft open nbhd of for all . Put , then is soft open nbhd of .

.

Since for all and for all .

Then , this implies that:

.

**Theorem (2.1.22) :**

Let be a over. Then:

For any soft sets and the following is true:

For every soft set in , then:

**Proof:**

Let and . By using theorem (1.2.37.i) , there are soft nets and , such that and .

By soft continuity of soft addition , it follows that . Hence , which proves .

Suppose that , then for every , we have :

.

i.e. .

Thus .

Conversely, suppose that . Then there is such that .

i.e. , hence .

**Theorem (2.1.23) :**

For any of a over , there is a soft nbhd of such that .

**Proof:**

Let is a soft compact set and is a soft open nbhd of of with and .

By using theorem (2.1.21), there is a soft open nbhd of such that .

It follows that .

By the definition (2.1.15) , there is a soft nbhd of such that:

.

Since is soft closed ,

.

**Definition (2.1.24) [2][3]:**

Let be a group and be a set of parameterse and then is called soft group over , if is subgroup of for all .

Let be a soft group over and , then is called soft subgroup of and written as , if:

• .

• is a subgroup of for all .

Let be a soft group over and . Let . Then the soft set defined as ( for all ) is called soft right coset of in generated by . Similarly, the soft set is called a soft left coset of in generated by .

**Theorem (2.1.25) [3]:**

Let be a soft group over and . Then:

forms a partion of over .

**Remark (2.1.26):**

Let be a soft group over and . Then:

forms a partion of over . i.e. .

**Definition (2.1.27) [20]:**

Let be a soft group over a universe and be a on. Then is called a soft topological soft group ( for short ) on , if for all, is a topological group on where is the relative topology on induced from as in [19].

**Theorem (2.1.28):**

Every is .

**Proof:** Clear.

**Theorem (2.1.29):**

Every soft convex and soft balanced subset of a over is soft subgroup.

**Proof:**

By using theorem (2.1.8.iv) and definition (2.1.24.ii).

**Theorem (2.1.30):**

Let be a soft open subgroup of a. Then soft closed.

**Proof:**

We have that equales the union of all soft left cosets of .

i.e. . Each left soft coset is soft open set in , and since:

, we see that is soft closed set in .

**§ (2.2) Theories related to :**

Throughout this section, we gave some core results which were not mentioned in the sources adopted in this work and its importance in next chapter.

**Theorem (2.2.1):**

Let be a soft convex subset of a over . If

and , then , for all .

**Proof:**

Let , we have to show that . By soft

translation if necessary, we can arrange that:

.

Now, with . Clear that is soft open nbhd of. Since , then , and there is , with w , ranslation if necessary and .

Clearl that , put , then:

……(\*) , .

Define a soft mapping as follows:

for all is soft homeomorphism and , from (\*).

Then is soft open nbhd of . Since and implies that (because be a soft convex set ) , thus and hence: .

**Theorem (2.2.2):**

Let be a soft subset of a over .

If is a soft convex set , then is soft convex.

If is a soft balanced set and , then is soft balanced.

**Proof:**

Let and , since , by using theorem (2.2.1) , implies that:

.

Thus is soft convex.

Assume that be a soft balanced and , thus:

and

This implies that is soft balanced.

Now, if , since . From hypothesis above

(), we have is soft balanced.

**Theorem (2.2.3):**

Let be a over. Then:

If is a soft convex set , then is soft convex .

If is a soft balanced set then is soft balanced.

**Proof:**

Since be a soft convex , then:

, for all .

Then (from soft multiplication), we have:

.

This implies that is soft convex.

Let such that . Since be a soft balanced , then:

.

Since (by using soft multiplication), thus is soft balanced.

**Proposition (2.2.4):**

Let be a over. Then:

If is a soft absorbing set then , is soft absorbing. If is a soft absorbing set , then is soft absorbing.

**Proof:** Clear.

**Theorem (2.2.5):**

Every proper soft linear subspace of a over has a null soft interior.

The only soft open linear subspace of a over is itself.

**Proof:**

Assume that is a soft linear proper subspace of and .

Thus there is a soft open nbhd of of and there is a soft point such that:

.

By using a soft translation , then , hence:

.

Thus is a soft nbhd of of. Since every soft nbhd of is soft absorbing , i.e.. This is a contradiction.

Assume that is a soft open linear subspace of . Then . Thus is a soft nbhd of . By using theorem (2.1.19), is soft absorbing. i.e. .

But , this implies that .

**Theorem (2.2.6):**

If is a soft convex and soft balanced set of over , then and , for all and for all where.

**Proof:** Clear.

**Theorem (2.2.7):**

Let  be a over , be a soft convex and soft balanced subset of and. Then is soft open and soft closed.

**Proof:**

Let be a non-null soft open subset of and . By using theorem (2.2.6) , clearl that is soft open set . From theorem (1.2.6.iii) , , is soft open set. We conclude that by using theorems (2.1.28) , (2.1.29) and (2.1.30) , is soft closed set.

**Corollary (2.2.8):**

Let be a over , be a soft convex and soft balanced nbhd of of . Then is soft open and soft closed.

**Proof:** Clear.

**Theorem (2.2.9):**

Let be a over , is a soft convex and soft balanced set of . Then the following statements are equivalent:

.

and .

**Proof:**

Since , there is . Let .

Since ince be a soft be a soft convex and soft balanced , we have:

and ; such that .

It is clear that is a soft open .

Since . By using theorems (2.1.14.i) and (2.1.14.ii), we have: is soft open set.

Since and is largest soft open set in .

Then, thus:

*.*

Since , we have .

**Theorem (2.2.10):**

Let be soft set of a over and. Then if and only if there is a soft net in such that .

**Proof:**

Let and be a soft open nbhd of of , then by using theorem (2.1.14.i) , we have is a soft open nbhd of .

Since , then by using theorem (1.2.6.i) , implies that:

..…. (\*)

Now, from (\*), there is a soft point of . It is clear that is directed set.

Define by for all Hence is a soft net in .

To prove . Let . Then for all , , all e r that implies that.

Conversely, suppose, there is a soft net in such that To prove ) let be a soft open nbhd of Since there is such that for all But for all ., so that for all

softopen nbhd of .

Thus by using theorem (1.2.6.i) ,.

**Theorem (2.2.11):**

A soft closed , soft balanced nbhds of forms a soft local base at

of a over .

**Proof:**

Let be a soft nbhd of of , by using theorem (2.1.20) , there is a soft symmetric nbhd of such that:

.

Also, by using theorem (2.1.16) , there is a soft balanced nbhd of such that , thus . Then by using theorem (2.1.22.ii), implies that: , where is a soft nbhd of . By using theorem (2.2.3 .ii) is a soft balanced.

**Theorem (2.2.12):**

Every over is soft Hausdorff space.

**Proof:**

Let with . Both soft sets and are soft closed sets (from definition (2.1.10)) and theorem (1.2.14.iii) are soft compact sets . Then is soft Hausdorff space by theorem (2.1.21).

**§(3.1) Soft topologies induced by a families of soft seminorms:**

In this section we introduce the important concept which is called soft seminorm on a over and we give some results on this concept. Also, we study a new type of a soft topology by using a family of soft seminorms denoted by a soft topology which is induced by a family of soft seminorms.

**Definition (3.1.1) [29]:**

A soft mapping is called soft norm on a over , if satisfies the following conditions:

.

if and only if , .

, and .

, .

A soft linear space over with the soft norm is called a soft normed space and it is denoted by. It is clear that every soft normed space is soft metric space.

**Remark (3.1.2)[29]:**

Let be a normed space. Then for all , is soft norm.

**Definition (3.1.3):**

Let be a soft normed linear space and . We define the followings:

is called a soft open ball with center and a radius .

is called a soft closed ball.

**Remark (3.1.4):**

A soft sequence in a soft normed space is soft convergent to () if and only if , as .

A soft open ball of a soft normed space is soft convex , soft balanced and soft absorbing.

**Theorem (3.1.5):**

Every soft normed space on over is .

**Proof:**

Let be a soft normed space and be a soft convergent sequence in . This means that and in , by remark (3.1.4.i) , we have and , as . Now , to show that :

, hence soft convergence of and , we have the desired.

Also, let in. Thus and in and respectively, so and as . To show that in , we have:

, it is clear that , as .

It clear that is soft Hausdorff space , then is soft closed set for all . Thus , we have completed the proof.

**Definition (3.1.6):**

Let and be two 's over .

A soft mapping is called a soft linear, if it is having the following:

; , .

**Theorem (3.1.7):**

Let and be two 's over . If be a soft linear, then:

.

.

**Proof:** Is clear.

**Definition (3.1.8) [9]:**

A soft mapping is called soft seminorm on over , if it is having the following:

for all ;

for alland for all .

**Remark (3.1.9) [9]**

Let be a soft seminorm on a over . Then:

If , then .

, for all .

; for all .

**Theorem (3.1.10) [9]:**

If be a soft seminorm on a over, then:

is soft linear subspace of .

A soft open unit semiball , is soft convex , soft balanced and soft absorbing.

**Theorem (3.1.11):**

A soft closed unit semiball , is soft convex, soft balanced and soft absorbing.

**Proof:** Is clear.

**Theorem (3.1.12):**

Let be a family of soft seminorms on a over . For , let denote the collection of all soft open semiball of.

Let be the collection of all soft sets of consisting of together

with all those soft sets such that for any , there is such that . Then is a soft topology on compatible with .

**Proof:**

Evidently , and it is clear that the union of any family soft sets of is also a soft member of . We showthat:

if, then. If , there is no more to be done and so suppose that . Then , , and so there are , such that:

and.

Put, where , clearly that:

.

It follows that is soft topology on.

**Theorem (3.1.13):**

A soft topology on is linear soft topology determined by a family of soft seminorms on a over , then is a .

**Proof:**

Suppose in. We want to show that .

Let be a soft basic soft nbhd of. Since , there is such that:

whenever . For and , this implies that:

; and so . Thus we get the required.

Now, suppose that in.

Let be a soft basic nbhd of . For all and , there is such that:

.

Hence for all and, we have:

; if we choose such that and then such that . So that , whenever . We conclude that .

Now , to show that is soft closed set for all . Sufficient to show that is soft open. From theorem (3.1.12) , for all and ( i.e. ). Then , , let .

Thus , i.e. , this implies that , and is soft open set.

**Remark (3.1.14):**

Let be a family of soft seminorms on a over. We consider the soft open semiball:

; , .

; , .

Notice that:

.

.

and are soft convex, soft balanced and soft absorbing set.

**Theorem (3.1.15):**

In a , then:

Every soft open semiball is soft open set.

Every soft closed semiball is soft closed set.

**Proof:**

We prove that is soft open set.

Let , then , so . Put ; .

We must prove that .

Let . So ,

and then . This implies that:

Hence and its soft open set.

We prove that is soft closed set. Enough to prove that is soft open set.

Since .

Then . Let , then , and hence .

Put .

We must prove that.

Let. Then and so .

This implies that:

But

.

Thus we have completed the proof.

**Remark (3.1.16):**

and are non-null soft sets.

.

**Theorem (3.1.17):**

Let be a family of soft seminorms on a over . For a collection , constitute of a soft local base at for compatible with.

**Proof**: Clear.

**Definition (3.1.18):**

A family of soft seminorms on a over is said to be soft separated, if for each there is such that .

**Theorem (3.1.19):**

In a a family of soft seminorms on a over is soft separating family.

**Proof:**

For with , there is a soft open set such that and (from soft Hausdorffness of ). From theorem (3.1.12) there is soft nbhd.

It is clear that .

This implies that there is at least and , and such that and so certainly , we see that is soft separating family.

**Theorem (3.1.20):**

In a the soft closure of is the soft intersection of the soft kernels of all soft seminorms of the family .

.

**Proof:**

Since for all , then .

We have .

Now , to show that .

Suppose that , then for all . Since be a soft separated , theorem (3.1.19), then from definition (3.1.18) , there is such that . This implies that , which is a contradiction , then.

**Corollary (3.1.21):**

In a , the soft set is the soft linear subspace of .

**Proof:**

By using theorems (3.1.20) , (3.1.10.i) and (2.1.5.ii).

**Theorem (3.1.22):**

A soft net in a is soft converging to in a if and only if , for each where be a family of soft continuous soft seminorm on .

**Proof:**

Assume that in , by soft continuity of , then we have .

Conversely, suppose that the condition is holds. Let and. There is such that , whenever for all . Hence , whenever , for all be a soft nbhd of . It follows the result is obtain.

**Remark (3.1.23):**

The soft converge of a soft net to is not necessarily implied by a soft convergence of in for all .

Indeed , for all and any , , as , but it is not true that if soft separated.

**Definition (3.1.24):**

Let be a soft subset of a over . Then is said to be a soft bounded, if for every soft nbhd of , there is such that .

In other words :

The soft set in a over is soft bounded, if it is soft absorbed by every soft nbhd of of.

**Proposition (3.1.25):**

Let and be two soft sets in a over and . Then:

A soft set is soft bounded .

If is a soft bounded set and , then is soft bounded.

If is a soft bounded set , then is also for all .

If is a soft bounded set , then is soft bounded.

**Proof:**

Let be a soft nbhd of of . Then is soft absorbing set , i.e. there is such that , that is is soft bounded.

Let be a soft nbhd of of . Since is a soft bounded set , then there is such that for every soft nbhd of . Since , we have . We easily get that is soft bounded.

If , this follows immediately from . If , and be a soft nbhd of of . By using theorem (2.1.16) , then there is a soft balanced nbhd of such that:

.

Since is a soft bounded set , there is such that:

.

Put , since is a soft balanced, we have:

.

Implies that .

Since , so . i.e. , then , proving is soft bounded.

Let is a soft nbhd of in . Then there is a soft nbhd of , and by using theorem (2.1.23) , we get:

.

Since is a soft bounded , there is such that . By using corollary (2.1.13.ii), we have:

.

We end up we have reached that is soft bounded.

**Theorem (3.1.26):**

Let , be two soft bounded sets in a over . Then:

is soft bounded.

is soft bounded.

is soft bounded.

**Proof:**

Produced directly from proposition (3.1.25.ii).

let is a soft nbhd of of , then by using theorem (2.1.16) there is a soft balanced nbhd of such that . Since and be two soft bounded, then there are such that and . Take. Since be a soft balanced , from proposition (2.1.8.iii), . And so is soft bounded.

let is a soft nbhd of of , then by using theorem (2.1.20) , there is a soft symmetric nbhd of such that:

.

Then there a soft balanced nbhd of such that . Since and is two soft bounded sets , then there are such that and .

Take . Since is a soft balanced , then:

.

Thus is soft bounded.

**Theorem (3.1.27):**

Every soft compact set of a over is soft bounded.

**Proof:**

Suppose that is a soft compact set of and is a soft nbhd of from remark(2.1.17ii) there a soft balanced , soft open nbhd of of for which . Then for all , , since in , so for all . That

soft compactness, there are such that:

.

If , This implies that . Thus is soft bounded set.

**Theorem (3.1.28):**

Let be a over . For every. The soft set is not soft bounded.

**Proof:**

Since . Then by soft Hausdorffness, there is a soft open nbhd of such that . Hence for all, this means that .

**Remark (3.1.29):**

Only soft bounded soft linear subspace of a over is .

**Definition (3.1.30):**

Aoverhas a soft Hine Borel property, if every soft closed and soft bounded set is a soft compact set.

**Definition (3.1.31):**

For two's and over .

A soft linear mapping is said to be a soft bounded , if is soft bounded set in for all soft bounded set of .

**Proposition (3.1.32):**

Let ,be two 's over and is a soft linear mapping . If is a soft continuous , then it is soft bounded.

**Proof:**

Suppose that is soft continuous. Let is a soft bounded set of . Let be a soft nbhd of of . Since , then there is a soft nbhd of of with . Since is soft bounded, there is , such that . By soft linearity , for all , we have , hence is soft bounded , which implies that is soft bounded.

**Example (3.1.33):**

For any soft identity linear mapping from is soft bounded.

A soft linear mapping , defined by , for every is soft bounded.

A soft linear mapping, defined by , for all is soft bounded.

**3.2) Soft locally convex spaces:)§**

In applications, it is often useful to define a soft locally convex space by means of a family of soft seminorms on a over . In this section we will investigate the relation between soft locally convex soft topological linear space and soft seminorms.

**Definition (3.2.1):**

Let be a over . A soft linear of is a soft linear combination of the soft linears of , if can be expressed as:

for some .

**Definition (3.2.2)[13]:**

The soft convex hull of a soft set of , which is denoted by is the smallest soft convex set which is containing .

In other words:

is the intersection of all soft convex sets containing .

**Theorem (3.2.3):**

Let be a soft set of a over . Then:

*.*

**Proposition (3.2.4):**

Let and be two soft sets of a over . Then:

.

is soft convex.

is soft convex if and only if .

If , then .

.

The proof of this proposition is follows immediately from the definition (3.2.2).

**Definition (3.2.5):**

Let be a over . The soft closed convex hull of a soft set , denoted by is the smallest soft closed soft convex set containing .

In other words:

is the intersection of all soft closed, soft convex sets which containing.

**Theorem (3.2.6):**

Let is a soft set of a over . Then:

If is a soft open , then is soft open.

If is a soft balanced , then is soft balanced.

If is a soft symmetric , then is soft symmetric.

If is a soft absorbing , then is soft absorbing.

If is a soft nbhd of , then is soft nbhd of .

**Proof:**

Since is a soft open set of , then, and

, we have:

.

By using theorem (2.2.2.i), implies that is soft convex containing .

But is the smallest soft convex set containing , so , but . And so we get the required.

Since is a soft balanced, then , for all with . From proposition (3.2.4.iv) , we have:

.

Since , for all such that . Thus is soft balanced.

Since is a soft symmetric, then and we have . Thus issoft symmetric.

Since is a soft absorbing set , then.

In general , then . Implies that , hence is soft absorbing set.

By using proposition (1.2.5.i).

**Definition (3.2.7) [6]:**

A soft set of a over is called an absolutely soft convex, if it is soft convex and soft balanced.

**Example (3.2.8):**

A soft open unit semiball is an absolutely soft convex.

**Definition (3.2.9):**

An absolutely soft convex hull of a soft set of a over , denoted by is the intersection of all absolutely soft convex sets which is containing .

In other words:

.

**Definition (3.2.10):**

A over is called a soft locally convex space , if there is a soft local base at , whose every members are soft convex.

**Example (3.2.11):**

is soft locally convex.

By using the remark (3.1.14.iii) and theorem (3.1.17) .

**Theorem (3.2.12):**

A soft set of a (which is soft locally convex space ) is soft bounded if and only if is soft bounded of for all in , where be a family of soft continuous soft seminorm on .

**Proof:**

Suppose that is a soft bounded set and . It is clear that:

is soft nbhd of of . Since is a soft bounded, then there is such that.

If, then , so.

Thus for all , by definition (1.1.24), implies that is soft bounded.

Conversely, suppose that is a soft bounded for each soft seminorms on . Let be a soft nbhd of .

Since is a (soft locally convex space), there is a family of soft seminorms on , which determine the soft topology . Thus there is and soft seminorms

such that:

. From hypothesis above, each is soft bounded for all , so there is such that for all and .

So, for all , , whenever .

i.e.

, whenever . It follows that

for all and we conclude that is soft bounded, as required.

**Theorem (3. 2.13):**

A over (which is soft locally convex) has a soft local base at consisting every members are soft open, soft absorbing and absolutely soft convex.

**Proof:**

Let be a soft local base at and be a soft nbhd of of , which is soft locally convex. Then there is a soft convex nbhd of in such that . By using theorem (2.1.16) , there is a soft balanced nbhd of of such that .

Then using that is a soft convex set containing , we get:

.

Assume that:

Clearly that: .

From the soft conclusion , implies that is soft open, soft balanced (because , see theorem (2.2.3.ii)) and soft convex nbhd of of .

**Example (3.2.14):**

is not soft locally convex space.

**Proof:**

It is easily that is a soft norm on over , for all . This soft norm induces a soft topology on making into a .

We now show that is not soft locally convex. Suppose it was , let:

,

is a soft open unit ball . Then there is a soft convex , soft balanced set:

in , which is a soft nbhd of such that:

.

Also, there is a soft open ball , such that:

.

Thus:

.

The soft linears:

; ,… , then by an absolutely soft convexity:

.

But

conflicting the soft boundedness of .

**Remark (3.2.15):**

It is known that there is a which is not soft locally convex space, see example (3.2.14).

**Theorem (3.2.16):**

A soft convex hull of a soft bounded set of a over (which is soft locally convex) is soft bounded.

**Proof:**

Let be a soft nbhd of of a soft locally convex space , then there is a soft convex nbhd of such that.

Since be a soft bounded , then there is such that:

.

From proposition (3.2.4.iv) , we have:

.

Also, by using proposition (3.2.4.iii) , then and so

.

This means that is soft bounded.

**Remark (3.2.17):**

In a (which is not soft locally convex space) a soft convex hull of a soft bounded set, need not to be a soft bounded set, as the example (3.2.14).

A soft convex hull of a soft closed set need not to be a soft closed set.

It does not have to be ; and although .

**§(3.3): Other types of soft locally convex spaces:**

In this section, we discuss the elementary properties of two types of a soft locally convex spaces that occur frequently in applications.

**Definition (3. 3.1):**

A soft set of a over is said to be a soft barrel set, if it has the following properties:

Soft absorbing.

Absolutely soft convex .

Soft closed.

**Example (3.3.2):**

In a , then:

A soft closed unit semiball is soft barrel.

**Theorem (3.3.3):**

Every soft nbhd of of a over is contained in a soft nbhd of which is soft barrel.

**Proof**:

Let be a soft nbhd of of .

.

It is clear that. Thus is soft nbhd of and so by theorem (2.1.19), it is soft absorbing. By construction is also soft closed and soft convex. To prove that is soft barrel it remains to show that it is soft balanced.

It is easy to see that any can be written as:

, with and , for some , such that and . Then for any with , we have:

,since and . This proves that is soft balanced. Now, by using theorem (2.2.3.ii), we have is soft balanced.

**Corollary (3.3.4):**

Every soft nbhd of of a over is contained in a soft nbhd of which is absolutely soft convex.

**Proof:** Clear.

**Theorem (3.3.5) :**

An absolutely soft convex (it is soft nbhd of ) of a over is soft barrels.

**Proof:**

Let an absolutely soft convex, soft nbhd of of . By using theorem (2.1.19), is soft absorbing set . Now, since be a soft nbhd of , then from definition (1.2.2.iii), we have , this implies that . Then from theorem (2.2.7), we have,is soft closed set.

**Corollary (3.3.6) :**

In a , then:

A soft open unit semiball is soft barrel.

**Proof:** Clear.

**Theorem (3.3.7) :**

A over (which is soft locally convex space) has a soft local base at consisting of soft barrels.

**Proof:**

Let be a soft local base at and a soft nbhd of of . By using theorem (2.1.23), there is a soft nbhd of such that:

.

Put , clearly that is soft closed nbhd of . Since be a soft locally convex , then there is a soft convex nbhd of such that . By using theorem (2.1.16), there is a soft balanced nbhd of of such that .

Summing up, we have:

for some and soft closed nbhds of such that soft balanced soft convex.

Then using that is a soft convex set containing, we get:

.

Passing to the soft closure and using that , we get:

Hence, the soft conclusion holds , because we have already showed in theorems (3.2.6.ii) , (3.2.6.v) and (2.2.3), implies that is soft barrel set.

**Definition (3.3.8):**

A over (which is soft locally convex space) is called a soft barrelled, if each soft barrel set of is soft nbhd of .

Equivalently, a soft barrelled space is a over (which is soft locally convex space) in which the family of all soft barrels form a soft nbhd of .

**Example (3.3.9):**

A soft locally convex space is not soft barrelled space.

**Theorem (3.3.10) :**

Let and be two 's over ( both are soft locally convex spaces), be a soft continuous, soft linear, soft open and soft surjective mapping. If be a soft barrelled space, then is soft barrelled.

**Proof:**

Let be a soft barrel set of , we prove that is soft nbhd of of . Since be a soft continuous and soft linear mapping, then is soft barrel set of . Since be a soft barrelled space , then is a soft nbhd of of .

Thus is soft nbhd of of , since is soft open and soft surjective mapping. Hence is soft barrelled space.

**Theorem (3.3.11):**

Let and be two 's over ( both are soft locally convex spaces ), be a soft linear, soft closed, soft continuous and soft bijective mapping. If be a soft barrelled space, then is soft barrelled.

**Proof:**

Let be a soft barrel set of , we prove that is soft nbhd of of . Since be a soft linear , soft closed and soft surjective mapping , then is soft barrel set of . Since be a soft barrelled space, then is a soft nbhd of of .

Thus is soft nbhd of of , ( because is soft continuous and soft injective mapping). Hence is soft barrelled space.

**Theorem (3.3.12) :**

Let and be two 's over ( both are soft locally convex spaces ), be a soft linear, homeomorphism mapping. Then is soft barrelled space if and only if is soft barrelled.

**Proof:** By using theorems (1.2.29) , (3.3.10) and (3.3.11) .

**Theorem (3.3.13) :**

A over (which is soft locally convex and soft second category space ) is soft barreled space.

**Proof:**

Let be a soft barrel set of . Since be a soft absorbing, we have:

, .

Hence for some (soft second category), see theorem (1.2.18.iii).

But , since a soft multiplication by is soft homeomorphism. By using theorem (2.2.9), implies that is soft nbhd of .

**Theorem (3.3.14):**

A over (which is soft locally convex and soft Baire space) is soft barreled space.

**Proof:**

Let be a soft barrel set of . Since be a soft absorbing , we have , . Since be a soft Baire space , then there is such that (which is soft closed) has soft interior point, see definition (1.2.19.i). Hence has a soft interior point. Since be a soft balanced, then. Implies that:

, is soft interior to from theorem (2.2.1), because is soft convex. Therefore is soft nbhd of.

**Definition (3.3.15):**

A over (which is soft locally convex space) is called soft bornological space, if each absolutely soft convex set of that soft absorbs all soft bounded sets is a soft nbhd of .

**Theorem (3.3.16):**

Let be a over (which is soft locally convex space). Then the following conditions are equivalent:

is soft bornological space .

every soft semi norm on which is soft bounded on a soft bounded subsets of is soft continuous.

**Proof:**

It is immediate that (i) and (ii) are equivalent in view of the correspondence between soft semi norms and soft convex , soft balanced and soft absorbing subsets. If be a soft semi norm on , which is soft continuous on each soft subset of , then it is soft bounded on a soft bounded subset of .

**Theorem (3.3.17):**

A over (which is soft locally convex and soft first countable space) is soft bornological space.

**Proof:**

Let be countable soft local base at such that . Suppose be an absolutely soft convex set of that soft absorbs all soft bounded sets, then proving that is soft nbhd of .

Assume that is not soft nbhd of and construct a soft bounded (in fact, soft compact) set that does not soft absorb. Then for all , we have:

.

If.

Since , , then , so by using theorem (1.2.14.iii), is soft compact, by using theorem (3.1.27) it is soft bounded.

But , says that cannot soft absorbs.

**Example (3.3.18):**

A soft locally convex space is soft bornological space.

**Theorem (3.319) :**

A over ( is soft locally convex and soft metrizable )

is soft bornological space.

**Proof:**

Since be a soft metrizable from definition (1.2.43)and theorem (1,2,44) , there is a countable soft local base at ℬ ing oft metrizable , ble , from remark . Suppose that be an absolutely soft convex of that absorbs all soft bounded sets , then proving that is soft nbhd of . Assume that is not soft nbhd of and construct a soft bounded (in fact, soft compact) set that does not absorb.

Then for all , we have:

and so .

Choose.

Since , then . Since be a soft compact, by using theorem (3.1.27) , then is soft bounded. But , says that cannot soft absorbs.

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**كجزء من متطلبات نيل درجة ماجستير في علوم الرياضيات من قبل**

**خلود محمد حسن عباس**

**بإشراف**

# د. نوري فرحان المياحي

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