



On Some Generalized Continuous Mappings in Fuzzy Topological Spaces

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Abstract: The purpose of this paper is to introduce a new class of intuitionistic fuzzy closed sets called intuitionistic k-continuous k-irresolute functions in intuitionistic fuzzy topological space (for short, fkt). Finally, we introduce the concepts of k-open (k-closed) mapping with some properties in fuzzy topological spaces.

Keywords: Intuitionistic fuzzy K-closed sets Intuitionistic fuzzy K-connectedness, Intuitionistic fuzzy K-compactness.

1 Introduction

Atanassov [1,2] was introduced the concept of intuitionistic fuzzy sets (IFS). After that Chang [3] gave the fuzzy topological spaces. Also appeared like such as fuzzy fuzzy compactness [4], fuzzy connectedness [10], fuzzy g-closed sets [7, 9] fuzzy rg-closed sets [8] have been generalized for IF topological spaces. Further in 2012 Vadivel and Sivakumar [13] were introduced the concept $\mathcal{G}^*$ -continuous mapping in fuzzy topology.

2. Preliminaries

We give the following definitions which needed in this paper

Definition 2.1 A fuzzy set ρ in (M, τ) is called:

- (1) a fuzzy pre-open set [10] if $\rho \leq cl(int(\rho))$ -
- (2) a fuzzy α -open set [8] if $\rho \leq int(cl(int(\rho)))$
- (3) a fuzzy s-open set [14] if $\rho \leq cl(int(\rho))$

Definition 2.2 A fuzzy set ρ in a fts (M, τ) is : (1) Fg- closed set (for short, FGC set) [1] if $cl(\rho) \leq v$ whenever $\rho \leq v$ and v is FO in M,

(2) a Fgpre-regular closed set [3] (for short, fgpr-closed fuzzy set) if $pcl(\rho) \subseteq v$ whenever $\rho \leq v$ and v is fuzzy regular open set in M.

(3) a Fgsemi-regular closed set [] (for short, fgssr-closed fuzzy set) if $scl(\rho) \subseteq v$ whenever $\rho \leq v$ and v is fuzzy semi open set in M.

Definition 2.3 Let M, N are two FTS. A mapping $L : M \rightarrow N$ is :

- (1) F-continuous (for short, F-cont) [9] if $L^{-1}(\rho)$ is Fo in M, $\forall$ Fo set ρ of N,
- (2) Fa-continuous (for short, fa-cont) [8] if $L^{-1}(\rho)$ is Fac -set in M, $\forall$ Fc set ρ of N,
- (3) F-pre-continuous [8] if $L^{-1}(\rho)$ is F-pre-closed set in M, $\forall$ Fcp of N,
- (4) F- gp-continuous (for short, Fgp-cont) [6] if $L^{-1}(\rho)$ is Fgp-closed set in M, $\forall$ Fcp of N,
- (5) Fg semi-irresolute (for short, Fgs-irresolute) [12] if $L^{-1}(\rho)$ is fgc- set in M, $\forall$ Fc ρ of N,
- (6) Fag irresolute (for short, fag-irresolute) [12] if $L^{-1}(\rho)$ is fagc set in M, $\forall$ fagc ρ of N,
- (7) F-perfectly continuous (for short, fp-cont) [12] if $L^{-1}(\rho)$ is Fo and Fc set in M, $\forall$ Fo ρ in N.

Definition 2.4 Let M, N are two fts. A mapping $L : M \rightarrow N$ is called:

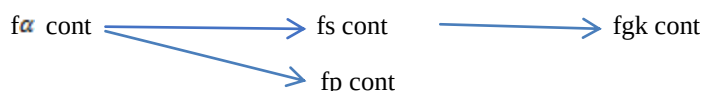
- (1) F-open (for short, f-open) [8] if $L(p)$ is Fo set in N , $\forall$ f-open set of M ,
(2) FG-open (for short, FG-open) [8] iff $L(p)$ is FGo set in N , $\forall$ F-open in M .

Definition 2.5 Let M, N are two Fts. A bijective map $L: M \rightarrow N$ is called F-homeomorphism [14] (for short, f-homeomorphism) if L and L^{-1} are f-continuous.

3. Fuzzy Generalized K-Continuous Mappings

Definition 3.1 Let O and R be two fts. A function $Q: O \rightarrow R$ is F generalized K-continuous (for short, fgk-cont) if the inverse image of $\forall$ FOS in N is fgko set in M .

Proposition 3.2 Let $L: M \rightarrow N$ be a mapping. Then following implications are true :



Proof: $f\alpha\text{cont} \rightarrow fs \text{ cont}$: Let p be Fac set in N . L is $f\alpha$ -continuous, L^{-1} is fac set in M , so, L^{-1} is fsc set in M . So that L is fs-cont.

$f\alpha$ continuous $\rightarrow fp$ continuous : Let p Fa closed set in N . L is $f\alpha$ -cont, L^{-1} is fac set in M , so, L^{-1} is fp closed set in M . Therefore L is fs-cont.

fs continuous $\rightarrow fgk$ continuous : it's clear. The counter examples show the cases.

Example 3.3. Let $M = Z = \{i_1, i_2\}$ and Fsets δ and ξ defined as following :

$$\delta = \frac{0.1}{i_1} + \frac{0.5}{i_2}, \xi = \frac{1}{i_1} + \frac{0.9}{i_2}. \text{ Let } \lambda = \gamma = \{0, \xi, 1\}. \text{ Let}$$

$Q: (M, \lambda) \rightarrow (Z, \gamma)$ by $Q(i_1) = i_2$ and $Q(i_2) = i_1$. Then L is not $f\alpha$ -cont because ξ is fac in Z and $Q^{-1}(\xi) = \xi$ is not fac- set in M but Q is fsc in M . Thus Q is fs-cont. Also Q is fp-cont but Q is not $f\alpha$ -cont.

Example 3.4 .Let $M = D = \{i_1, i_2, i_3\}$ and Fsets δ, ξ and ω defined as following

$$\delta = \frac{0}{i_1} + \frac{0.4}{i_2} + \frac{0.2}{i_3}, \xi = \frac{0.6}{i_1} + \frac{0.7}{i_2} + \frac{0.9}{i_3}, \omega = \frac{0.1}{i_1} + \frac{0.2}{i_2} + \frac{0.3}{i_3}.$$

Let $\lambda = \gamma = \{0, \xi, 1\}$. Let $Q: (M, \lambda) \rightarrow (D, \gamma)$ by $Q(i_1) = i_1$, $Q(i_2) = i_2$ and $L(i_3) = i_3$. Then Q is not fs-cont because ξ is fsc in D and $Q^{-1}(\xi) = \xi$ is not fsc in M but it is fgkc set in M . Therefore Q is fgk- cont.

Example 3.5 .Let $M = D = \{i_1, i_2, i_3\}$ and Fsets δ, ξ and ω defined as following : $\delta = \frac{0.8}{i_1} + \frac{0.9}{i_2} + \frac{1}{i_3}$, $\xi = \frac{0.2}{i_1} + \frac{0.6}{i_2} + \frac{0.2}{i_3}$, $\omega = \frac{0.4}{i_1} + \frac{0.5}{i_2} + \frac{0.6}{i_3}$. Let $\lambda = \gamma = \{0, \xi, 1\}$. Let $Q: (M, \lambda) \rightarrow (D, \gamma)$ by $Q(i_1) = i_3$, $Q(i_2) = i_1$ and $Q(i_3) = i_2$. Then Q is not fgk-cont because δ is fgkc in D and $Q^{-1}(\delta) = \delta$ is not fgkc set in M but it is fgkc in M . Therefore Q is fp-cont.

Example 3.6 .Recall example 3.4 We see that Q is not fp-continuous because ξ is fgkc in N and $Q^{-1}(\xi) = \xi$ is not fp-closed set in M but it is fgkc set in M . Therefore Q is fgk-continuous.

Example 3.7 .Let $M = O = \{i_1, i_2, i_3\}$ and Fsets δ, ξ and ω defined as following

$$\delta = \frac{0.1}{i_1} + \frac{0.2}{i_2} + \frac{0.3}{i_3}, \xi = \frac{0.3}{i_1} + \frac{0.4}{i_2} + \frac{0.5}{i_3}, \omega = \frac{0}{i_1} + \frac{0.2}{i_2} + \frac{0}{i_3}. \text{ Let } \lambda = \gamma = \{0, \xi, 1\}. \text{ Let}$$

$L: (M, \lambda) \rightarrow (O, \gamma)$ by $L(i_1) = i_2$, $L(i_2) = i_3$, $L(i_3) = i_1$. Then L is not fs-cont because δ is fsc in O and $L^{-1}(\delta) = \delta$ is not fpc set in M but it is fs-closed set in M . Therefore L is fs-cont.

Example 3.8 .Let $M = N = \{i_1, i_2, i_3\}$ and Fsets δ , ξ and ω defined as following : $\delta = \frac{0}{i_1} + \frac{0.9}{i_2} + \frac{0.2}{i_3}$, $\xi = \frac{0.8}{i_1} + \frac{0.7}{i_2} + \frac{1}{i_3}$, $\omega = \frac{0.3}{i_1} + \frac{0.6}{i_2} + \frac{0.7}{i_3}$. Let $\lambda = \gamma = \{0, \xi, 1\}$. Let $L : (M, \lambda) \rightarrow (O, \gamma)$ by $L(i_1) = i_3$, $L(i_2) = i_1$, $L(i_3) = i_2$. Then L is not **fp**-cont because δ is **fp**c in O and $L^{-1}(\xi) = \xi$ is not **fsc** in M but it is **fsc** in M . Therefore L is **fp**-cont.

Remark 3.9. By transitivity we get : **fa** continuous $\rightarrow$ **fgk** continuous .

Proposition 3.10. If $L : M \rightarrow N$ is **fgk**-continuous and $K : N \rightarrow W$ is L-continuous, then $K \circ L : M \rightarrow W$ is **fgk**-cont .

Proof: Let ρ Fc in W. Then $K^{-1}(\rho)$ is Fc in N , since K is f-cont, then $L^{-1}(K^{-1}(\rho))$

is **fgk**c in M , since L is **fgk** -cont. So $(K \circ L)^{-1}(\lambda) = L^{-1}(K^{-1}(\rho))$ is **fgk**c in M. Therefore $K \circ L : M \rightarrow W$ is **fgk**- cont.

Definition 3.11 $L : K \rightarrow H$ is said to be Fgk-irresolute (for short **fgkir**) if $L^{-1}(h) \forall$ **fgk**c in N is **fgk**c in M .

Proposition 3.12 Let $L : K \rightarrow H$ be a mapping .Then every **fgkir** is **fgk** -cont.

Proof: Let ρ Fc in H . So ρ is **fgk**c in H . Since L is **fgkir** , $L^{-1}(\rho)$ is **fgk**c in K . Therefore L is **fgk** -cont.

Example 3.13 . Recall example 3.4 We see that L is **fgk**-cont but not **fgkir** because the FC ω in N is $L^{-1}(\omega) = \omega$ which is not **fgk**c in M .

Proposition 3.14. If $L : M \rightarrow N$ and $Q : N \rightarrow W$ are two mappings . If L and Q are **fgkir** , mapping ,then $Q \circ L : M \rightarrow W$ is **fgkir** mapping .

Proof: Let ρ Fc in M . Then $K^{-1}(\rho)$ is Fc in N , since Q is **fgkir**, then $L^{-1}(Q^{-1}(\rho))$

is **fgk**c in M , since L is **fgkir**. So $(Q \circ L)^{-1}(\lambda)$ is **fgk**c in M . Therefore $Q \circ L : M \rightarrow W$ is **fgkir** .

Proposition 3.15. Let $L : M \rightarrow N$ and $Q : N \rightarrow W$ are two mappings . If L is **fgk** ir and Q is **fgk** -cont, then $Q \circ L : M \rightarrow W$ is **fgk**-cont.

Proof : Let ρ Fc in M . Then $Q^{-1}(\rho)$ is **fgk**c in N , since Q **fgk** -cont. Since L **fgkir**, $L^{-1}(Q^{-1}(\rho))$ is **fgk**c in M . Hence $Q \circ L : M \rightarrow W$ be **fgk** -cont.

Definition 3.16 $L : J \rightarrow W$ is called to be Fgk-regular open (for short, **fgkro**) if $L(j) \forall$ Fo in M is **fgkro** in N .

Definition 3.17 $L : J \rightarrow W$ is said to be Fgk-regular closed (for short, **fgkrc**)

if $L(j) \forall$ Fc in M is **fgkrc** in N .

Proposition 3.18 If $L : M \rightarrow N$ is Fc map and $K : N \rightarrow W$ is **fgk**c map , then $K \circ L : M \rightarrow W$ is **fgk**c map.

Proof: Let ρ Fc in M . So $L(\rho)$ Fc in N . Since K is **fgk**c map and since $K(L(\rho))$ is **fgk**c in W. So $(K \circ L)(\rho) = K(L(\rho))$ is **fgk**c in W. Therefore $K \circ L : M \rightarrow W$ is **fgk**c map.

Proposition 3.19 Let $L : M \rightarrow N$ and $K : N \rightarrow W$ are two mappings s.t, $K \circ L : M \rightarrow W$ is **fgkrc** map.

(1) If L is f-continuous and surjective, thus K is **fgkrc**-mapping.

(2) If h is **fgkir** and 1-1 , thus f is **fgkrc** mapping.

Proof: (1) Let ρ Fc in N. So $L^{-1}(\rho)$ is Fc in M. Thus $K \circ L$ is **fgkrc** mapping, $(K \circ L)(L^{-1}(\rho)) = K(\rho)$ is **fgkrc** in W. Therefore K is **fgkrc** mapping .

(2) Let v Fc in N. Then $(h \circ f)(\mu)$ is **fgk** c in W, hence $(K^{-1}(K \circ L)(v))$ is **fgk** c in N. So that K is 1-1, $L(v) = K^{-1}(K \circ L)(v)$ is **fgkrc** in N . Therefore L is **fgkrc** mapping.

Definition 3.20 Let M and N be two fts. A bijective map $L : M \rightarrow N$ is called Fgk-regular homeomorphism (for short, **fgkr**-homeomorphism) if L and L^{-1} are **fgkr**-continuous.

Proposition 3.21 Every **fa**-homeomorphism is **fgkr**-homeomorphism.

Proof: Let $L : M \rightarrow N$ be a **fa**-homeomorphism. Then Land L^{-1} are f-cont. Therefore L and L^{-1} are **fgkr** -cont. So L is **fgkr** -homeomorphism.

Example 3.22 Let $M = N = \{m_1, m_2\}$ and the fuzzy sets δ , ξ defined as following :

$$\delta = \frac{0.2}{m_1} + \frac{0}{m_2}, \xi = \frac{1}{m_1} + \frac{0.8}{m_2}. \text{ Let } \lambda = \gamma = \{0, \xi, 1\}. \text{ Define } L : (M, \lambda) \rightarrow (N, \gamma)$$

by $L(m_1) = m_1$ and $L(m_2) = m_2$. Then L is $fgkr$ -homeomorphism but not fa -homeomorphism because the fuzzy set δ is open in M and its image $L(\delta) = \delta$ is not fa_0 in N , $L^{-1} : M \rightarrow N$ is not fa -cont.

Definition 3.23. A bijective map $L : M \rightarrow N$ is called Fgk -regular-semi-homeomorphism (for short, $fgkrs$ -homeomorphism) if L and L^{-1} are $fgkrir$.

Proposition 3.24 Let $L : M \rightarrow N$, $K : N \rightarrow W$ are two $fgkrs$ -homeomorphism, then $K \circ L : M \rightarrow W$ is $fgkrs$ -homeomorphism.

Proof : Let ρ $fgkro$ in W , and since $K : N \rightarrow W$ is $fgkrir$, $K^{-1}(\rho)$ is $fgkro$ in N . Also $L : M \rightarrow N$ is $fgkrir$, and $L^{-1}(K^{-1}(\rho)) = (K \circ L)^{-1}(\rho)$ is $fgkro$ in M . So that $K \circ L : M \rightarrow W$ is $fgkrsir$.

Now, let ρ $fgkro$ in M . Then $(L^{-1})^{-1}(\rho) = L(\rho)$ is $fgkro$ in N . Also $K^{-1} : W \rightarrow N$ is $fgkrir$, $(K^{-1})^{-1}(L(\rho)) = K(L(\rho)) = (K \circ L)(\rho)$ is $fgpro$ in W . Hence $(K \circ L)^{-1} : W \rightarrow M$ is $fgkrir$. Thus $K \circ L$ is $fgkrs$ -homeomorphism.

Proposition 3.25 Let $L : M \rightarrow N$, $K : N \rightarrow W$ are two $fgkr$ -homeomorphism, then $K \circ L : M \rightarrow W$ is $fgkrs$ -homeomorphism.

Proof : it's obvious.

Conclusion

We studied Fgk -cont, $Fgkc$ mappings and studied some properties. It is observed that every Fa -cont and Fs -cont is a Fgk -cont but not conversely. Also every fgk -cont is a $Fgkr$ -cont but not conversely. And we get results of composition of $Fgkr$ -cont, $Fgkrc$ maps, and $Fgkr$ -homeomorphisms maps are obtained. Finally Fc map is $fgkrc$ map but not conversely.

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