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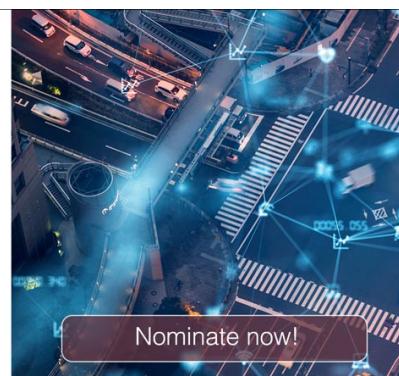


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Some Properties of Regular and Normal Space on Topological Graph Space

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Abstract. The goal of this article is to give the concepts of Regular space and Normal space in the topological graph space also generalize α and β that to Regular space and Normal space where the relationship between these concepts were given. Finally, some characteristics of these concepts were investigated.

1. Introduction

Graph theory [3] is one of the fundamental topics of modern mathematics. This theory is also used in most branches of knowledge, as it has applications and uses of great benefits in topics of scientific and economic importance such as game theory, mathematical programming, connection theory and electrical networks in addition to its use in physics, organic chemistry, economics, civil engineering, biology and many other fields.

Kosterlitz [5] given the relationship between graph theory and topology. The topology is created by graph theory and the study of topological properties in graph theory. Some researchers make the relationship only on the vertices of the graph, while others make it on the edges. But the process is to create a schematic diagram of a particular topology that was not interested before ([6] and [8]).

2. Preliminaries

Definition 2.1 [3]: A graph H is a set $I \neq \emptyset$ of elements called vertices graph with a family L from unarranged pairs of vertices a graph is called an edge. It is called set vertices graph $I(H)$ and family edge $m(H)$ and We express a graph of ordered pairs $H(I, M)$.

Definition 2.2 [7]: A directed graph or digraph D as set w that is non-empty of elements called vertices I and edges M with a family of ordered pairs of vertices denoted by (I, M) .

A directed graph is expressed as an ordered pair (I, D) and denoted set vertices $I(D)$ and family directed edge $M(D)$.

Definition 2.3 [6]:

Let $H(I, m)$ be a graph, we call K is a sub graph from H and can write by $K \subseteq H$ if



$l(K) \subseteq l(H), m(K) \subseteq m(H)$. The spanning sub graph from a graph is a sub graph acquired by edge deletions only.

Definition 2.4 [4]: A topological space (S, σ) is called regular space if for every non-empty closed set F and a point m which does not belong to F , there are open sets U_1, U_2 s.t. $m \in U_1, F \not\subseteq U_2$ and $U_1 \cap U_2 = \emptyset$.

Definition 2.5 [4]: A topological space (S, σ) is called normal space iff for every non-empty closed set F_1, F_2 , there is a pair U_1, U_2 of open set s.t. $F_1 \subseteq U_1, F_2 \subseteq U_2$ and $U_1 \cap U_2 = \emptyset$.

Definition 2.6: A subset K of a space S is called:

(4) α -open [2] if $K \subseteq \text{int}(\text{cl}(\text{int}(K)))$.

(5) β -open [9] if $K \subseteq \text{cl}(\text{int}(\text{cl}(K)))$.

Definition 2.7[1]: Let (L, H) be a graph, $l \in L(H)$ then we define the post stage is the set of all vertices which is not a neighborhood of l . QH is the collection of (lR) is called sub basis of graph $B_H = \bigcap_{j=1}^n QH_j$ is called bases of the graph. Then the union of B_H is form a topology on H and $(L(H), \sigma_H)$ is called a topological graph.

Example 2.8[1]: Let (L, H) be a graph, We'll create a topology via graph as follows:

$l_1(H) = \{l_1, l_2\}, l_2(H) = \{l_3, l_4\}, l_3(H) = \{l_1\}, l_4(H) = \{l_2\}, l_5(H) = \{l_2, l_5\}$. Then a subbase is a topology is $S_H = \{\{l_1, l_2\}, \{l_3, l_4\}, \{l_1\}, \{l_2\}, \{l_2, l_5\}\}$. The base is $B_H = \{L(H), \emptyset, \{l_1, l_2\}, \{l_3, l_4\}, \{l_3\}, \{l_1\}, \{l_2\}, \{l_2, l_5\}\}$. Therefore, the topological graph of H is $\sigma_H = \{L(H), \emptyset, \{l_1, l_2\}, \{l_3, l_4\}, \{l_3\}, \{l_1\}, \{l_2\}, \{l_2, l_5\}, \{l_1, l_3\}, \{l_1, l_2, l_3, l_4\}, \{l_1, l_2, l_3\}, \{l_1, l_3, l_4\}, \{l_2, l_3, l_4\}, \{l_1, l_2, l_5\}, \{l_3, l_2, l_5\}, \{l_3, l_4, l_2, l_5\}\}$

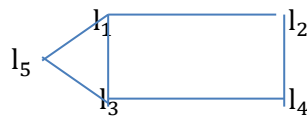


Figure 1 Topology via graph

3- Regular and Normal Space on Topological Graph Space

In this section, we give new definitions of Regular and Normal space on topological graph space with relation among them.

Definition 3.1: A topological graph $(L(H), \sigma_H)$ is called H-Regular space if for any vertices l from $L(H)$ and F is closed set vertices s.t. $l \notin F$, there exist post two stages from any vertices U_1, U_2 s.t. $l \in U_1, F \not\subseteq U_2$ and $U_1 \cap U_2 = \emptyset$.

Definition 3.2: A topological graph $(L(H), \sigma_H)$ is called H-Normal space if for any two closed vertices F_1, F_2 from $L(H)$, there exists two post stage from any vertices U_1, U_2 s.t. $F_1 \subseteq U_1, F_2 \not\subseteq U_2$ and $U_1 \cap U_2 = \emptyset$.

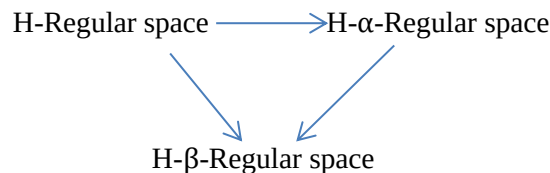
Definition 3.3: A topological graph $(L(H), \sigma_H)$ is called H- α -Regular space if for any vertices l from $L(H)$ and F is α -closed vertices s.t. $l \notin F$, there exist α -post two stages from any vertices U_1, U_2 s.t. $l \in U_1, F \not\subseteq U_2$ and $U_1 \cap U_2 = \emptyset$.

Definition 3.4: A topological graph $(L(H), \sigma_H)$ is called H - β -Regular space if for any vertices l from $L(H)$ and F is β -closed set vertices s.t. $l \notin F$, there exist two β -post stage from any vertices U_1, U_2 s.t. $l \in U_1, F \notin U_2$ and $U_1 \cap U_2 = \emptyset$.

Definition 3.5: A topological graph $(L(H), \sigma_H)$ is called H - α -Normal space if for any two α -closed vertices F_1, F_2 there exist two α -post stage from any vertices U_1, U_2 s.t. $F_1 \in U_1, F_2 \notin U_2$ and $U_1 \cap U_2 = \emptyset$.

Definition 3.6: A topological graph $(L(H), \sigma_H)$ is called H - β -Normal space if for any two β -closed vertices F_1, F_2 , there exist two β -post stage from any vertices U_1, U_2 s.t. $F_1 \in U_1, F_2 \notin U_2$ and $U_1 \cap U_2 = \emptyset$.

Theorem 3.7. Let $(L(H), \sigma_H)$ be a topological graph space. Then $L(H)$ is satisfied following:



Proof: H -regular space $\Rightarrow$ H - α -Regular space: Let $(L(H), \sigma_H)$ be H -Regular space and let each of l is vertices and F is closed vertices s.t. $l \notin F$, since $(L(H), \sigma_H)$ be H -Regular space, then F is α -closed vertices (every closed vertices are α -closed vertices). Thus, there exist α -post two stages from any α vertices U_1, U_2 s.t. $l \in U_1, F \notin U_2$ and $U_1 \cap U_2 = \emptyset$. Therefore $(L(H), \sigma_H)$ is H - α -Regular space.

H -regular space $\Rightarrow$ H - β -Regular space: It is clear.

H -regular space $\Rightarrow$ H - β -Regular space: Let $(L(H), \sigma_H)$ be H - α -Regular space and let each of l is vertices and F is α -closed vertices s.t. $l \notin F$, since $(L(H), \sigma_H)$ be H - α -Regular space, then F is β -closed vertices (every α -closed vertices is β -closed vertices). Thus, there exist β -post two stages from any β vertices U_1, U_2 s.t. $l \in U_1, F \notin U_2$ and $U_1 \cap U_2 = \emptyset$. Therefore $(L(H), \sigma_H)$ is H - β -Regular space.

Example 3.8: Let $(L(H), \sigma_H)$ be a topological graph space. We'll create a topology via a graph.

We take $l_1(H) = \{l_1, l_2\}, l_2(H) = \{l_3, l_4\}, l_3(H) = \{l_1\}, l_4(H) = \{l_2\}, l_5(H) = \{l_2, l_3\}$.

Then a subbase is a topology is $S_H = \{\{l_1, l_2\}, \{l_3, l_4\}, \{l_1\}, \{l_2\}, \{l_2, l_3\}\}$. The base is $B_H = \{L(H), \emptyset, \{l_1, l_2\}, \{l_3, l_4\}, \{l_3\}, \{l_1\}, \{l_2\}, \{l_2, l_3\}\}$. Therefore, the topological graph on H is $\sigma_H = \{L(H), \emptyset, \{l_1, l_2\}, \{l_3, l_4\}, \{l_3\}, \{l_1\}, \{l_2\}, \{l_2, l_3\}, \{l_1, l_3\}, \{l_1, l_2, l_3, l_4\}, \{l_1, l_2, l_3\}, \{l_1, l_3, l_4\}, \{l_2, l_3, l_4\}\}$.

Then $(L(H), \sigma_H)$ is H - α -Regular space, but $(L(H), \sigma_H)$ is not H -Regular space, because there is not exist disjoint post two stages for any vertices and closed set vertices in this graph.

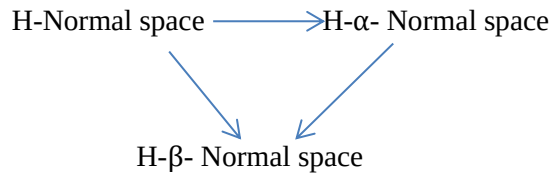
Example 3.9: Let $(L(H), \sigma_H)$ be a topological graph space We'll create a topology via a graph.

We take $l_1(H) = \{l_1, l_2\}, l_2(H) = \{l_3, l_4\}$. Then a sub base is a topology is

$S_H = \{\{l_1, l_2\}, \{l_3, l_4\}\}$. The base is $B_H = \{L(H), \emptyset, \{l_1, l_2\}, \{l_3, l_4\}, \{l_2\}\}$. Therefore, the topological graph on H is $\sigma_H = \{L(H), \emptyset, \{l_1, l_2\}, \{l_3, l_4\}, \{l_2\}, \{l_2, l_3, l_4\}\}$.

Then $(L(H), \sigma_H)$ is H - β -Regular space, but $(L(H), \sigma_H)$ is not H - α -Regular space, because there is not exist disjoint post two stages for any vertices and α -closed set vertices in this graph. Also $(L(H), \sigma_H)$ is not H -Regular space, because there is not exist disjoint post two stages for any vertices and closed set vertices in this graph.

Proposition 3.10. Let $(L(H), \sigma_H)$ be a topological graph space. Then $L(H)$ is satisfied the following:



Proof: $H\text{-Normal space} \Rightarrow H\text{-}\alpha\text{- Normal space}$: Let $(L(H), \sigma_H)$ be $H\text{-normal space}$ and let and two α -closed vertices F_1, F_2 , since $(L(H), \sigma_H)$ then F is α -closed vertices (every closed vertices are α -closed vertices). Thus, there exist α -post two stages from any vertices U_1, U_2 s.t. $F_1 \in U_1, F_2 \notin U_2$ and $U_1 \cap U_2 = \emptyset$. Therefore $(L(H), \sigma_H)$ is $H\text{-}\alpha\text{-Normal space}$.

$H\text{-Normal space} \Rightarrow H\text{-}\beta\text{-Normal space}$: it is same above.

$H\text{-}\alpha\text{-Normal space} \Rightarrow H\text{-}\beta\text{-Normal space}$: it is same above.

Example 3.11: Let $(L(H), \sigma_H)$ be a topological graph space. We'll create a topology via a graph. We take

$$l_1(H) = \{l_1\}, l_2(H) = \{l_2, l_4\}, l_3(H) = \{l_3\}, l_4(H) = \{l_4\}.$$

Then a subbase is a topology is $S_H = \{\{l_2, l_4\}, \{l_1, l_3\}, \{l_3\}, \{l_4\}\}$. The base is $B_H = \{L(H), \emptyset, \{l_2, l_4\}, \{l_1, l_3\}, \{l_3\}, \{l_4\}\}$. Therefore, the topological graph on H is $\sigma_H = \{L(H), \emptyset, \{l_2, l_4\}, \{l_1, l_3\}, \{l_3\}, \{l_4\}, \{l_1, l_3, l_4\}, \{l_2, l_3, l_4\}, \{l_3, l_4\}\}$.

Then $(L(H), \sigma_H)$ is $H\text{-}\alpha\text{normal space}$, but $(L(H), \sigma_H)$ is not $H\text{-normal space}$, because there is not exist disjoint post two stages contain two closed vertices for any two vertices in this graph.

Example 3.12: Let $(L(H), \sigma_H)$ be a topological graph space. We'll create a topology via a graph.

We take $l_1(H) = \{l_1\}, l_2(H) = \{l_2, l_4\}, l_3(H) = \{l_1, l_3\}$. Then a sub base is a topology

$S_H = \{\{l_1\}, \{l_2, l_4\}, \{l_2, l_3\}\}$. The base is $B_H = \{L(H), \emptyset, \{l_1\}, \{l_2, l_4\}, \{l_2, l_3\}, \{l_2\}\}$.

Therefore, the topological graph on $L(H)$ is $\sigma_H = \{L(H), \emptyset, \{l_1\}, \{l_2, l_4\}, \{l_2, l_3\}, \{l_2\}, \{l_1, l_2, l_4\}, \{l_1, l_2, l_3\}, \{l_1, l_2\}, \{l_2, l_3, l_4\}\}$.

Then $(L(H), \sigma_H)$ is $H\text{-}\beta\text{-Normal space}$, but $(L(H), \sigma_H)$ is not $H\text{-}\alpha\text{-Normal space}$, because there is not exist disjoint post two stages contain two β -closed vertices for two α -vertices in this graph.

Remark 3.13. The relation between $H\text{-Normal space}$ with $H\text{-Regular space}$ ($H\text{-}\alpha\text{-Normal space}$ with $H\text{-}\alpha\text{-Regular space}$, and $H\text{-}\beta\text{-Normal space}$ with $H\text{-}\beta\text{-Regular space}$) are independent relations. We show this following:

- 1- Recall Example 3.8. We see that $(L(H), \sigma_H)$ is $H\text{-}\alpha\text{-Regular space}$, but it is not $H\text{-}\alpha\text{-Normal space}$.
- 2- Recall Example 3.11. We see that $(L(H), \sigma_H)$ is $H\text{-}\beta\text{-Regular space}$, but it is not $H\text{-}\beta\text{-Normal space}$.
- 3- Recall Example 3.8. We see that $(L(H), \sigma_H)$ is $H\text{-}\alpha\text{-Normal space}$, but it is not $H\text{-}\alpha\text{-Regular space}$.
- 4- Recall Example 3.8. We see that $(L(H), \sigma_H)$ is $H\text{-}\beta\text{-Normal space}$, but it is not $H\text{-}\beta\text{-Regular space}$.

Example 3.14: Let $(L(H), \sigma_H)$ be a topological graph space. We'll create a topology via a graph.

We take $l_1(H) = \{l_1\}, l_2(H) = \{l_2, l_3\}$. Then a sub base is a topology is

$S_H = \{\{l_1\}, \{l_2, l_3\}\}$. The base is $B_H = \{L(H), \emptyset, \{l_1\}, \{l_2, l_3\}\}$. Therefore, the topological graph on $L(H)$ is $\sigma_H = \{L(H), \emptyset, \{l_1\}, \{l_2, l_3\}\}$. Then $(L(H), \sigma_H)$ is $H\text{-Normal space}$, but $(L(H), \sigma_H)$ is not $H\text{-Regular space}$, because there is not exist disjoint post two stages for any two α -vertices in this graph.

Example 3.15: Let $(L(H), \sigma_H)$ be a topological graph space. We'll create a topology via a graph.

We take $l_1(H) = \{l_2\}, l_2(H) = \{l_2, l_3\}, l_3(H) = \{l_3\}$.

Then a sub base is a topology is $S_H = \{\{l_2\}, \{l_2, l_3\}, \{l_3\}\}$. The base is $B_H = \{L(H), \emptyset, \{l_2\}, \{l_2, l_3\}, \{l_3\}\}$.

Therefore, the topological graph on $L(H)$ is $\sigma_H = \{L(H), \emptyset, \{l_2\}, \{l_2, l_3\}, \{l_3\}\}$. Then $(L(H), \sigma_H)$ is $H\text{-}$

regular space, but $(L(H), \sigma_H)$ is not H- normal space, because there is not exist disjoint post two stages for any two α -vertices contain two closed vertices in this graph.

4-Some Properties of Regular and Normal Space on Topological Graph

In the section, we introduce some properties of Regular and Normal Space on Topological Graph. We start by the following proposition:

Proposition 4.1. Let $(L(H), \sigma_H)$ be a topological graph space. Then the following are equivalents:

- 1) $L(H)$ is H-Regular space.
- 2) For each $l \in l(H)$, F is closed a post stage and $F \subseteq M(F)$, there exists a post stage K s.t, $l \in l(H) \subseteq \text{cl}_H(l(H)) \subseteq M(F)$.

Proof: (1) $\Rightarrow$ (2) Assume that $L(H)$ is H-Regular space, F is closed a post stage and

$l \in M(F)$, then there exists a post stage K s.t, $l \in l(H) \subseteq \text{cl}_H(l(H))$, so that

$$M(\text{cl}_H(l(H))) \text{ and } \text{cl}_H(l(H)) \subseteq M(F). \text{ Thus } l \in l(H) \subseteq \text{cl}_H(l(H)) \subseteq M(F).$$

(2) $\Rightarrow$ (1) Assume that F is closed a post stage, $l \in M(F)$, so that there exists a post stage K s.t. $l \in l(H) \subseteq \text{cl}_H(l(H)) \subseteq M(F)$. Thus $l \in l(H) \subseteq \text{cl}_H(l(H)) \subseteq M(F)$, so $F \subseteq M(\text{cl}_H(l(H)))$ which is a disjoint post stage. Thus $L(H)$ is H-Regular space.

Remark 4.2. The converse of above Proposition is not true. The following example shows that the opposite of the above theorem is incorrect.

Example 4.3. Let (L, H) be a graph, We'll create a topology via graph as follows:

$l_1(H) = \{l_1\}$, $l_2(H) = \{l_3\}$, $l_3(H) = \{l_2\}$. Then a sub base is a topology is

$S_H = \{\{l_1\}, \{l_3\}, \{l_2\}\}$. The base is $B_H = \{L(H), \emptyset, \{l_1, l_2\}, \{l_1, l_3\}, \{l_3, l_2\}, \{l_3\}, \{l_1\}, \{l_2\}\}$. Therefore, the topological graph on H is $\sigma_H = \{L(H), \emptyset, \{l_1, l_2\}, \{l_1, l_3\}, \{l_3, l_2\}, \{l_3\}, \{l_1\}, \{l_2\}\}$

Then $\{l_1, l_2\}$ is closed a post stage and $\{l_1, l_2\} \subseteq M(\{l_1, l_2\})$, there exists a post stage K s.t, $\{l_1, l_2\} \in l(H) \subseteq \text{cl}_H(\{l_1, l_2\}) \subseteq M(\{l_1, l_2\})$, but $L(H)$ is not H-Regular space.

Proposition 4.4. Let $(L(H), \sigma_H)$ be a topological graph space. Then the following are equivalents:

- 1) $L(H)$ is H- α -Regular space.
- 2) For each $l \in l(H)$, F is α -closed a post stage and $F \subseteq M(F)$, there exists a post stage αK s.t. $l \in l(H) \subseteq \alpha \text{cl}_H(l(H)) \subseteq M(F)$.

Proof: it is obvious.

Remark 4.5 the converse of above Proposition is not true. The following example shows that the opposite of the above theorem is incorrect.

Example 4.6: Recall Example 3.3. Then $\{l_1\}$ is α -closed a post stage and $\{l_1\} \subseteq M(\{l_1\})$, there exists a post stage K s.t, $\{l_1\} \in l(H) \subseteq \text{cl}_H(\{l_1\}) \subseteq M(\{l_1\})$, but $L(H)$ is not H- α -Regular space.

Proposition 4.7. Let $(L(H), \sigma_H)$ be a topological graph space. Then the following are equivalents:

- 1) $L(H)$ is H- β regular space.
- 2) For each $l \in l(H)$, F is β -closed a post stage and $F \subseteq M(F)$, there exists a post stage βK s.t, $l \in l(H) \subseteq \beta \text{cl}_H(l(H)) \subseteq M(F)$.

Proof: it is obvious.

Remark 4.8 the converse of above Proposition is not true. The following example shows that the opposite of the above theorem is incorrect.

Example 4.9: Recall Example 3.3. Then $\{l_1, l_3\}$ is β -closed a post stage and $\{l_1, l_3\} \subseteq M(\{l_1, l_3\})$, there exists a post stage K s.t. $\{l_1, l_3\} \in l(H) \subseteq cl_H(\{\{l_1, l_3\}\}) \subseteq M(\{\{l_1, l_3\}\})$, but $L(H)$ is not H - β -Regular space.

Proposition 4.10. Let $(L(H), \sigma_H)$ be H -Regular topological graph space and $l_1, l_2 \in l(H), l_1 = l_2$. Then $cl_H(l_1) = cl_H(l_2)$.

Proof: Assume that $(L(H), \sigma_H)$ be H -Regular and $l_1, l_2 \in l(H), l_1 = l_2$. Suppose that $cl_H(l_1) \neq cl_H(l_2)$. Then $l_1 \notin cl_H(l_2)$ and $l_2 \notin cl_H(l_1)$, since $l_1 \notin cl_H(l_2)$ then there exists a post K s.t. $cl_H(l_1) \subseteq K$, so $l_1 \in M(cl_H(l_2))$ and $l_1 \in M(K)$, which is a closed post stage. So $cl_H(l_1) \subseteq M(K)$. Thus $cl_H(l_1) = cl_H(l_2)$.

Proposition 4.11. Let $(L(H), \sigma_H)$ be H - α -Regular topological graph space and $l_1, l_2 \in l(H), l_1 = l_2$. Then $\alpha cl_H(l_1) = \alpha cl_H(l_2)$.

Proof: it is clear.

Proposition 4.12. Let $(L(H), \sigma_H)$ be H - β -Regular topological graph space and $l_1, l_2 \in l(H), l_1 = l_2$. Then $\beta cl_H(l_1) = \beta cl_H(l_2)$.

Proof: it is clear.

Proposition 4.13. Let $(L(H), \sigma_H)$ be a topological graph space. Then the following are equivalents:

- 1) $L(H)$ is H -Normal space.
- 2) For each $l \in l(H)$, F is closed a post stage and $F \subseteq M(F)$, there exists a post stage K s.t. $l \in l(H) \supseteq int_H(l(H)) \subseteq M(F)$.

Proof: (1) $\Rightarrow$ (2) Assume that $L(H)$ is H -Regular space, F is closed a post stage and $l \in M(F)$, then there exists a post stage K s.t. $l \in l(H) \supseteq int_H(l(H))$, so that

$$M(int_H(l(H))) \text{ and } int_H(l(H)) \subseteq M(F). \text{ Thus } l \in l(H) \supseteq int_H(l(H)) \subseteq M(F).$$

(2) $\Rightarrow$ (1) Assume that F is closed a post stage, $l \in M(F)$, so that there exists a post stage K s.t. $l \in l(H) \supseteq int_H(l(H)) \subseteq M(F)$. Thus $l \in l(H) \supseteq int_H(l(H)) \subseteq M(F)$, so $F \subseteq M(int_H(l(H)))$ which is a disjoint post stage. Thus $L(H)$ is H -Normal space.

Remark 4.14 the converse of above Proposition is not true. The following example shows that the opposite of the above theorem is incorrect.

Example 4.15: Recall Example 3.15. Then $\{l_1\}$ is closed a post stage and $\{l_1\} \subseteq M(\{l_1\})$, there exists a post stage K s.t. $\{l_1\} \in l(H) \subseteq cl_H(\{\{l_1\}\}) \subseteq M(\{\{l_1\}\})$, but $L(H)$ is not H -Normal space.

Proposition 3.16. Let $(L(H), \sigma_H)$ be a topological graph space. Then the following are equivalents:

- 1) $L(H)$ is H - α -Normal space.
- 2) For each $l \in l(H)$, F is α -closed a post stage and $F \subseteq M(F)$, there exists a post stage αK s.t. $l \in l(H) \supseteq \alpha int_H(l(H)) \subseteq M(F)$.

Proof: it is obvious.

Remark 4.17 the converse of above Proposition is not true. The following example shows that the opposite of the above theorem is incorrect.

Example 4.18: Recall Example 3.8. Then $\{l_2\}$ is α -closed a post stage and $\{l_2\} \subseteq M(\{l_2\})$, there exists a post stage K s.t. $\{l_2\} \in l(H) \subseteq cl_H(\{\{l_2\}\}) \subseteq M(\{\{l_2\}\})$, but $L(H)$ is not H - α -Regular space.

Proposition 4.19. Let $(L(H), \sigma_H)$ be a topological graph space. Then the following are equivalents:

- 1) $L(H)$ is H - β -Normal space.
- 2) For each $l \in l(H)$, F is β -closed a post stage and $F \subseteq M(F)$, there exists a post stage βK s.t. $l \in l(H) \supseteq \beta \text{int}_H(l(H)) \subseteq M(F)$.

Proof: it is obvious.

Remark 4.20 the converse of above Proposition is not true. The following example shows that the opposite of the above theorem is incorrect.

Example 4.21: Recall Example 3.11. Then $\{l_1, l_2\}$ is β -closed a post stage and $\{l_1, l_2\} \subseteq M(\{l_1, l_2\})$, there exists a post stage K s.t. $\{l_1, l_2\} \in l(H) \subseteq cl_H(\{\{l_1, l_2\}\}) \subseteq M(\{\{l_1, l_2\}\})$, but $L(H)$ is not H - β -Regular space.

Proposition 4.22. Let $(L(H), \sigma_H)$ be H - normal topological graph space and $l_1, l_2 \in l(H), l_1 = l_2$. Then $\text{int}_H(l_1) = \text{int}_H(l_2)$.

Proof: Assume that $(L(H), \sigma_H)$ be H -Normal and $l_1, l_2 \in l(H), l_1 = l_2$. Suppose that $\text{int}_H(l_1) \neq \text{int}_H(l_2)$. Then $l_1 \notin \text{int}_H(l_2)$ and $l_2 \notin \text{int}_H(l_1)$, since $l_1 \notin \text{int}_H(l_2)$ then there exists a post K s.t. $\text{int}_H(l_1) \supseteq K$, so $l_1 \in M(\text{int}_H(l_2))$ and $l_1 \in M(K)$, which is a closed post stage. So $\text{int}_H(l_1) \supseteq M(K)$. Thus $\text{int}_H(l_1) = \text{int}_H(l_2)$.

Proposition 4.23. Let $(L(H), \sigma_H)$ be H - α -Normal topological graph space and $l_1, l_2 \in l(H), l_1 = l_2$. Then $\alpha \text{int}_H(l_1) = \alpha \text{int}_H(l_2)$.

Proof: it is clear.

Proposition 4.24. Let $(L(H), \sigma_H)$ be H - β -Normal topological graph space and $l_1, l_2 \in l(H), l_1 = l_2$. Then $\beta \text{int}_H(l_1) = \beta \text{int}_H(l_2)$.

Proof: it is clear.

5. Conclusion

The regular space and the normal space were generalized to the space of the topological graph statement, as well as the α and β were generalized to both spaces, and we studied the relationships among these generalizations and gave us opposite examples of cases in which the opposite is not valid. We also obtained cases for all these generalizations.

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