Impact of Poisson parameter of Impulsive Multiplicative Noise on Impairment of Single-tone Sinusoid Signals

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Abstract: *Performance of FFT frequency estimation of single tones based instantaneous frequency estimators for noisy single-tone sinusoid signal is investigated under impulsive multiplicative noise with different value of Poisson parameter as well as comparative study on the performance of the FFT frequency estimation methods under impulsive, Gaussian and uniform multiplicative noise with different values of power is presented. This paper presents simulation of impulsive noise and analysis study of its damage effect on FM signal in time domain. Effect of Poisson parameter in damage effect of impulsive noise on FM signal has been investigated. Simulation results show that impulsive noise has less impairment effect at high value of Poisson parameter. FFT frequency estimation method is the best estimator under impulsive multiplicative noise in terms of minimum mean squared estimation error, especially at high value of Poisson parameter.*

Keywords: *multiplicative noise; impulsive noise; FM signal; Poisson parameter; AWGN.*

1. Introduction

In wireless communication systems common kinds of noise is impulsive noise, which is modelled as Poisson-Gaussian model [Ghadimi *et al.*, 2012]. Source of impulsive noise is human-made (physical or industrial) source such as mechanical switches, even light switches, power lines [Ghadimi et al., 2012; Gurubelli et al., 2014]. Unlike additive noise, which effect amplitude of signal, multiplicative noise power is affecting both the amplitude and phase of the signal therefore, it can be considered as most severe degradation noise that affects signal and system [Tuzlukov, 2002]. There are many type of multiplicative noise; impulsive noise is one of these types that damages signal and weak performance and reliability of system especially image and communication system in spite of a high signal-to-noise ratio. [Tuzlukov, 2002; Al-Mawali *et al.*, 2010]. Impulsive noise has very high amplitude (hundred micro-volts) and short duration so that it causes great impairments and high error rate during transmission data in power line communication system (PLC) according to these features where noise in network is modelled as background Gaussian noise and Poisson-Gaussian noise (impulsive noise) [Ghadimi *et al.*, 2012; Al-Mawali, 2011]. Signals in most important applications such as communication, biomedical, sonar and radar, are nonstationary signals [König, 1996]. Frequency modelled (FM) signals are nonstationary signal which their frequency content is varying with time and defined by instantaneous frequency (IF), therefore IF estimation of signal under noise is considered major problem in important fields [Cohen, 1995].

[Liu et al, 2016] proposed estimation methods to estimate carrier frequency of signal under impulsive noise, this proposal method exceed the conventional DFT method.

[Gurubelli et al., 2014] proposed effective and robust frequency estimator, which estimated exact frequency of signal under impulsive noise without iteration search.

Few literatures focus on multiplicative noise especially when it modelled as impulse process. This paper analyses the effects of impulsive multiplicative noise on FM signals.

The paper is organized as follows: The problem formulation is defined in section 2. In section 3, Methodology and Simulation Experiments are presented. In section 4, simulation details and results are explained. Finally, In section 5 conclusions are presented.

2. Problem Formulation

In this paper the noisy signal can be modelled as:

$$y(t) = n(t)\, sin\big(\phi(t)\big) + \epsilon(t) \qquad (1)$$

where $n(t)$ is multiplicative noise, $\phi(t)$ is the initial phase and $\epsilon(t)$ is an additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_a{}^2$; with $n(t)$ and $\epsilon(t)$ considered as independent processes.

First, consider $n(t)$ as white Gaussian noise with zero mean and variance $\sigma_m{}^2$. And then $n(t)$ consider as impulsive noise with Poisson-Gaussian model.

Linear frequency modulation (LFM) law of signal model in equation (1) has been modelled as [Boashash *et al.*, 1990]:

$$s(t) = e^{\{j2\pi\left(f_0 t+\frac{\alpha}{2}t^2\right)\}} \quad (2)$$

where α is the linear modulation index, and f_0 is the initial frequency (in Hertz).

In this paper, Gaussian, impulsive and uniform models have been considered. Since impulsive noise sources are human -man sources environment, therefore it non Gaussian model, so it can be modelled as Poisson-Gaussian noise [Gurubelli et al., 2014] as bellow:

$$i_k = b_k \cdot g_k \quad (3)$$

Where b_k is Poisson process that is modeling the arrival time of impulsive noise at instant k with parameter λ which denote the rate of unit per second.

A random variable X is said to be Poisson if its pdf is given by [Hogg and Craig, 1978]:

$$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}\ ; \quad x = 0,1,2,\ldots.. \quad (4)$$

$\mathcal{E}\{X\} = \lambda$; var(X) $= \lambda$.

Where $P(X = x)$ is the probability of event of x arrivals in unit time, thereby when X represents the time count of arrival of impulsive noise, then it distributed with above Poisson PDF.

And g_k is Gaussian process that is used to model the amplitude of impulsive noise with zero mean and variance (power) σ^2, so the total power of impulsive noise is [Al-Mawali *et al.*, 2010]:

$$n_p = \frac{\sigma^2}{\lambda} \quad (5)$$

Gaussian process can be simulated by using probability density function (pdf) with zero mean and variance (power) σ^2 as follows [Hussain *et al*., 2011]:

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}}e^{-n^2/2\sigma^2} \quad (6)$$

In this paper, uniform noise also consider by using its probability density function (pdf) as:

$$p(x) = \frac{1}{b-a};\ -\infty < a < b < \infty$$

The mean and variance of this distribution are given by:

$$\mu = \mathcal{E}\{X\} = \frac{a+b}{2};\ \text{var} = \mathcal{E}\{(X-\mu)^2\} = \frac{(b-a)^2}{12}$$

3. Methodology and Simulation Experiments

I. Estimation methods

In this paper, maximum likelihood (ML) estimator using Fast Fourier Transform (FFT) with interpolated peak estimation based frequency estimation is used to estimate frequency of noisy signal under three kinds of multiplicative noise (Gaussian, Uniform and impulsive) where estimated frequency can be given by the peak of the Fourier transform occurs (Rife and Boorstyn, 1974):

$$f_{ML} = arg(max|X(f)|) \quad (7)$$

Where f_{ML} was estimated frequency, arg return the index of peak of $X(f)$ and $X(f)$ is the Fourier transform of the single-tone signal $x(t)$, computed from the sampled version of the input signal $x(n)$ of length N by the DFT as:

$$X(k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x(n)e^{(-\frac{2\pi kn}{N})};\ 0 \le k \le N \quad (8)$$

II. Signal to noise ratio (SNR) of noisy FM signal

Since the multiplicative noise power is affecting both the amplitude and phase of the signal, the signal-to-noise ratio (SNR) of noisy FM under effect of both AWGN and MN is defined as follows:

$p_{mn} = (p_n + p_m)$, then

$$SNR = \frac{p_x}{p_{mn}} \quad (9)$$

where p_x being the signal power, p_m being the MN power, and p_n is the additive noise power.

The relative squared-error under each SNR and MN power is calculated as follow:

$$e = |\,((F_O - f_o)/f_o)|^2 \quad (10)$$

Where F_O is estimated frequency and f_o is actual frequency of noiseless signal.

III. Simulation hypotheses

We simulated the above algorithms with signal model under multiplicative noise (MN) as per Equation (1) using MATLAB.

Linear frequency modulation (LFM) signal is simulated as follows:

$$y(t) = n(t)\sin\left\{j2\pi\left(f_o t + \frac{\alpha}{2}t^2\right)\right\} + \epsilon(t) \quad (11)$$

where $n(t)$ is multiplicative zero-mean noise and $\epsilon(t)$ is additive white Gaussian noise (AWGN) with zero mean, $\alpha = 0.5$ is slope of IF low of the signal. the simulated signal has total time length $L = 10$s, the sampling interval is $T_s = 0 \cdot 001$s, and the number of samples is given by $N = [L/T_s]$. The signal amplitude is $A = 1$ volt, ω_o is angler frequency $\omega_o = 2\pi f_o$, where $f_o = 23$Hz. First, MN has been modelled as zero-mean Gaussian, then it has been modelled as impulsive processes as per equation (3) and (4) with Poisson parameter $\lambda = 20$. Monte Carlo simulations were performed with $M = 20$ realizations.

4. Simulation results

From simulation results of implementing maximum likelihood (ML) estimator, using Fast Fourier Transform (FFT) with interpolated peak estimation using complex single-tone sinusoid affected by additive Gaussian and zero mean multiplicative in MATLAB. Comparative study of above estimation method performance in term of mean square error (MSE) using three models of multiplicative noise (Gaussian, impulsive and uniform) is present with different value of Poisson parameter as shown in figures (13-18).

Effect of impulsive noise on FM signal in time domain with different value of arrival time λ is shown as in figures (1-12)

Figures (1)-(4) show one realization of impulsive noise with different value of λ (Poisson parameter) in time domain.

Figures (5)-(8) show noisy FM signal under additive Gaussian noise and multiplicative impulsive noise with different value of λ (Poisson parameter) in time domain.

Figures (9) and (10) show one realization of Gaussian noise at power $= -3$dB and 3dB respectively.

Figures (11) and (12) show noisy FM signal under multiplicative Gaussian noise at power $= -3$dB and $=$ 3dB respectively in time domain.

It is clear that impulsive noise has less destructive effect at high value of λ. Since impulsive noise is modelled as Poisson-Gaussian representation, it clear from results in figure (1)-(4) that amplitude of impulsive noise increase and be more damage at high power of Gaussian noise. Nevertheless, at low value of λ and high power of Gaussian noise the impulsive noise effect approaches to the Gaussian noise as evidenced by simulation results in figures (1)-(12).

Figures (13)-(15) show the estimated frequency versus SNR using interpolated FT peak for various multiplicative noise models (Gaussian, impulsive and uniform) with different powers and $\lambda = 30$. It is clear that FT method hold at SNR=0 dB in case of impulsive noise with $p_m = 30$ while in case of uniform and Gaussian noise it hold at SNR more than10 dB.

Figures (16)-(18) show the estimated frequency versus SNR using interpolated FT peak for various multiplicative noise models (Gaussian, impulsive and uniform) with different powers and $\lambda = 3$.

Note that under Impulsive noise better frequency estimate is obtained at lower SNR under the same multiplicative noise power especially at high value of Poisson parameter λ (low value of arrival time of impulse), high signal-to-noise ratios and $p_m = 30$.

5. Conclusions

This paper presented a study on the impairment effect of Impulsive multiplicative noise (MN). Poisson parameter of impulsive noise is important factor that effect on damaged effect of impulsive noise on signals as well as on performance of frequency estimation method where the low value of Poisson parameter increase the severe effect of noise and damage of signal, thereby impulsive noise effect be closed to Gaussian noise. Impulsive noise is less destructive than Gaussian or uniform noise with the same power and high value of Poisson parameter.

6. Acknowledgements

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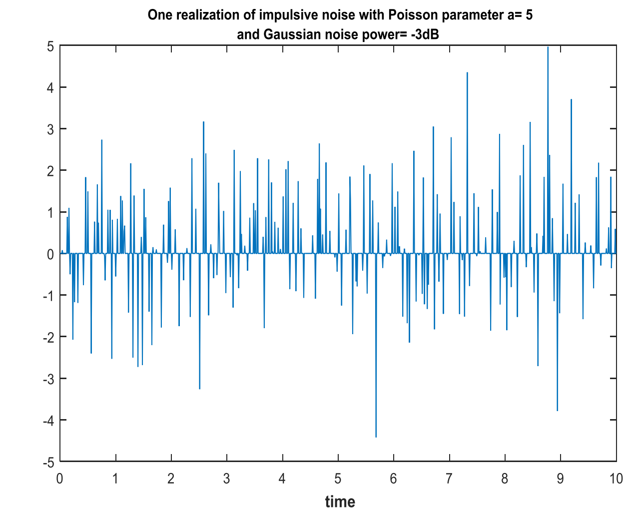

Fig. 1. Time domain representation of one realization of impulsive noise with $\lambda = 5$.

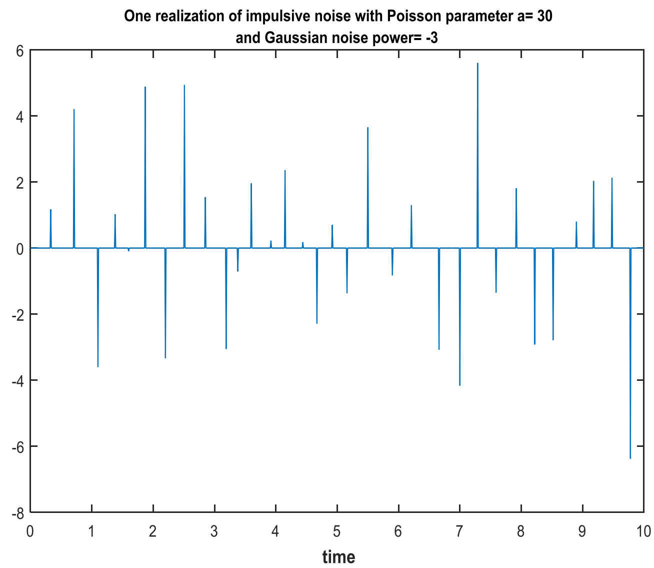

Fig. 2. Time domain representation of one realization of impulsive noise with $\lambda = 30$.

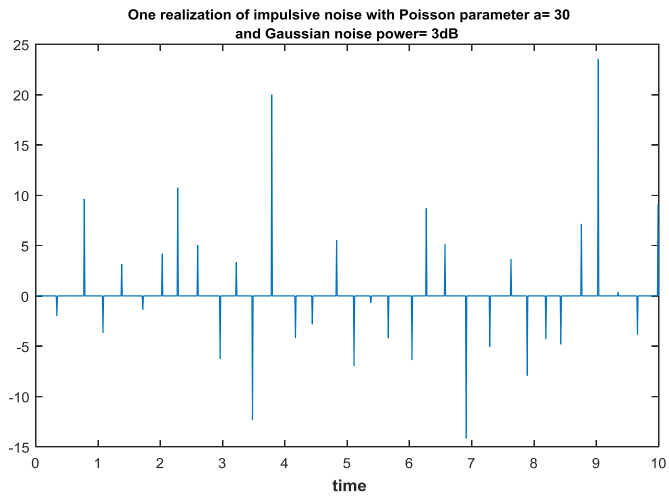

Fig. 3. Time domain representation of one realization of impulsive noise with $\lambda = 30$.

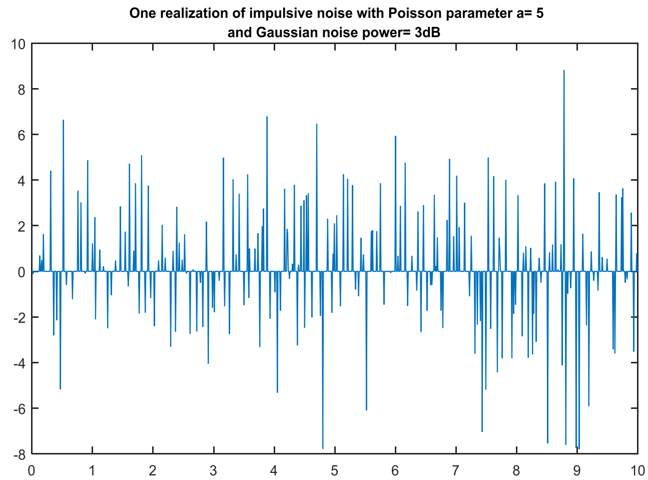

Fig. 4. Time domain representation of one realization of impulsive noise with $\lambda = 5$.

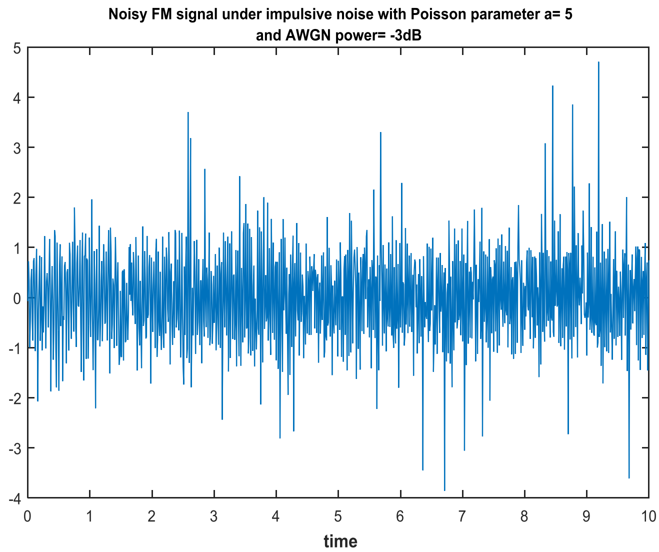

Fig. 5. Noisy FM signal under impulsive noise with $\lambda = 5$ and Gaussian noise power $= -3$.

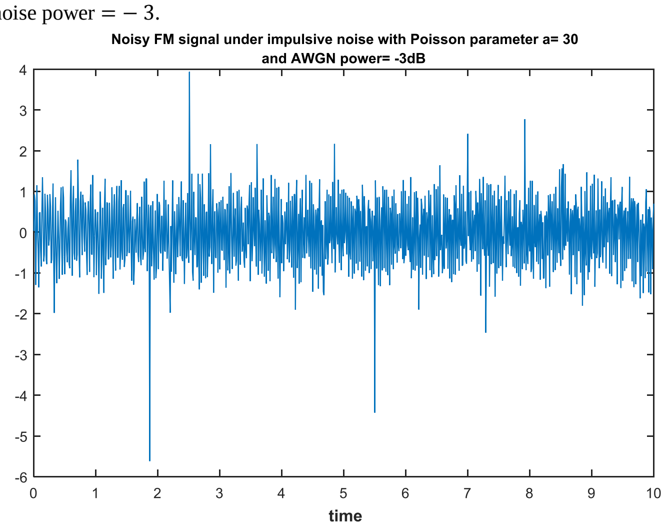

Fig. 6. Noisy FM signal under impulsive noise with $\lambda = 30$ and Gaussian noise power $= -3$.

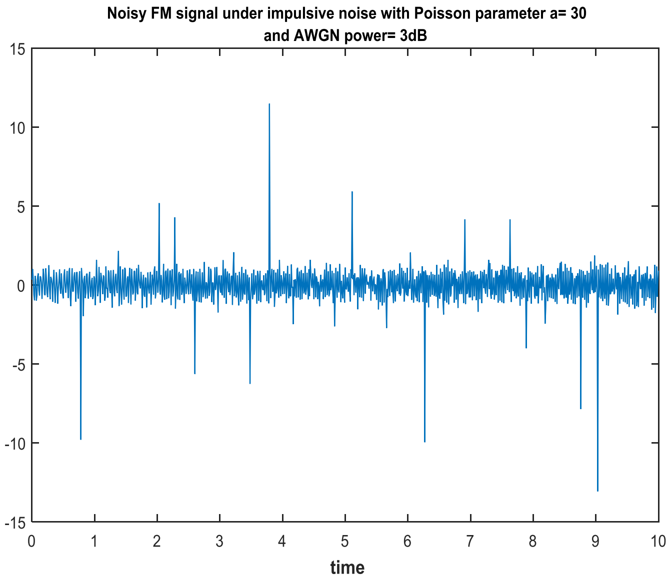

Fig. 7. Noisy FM signal under impulsive noise with $\lambda = 30$ and Gaussian noise power $=$ 3dB.

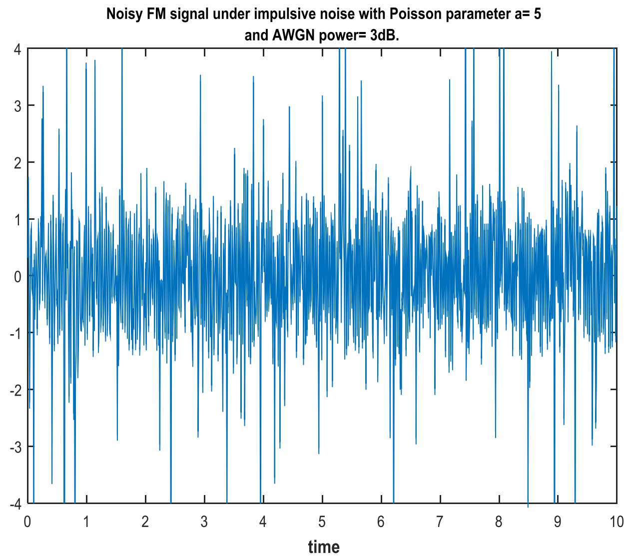

Fig. 8. Noisy FM signal under impulsive noise with $\lambda = 5$ and Gaussian noise power $=$ 3.

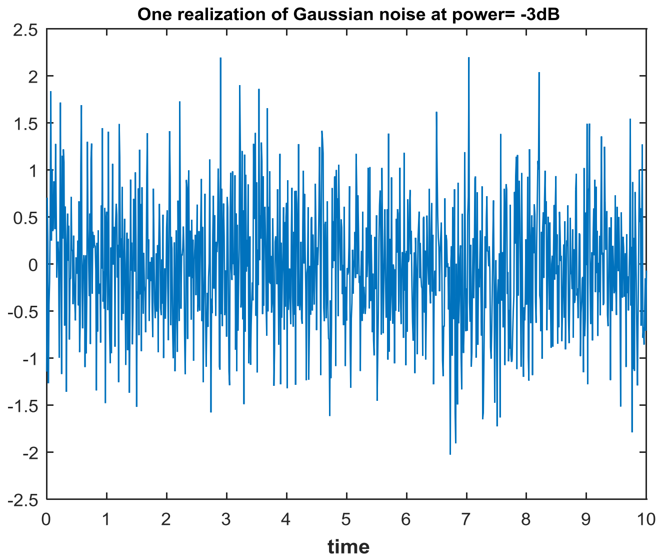

Fig. 9. Time domain representation of one realization of Gaussian noise at power $= -3$ dB.

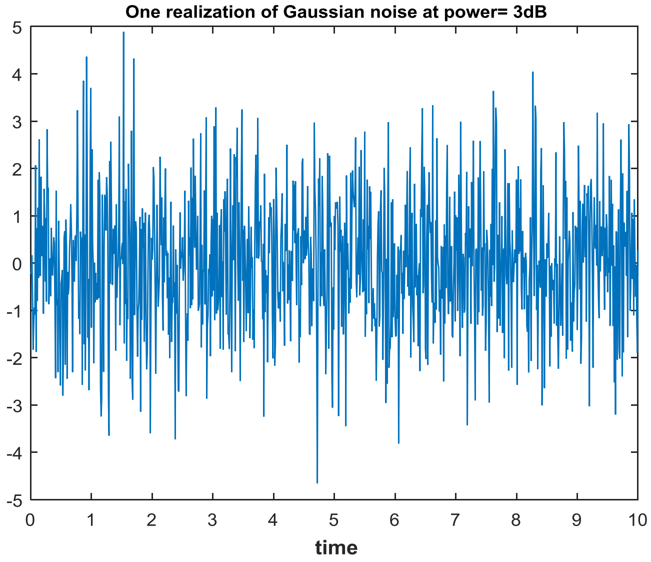

Fig. 10. Time domain representation of one realization of Gaussian noise at power $=$ 3 dB..

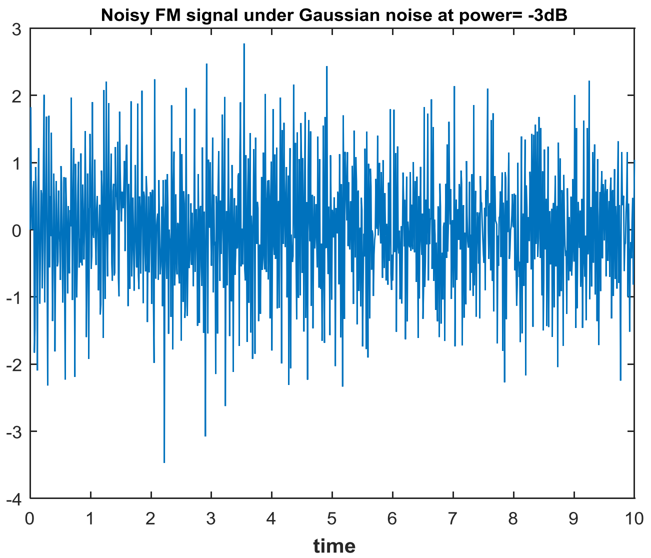

Fig. 11. Noisy FM signal under Gaussian noise at power $= -3$dB.

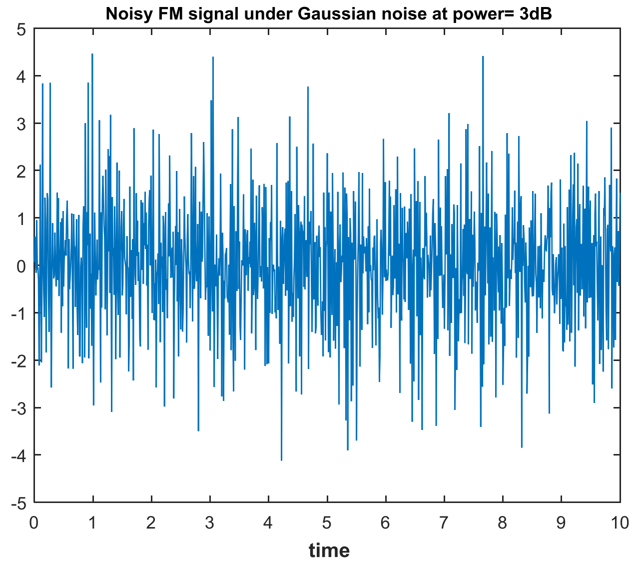

Fig. 12. Noisy FM signal under Gaussian noise at power $=$ 3dB.

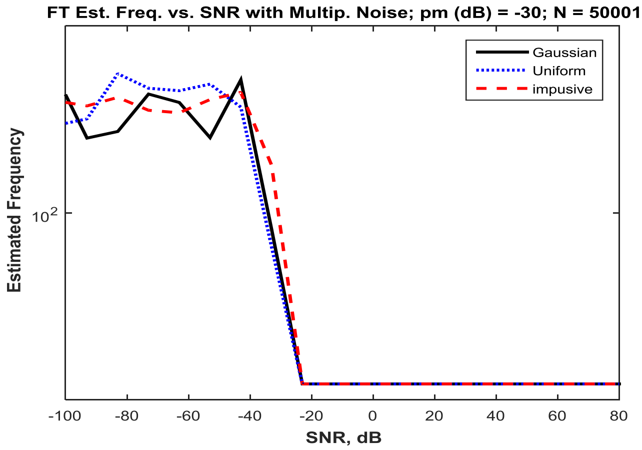

Fig. 13. Interpolated FT peak method for various multiplicative noise models.with power = −30dB and $\lambda = 30$.

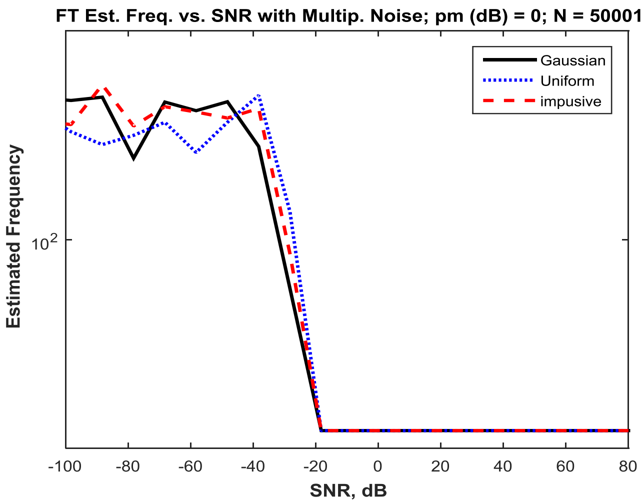

Fig. 14. Interpolated FT peak method for various multiplicative noise models.with power = 0dB and $\lambda = 30$.

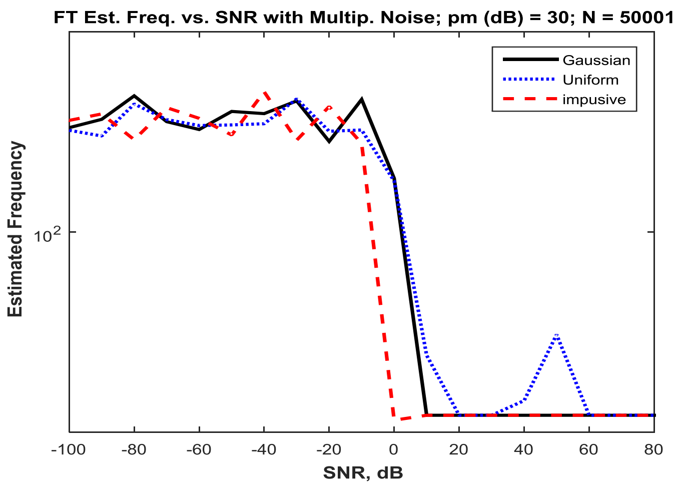

Fig. 15. Interpolated FT peak method for various multiplicative noise models.with power = 30dB and $\lambda = 30$.

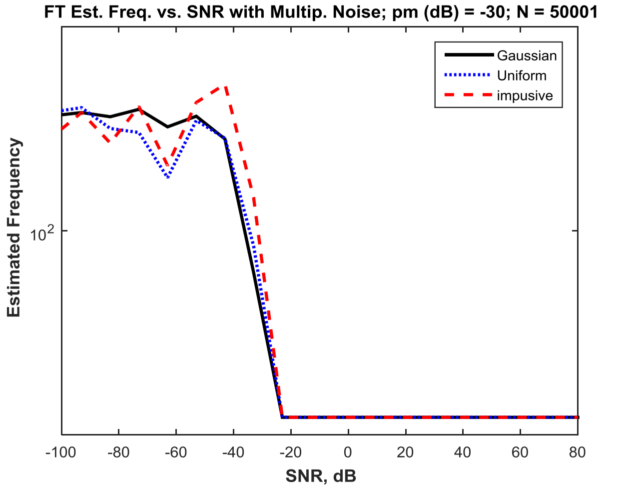

Fig. 16. Interpolated FT peak method for various multiplicative noise models.with power = −30dB and $\lambda = 3$.

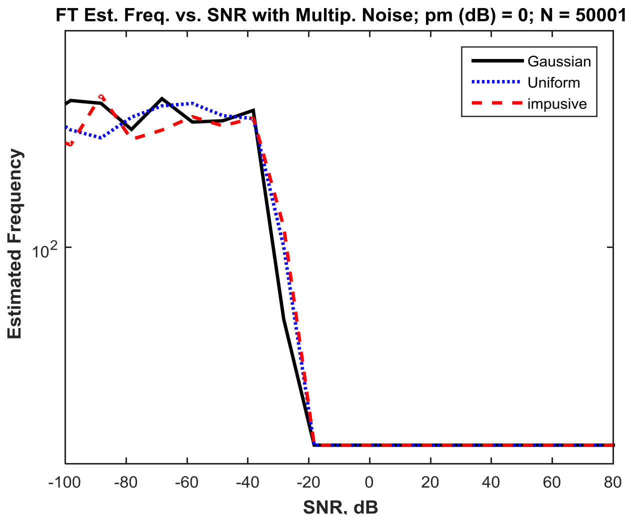

Fig. 17. Interpolated FT peak method for various multiplicative noise models.with power = 0dB and $\lambda = 3$.

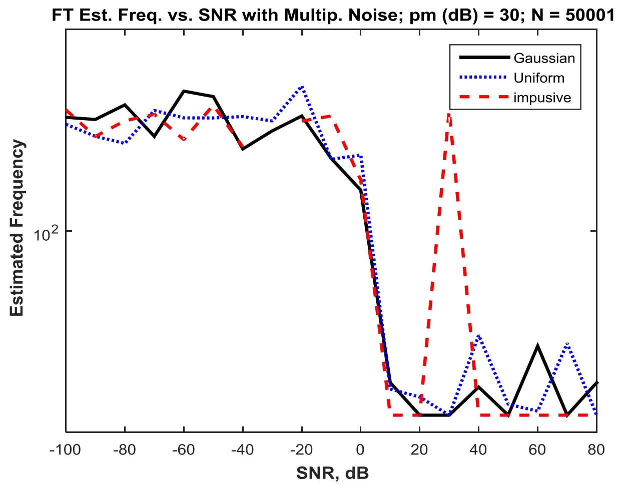

Fig. 18. Interpolated FT peak method for various multiplicative noise models.with power = 30dB and $\lambda = 3$.