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Instantaneous Frequency Estimators of FM Signals: Performance under Models of Multiplicative Noise

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Abstract

This paper presents a study on the performance of instantaneous frequency estimators of mono-component FM signals, including a single-tone sinusoid, under additive white Gaussian noise (AWGN) with different kinds of multiplicative noise. For single-tone signals, two main estimators are considered: maximum likelihood (ML) estimator using Discrete Fourier Transform (DFT) with interpolated peak estimation, and autocorrelation method. For linear and non-linear FM signals, peak of a specific time-frequency distribution (TFD), the periodogram, has been considered. Three statistical models of multiplicative noise are analyzed (Gaussian, Rayleigh, uniform). Simulation results showed that Rayleigh noise is less destructive than Gaussian or uniform noise with the same power; also, DFT is still better than correlation for IF estimation in terms of error and robustness against all noise models.

AMS subject classification:
Keywords: Frequency estimation, multiplicative noise, maximum likelihood estimator, time-frequency distribution, Rayleigh noise.

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1. Introduction

Frequency estimation appears in a wide range of engineering applications, e.g. in communications, radar, radio frequency identification and resonance sensor systems [1, 2]. Noise is a major problem in signal processing, and the type of noise (its statistical model) is a major factor that affects the performance of frequency estimation methods [3]. There are different kinds of noise encountered in application. According to its structural properties, noise can be additive, phase, and multiplicative [3]. According to its model, it can be Gaussian, Poisson, impulsive, non-Gaussian, among other models [3]. Many areas of signal processing suffer from multiplicative noise, i.e., noise that affects the amplitude of the signal. For example, in time-selective fading in communication channels, amplitude variations are usually modeled as Rayleigh or Rician distributed [4]. There are various methods for estimating the frequency and phase of sinusoidal signals, however, these methods widely differ in performance, especially in regards to accuracy and computational complexity [1, 2, 5]. A comparative study of different FFT estimation methods for the problems of detecting and measuring the frequency of a single tone in additive white noise have been presented in [6].

An important kind of sinusoidal signals is single-tone sinusoids, which are widely used as information carriers [7]. Using discrete Fourier transform (DFT) of the collected samples is a common method for frequency estimation [8], however, it is a costly method as one should first calculate a large number of DFT samples. On the other hand, using a few autocorrelation samples is more efficient in computation but less accurate [9, 10].

To achieve extra benefits for frequency estimation algorithms have been proposed in [11, 12] that use higher-order autocorrelation lags. Lei Zhu and Ji-Hong Shen in [13] proposed a new formulation of frequency estimation using the linear prediction (LP) property and high lags autocorrelation of sinusoidal signals enjoys low computational complexity because it only need the autocorrelation coefficients and its unwrapping processing. Ubolkosold [14] presented an estimator that relies on the nonlinear least-squares (NLS) principle in conjunction with the summation-by-parts formula. This estimator is simple and robust to the lack of information on the form of the correlation of the multiplicative noise. In [15], a comprehensive analysis of frequency estimation in the presence of multiplicative and additive noise both in time domain and frequency domain was provided, the multiplicative noise is assumed to be circular Gaussian process.

In many application, estimation of the instantaneous frequency of a nonstationary signal is important. Instantaneous frequency of nonstationary signals can be described by time-frequency distributions, such as the Wigner-Ville distribution (WVD) [16]. Boashash [17] developed the concept of instantaneous frequency (IF) and discussed different estimation methods based on time frequency distributions. The definition of IF has further been improved in [18].

Despite its important effects, there have been only a few works on multiplicative noise due its difficult modeling and analysis. This paper analyzes the effects of different statistical models of multiplicative noise (Gaussian, Rayleigh and uniform distribution) on frequency estimation methods of mono-component single-tone as well as FM signals. A comparative study is presented on the performance of the above frequency estimation methods (DFT, correlation, and TFD) under different models of multiplicative noise with different values of power.

The paper is organized as follows: The single-tone signal model is defined in Section 2. In Section 3, IF definition and FM signal model are presented. In Section 4, multiplicative noise models are presented. Section 5 presents frequency estimation methods based on spectral domain (Fourier transform), while Section 6 presents frequency estimation based on the signal autocorrelation. In Section 7, IF estimation based on time-frequency distribution is handled. Finally, in Section 8 simulation details and results are explained.

2. The Single-Tone Signal Model under Gaussian Noise

The single-tone noisy sinusoid model is given by:

$$y(t) = n(t) \cdot \cos(\omega_o t + \phi_o) + \epsilon(t) \tag{1}$$

where $n(t)$ is multiplicative noise, ω_o is the frequency of the signal, ϕ_o is the initial phase and $\epsilon(t)$ is an additive white Gaussian noise with zero mean and variance σ_a^2; with $n(t)$ and $\epsilon(t)$ considered as independent processes. First, consider $n(t)$ as white Gaussian noise with zero mean and variance σ_m^2. Without loss of generality consider $\phi_o = 0$. It can be shown that the product $n(t) \cdot \cos(\omega_o t)$ is Gaussian with zero mean and variance σ_m^2.

Proof. Consider the signals $u(t) = n(t)e^{j\omega_o t}$ and $v(t) = n(t)e^{-j\omega_o t}$. Their autocorrelations are given by:

$$\begin{aligned} R_u(\tau) &= \mathcal{E}\{u(t)u^*(t+\tau)\} \\ &= \mathcal{E}\{n(t)n(t+\tau)e^{j\omega_o t}e^{-j\omega_o(t+\tau)}\} \\ &= \mathcal{E}\{n(t)n(t+\tau)e^{-j\omega_o \tau}\} \\ &= \mathcal{E}\{n(t)n(t+\tau)\}\mathcal{E}\{e^{-j\omega_o \tau}\} \quad \text{[Independent]} \\ &= R_n e^{-j\omega_o \tau} \end{aligned}$$

where $\mathcal{E}$ is the statistical expectation functional. Similarly: $R_v(\tau) = R_u(\tau)e^{+j\omega_o \tau}$.

Since $\cos(\omega_o t) = \frac{1}{2}(e^{j\omega_o t} + e^{-j\omega_o t})$ [Euler], then

$$R_y(\tau) = e^{j\omega_o t}\frac{1}{2}[R_u(\tau) + R_v(\tau)] = R_n(\tau)\cos(\omega_o \tau).$$

Since $n(t)$ is wide-sense stationary (WSS), then $R_n(\tau) = \frac{\eta}{2}\delta(\tau)$; hence, $\mathcal{S}_n(f) = \frac{\eta}{2}$, where $\frac{\eta}{2}$ is a constant that represents the double-sided power spectral density (psd) of noise, $\mathcal{S}_n(f)$. Note that according to Wiener-Kinchin Theorem (WKT) [7], we have $\mathcal{S}_n(f) = \underset{\tau \to f}{\mathcal{F}}[R_n(\tau)]$, where $\mathcal{F}$ represents the Fourier transformation.

The psd of the signal $y(t)$ is given by:

$$S_y(f) = \frac{1}{2}[\mathcal{S}_n(f + f_o) + \mathcal{S}_n(f - f_o)] = \frac{\eta}{2}$$

Hence, by WKT,

$$R_y(\tau) = \frac{\eta}{2}\delta(\tau),$$

which means that $y(t)$ is white noise with the same power σ^2 as $n(t)$. On the other hand, the autocorrelation of the signal $y(t)$ can be obtained as:

$$\begin{aligned} R_y(\tau) &= \frac{\eta}{2}\delta(\tau)\cos(\omega_o\tau) \\ &= \frac{\eta}{2}\delta(\tau)\cos(0) = \frac{\eta}{2}\delta(\tau) \end{aligned}$$

where the multiplication property of the delta function has been used [7, 19]: $x(t)\delta(t) = x(0)\delta(t)$ [on condition that $x(t)$ is continuous at $t = 0$]. The mean of $n(t)$ is zero, i.e., $\mathcal{E}\{n(t)\} = 0$. Hence,

$$\begin{aligned} \mathcal{E}\{y(t)\} &= \mathcal{E}\{n(t)\cos(\omega_o t)\} \\ &= \mathcal{E}\{n(t)\} \cdot \mathcal{E}\{\cos(\omega_o t)\} = 0, \end{aligned}$$

which implies that $y(t)$ has zero mean. ■

3. IF and FM Signal Model

Many signals in practice are nonstationary, i.e., with frequency content that varies with time. Examples are those signals found in speech processing, biomedical applications, seismology, machine condition monitoring, radar, sonar, telecommunication, and many other applications [20]. In such signals, the instantaneous frequency (IF) that describes the variations of the frequency content with time is the most important characteristic of the signal. In the case of a frequency-modulated (FM) signals, the IF represents the FM modulation law and is often referred to as simply the IF law [1, 2].

The IF is defined for a given real signal, $s(t)$, as follows [20]:

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \tag{2}$$

where $\theta(t)$ is instantaneous phase of real signal $s(t)$ whose analytic associate is [7, 21]:

$$z(t) = a(t) \cdot e^j\theta(t) = s(t) + j\mathcal{H}[s(t)] \tag{3}$$

where $a(t)$ is instantaneous amplitude, and $\mathcal{H}$ is Hilbert transform of $s(t)$ defined as follows [7]:

$$\mathcal{H}(f) = \left\{ \begin{array}{ll} -j, & f \geq 0 \\ j, & f < 0 \end{array} \right\} = -j \cdot \text{sgn}(f) \tag{4}$$

Hence, in order to deal with IF estimation, the analytic associate is necessary to avoid aliasing.

In this work, the signal model having linear frequency modulation (LFM) law is [22]:

$$s(t) = e^{j2\pi(f_o t + \frac{\alpha}{2} t^2)} \tag{5}$$

where α is the linear modulation index, and f_o is the initial frequency (in Hertz).

Using Equation (2), the LFM signal IF will be:

$$f_i(t) = f_o + \alpha t \tag{6}$$

Quadratic frequency modulation (QFM) signal has also been considered in this work with quadratic IF law as follows:

$$s(t) = e^{j2\pi(f_o t + \frac{\alpha}{2} t^2 + \frac{\beta}{3} t^3)} \tag{7}$$

where β is the quadratic modulation index of the QFM signal, with the quadratic IF law:

$$f_i(t) = f_o + \alpha t + \beta t^2 \tag{8}$$

For FM signals, IF estimation can only be done through time-frequency analysis, as Fourier and correlation methods fail [20].

4. Multiplicative Noise Models

In signal processing systems, the integrity and quality can be realized by understanding the statistical characteristics of the noise process associated with the system. These noise processes are generated by electromagnetic or electronic sources. Considering multiplicative noise, there are different statistical models: Gaussian model, Rice model, Rayleigh model, Hoyt model, and Nakagami model [3]. In this paper uniform model has also been considered. A brief description of each model is given below.

Gaussian model

Multiplicative noise that is mostly encountered in electrical systems has Gaussian probability density function (pdf) with zero mean and variance (power) σ^2 as follows [7]:

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{n^2}{2\sigma^2}}$$

Rayleigh model

A random variable X is said to be Rayleigh if its pdf is given by:

$$p(x) = \frac{x}{B} e^{-\frac{x^2}{2B}} \quad x \geq 0;\ B = b^2$$

where b is a real positive parameter called Rayleigh parameter. This distribution has mean and variance given by:

$$\mathcal{E}\{X\} = b\sqrt{\frac{\pi}{2}}; \quad \text{var}(X) = B\frac{4-\pi}{2}.$$

Hence, Rayleigh noise has a non-zero mean. The second moment (power) of Rayleigh noise is given by:

$$p = \mathcal{E}\{X^2\} = 2B$$

where p denote the power of noise, hence, $b = \sqrt{\frac{p}{2}}$.

Uniformly-distributed model

A random variable X is said to have uniform distribution on $[a, b]$ if its pdf is given by:

$$p(x) = \frac{1}{b-a}; \quad -\infty < a < b < +\infty$$

The mean and variance of this distribution are given by:

$$\mu = \mathcal{E}\{X\} = \frac{a+b}{2}; \quad \text{var} = \mathcal{E}\{(X-\mu)^2\} = \frac{(b-a)^2}{12}$$

If the interval is symmetric, i.e., $a = -b$, then: $\mathcal{E}\{X\} = 0$; var = power = $b^2/3$.

A uniform random variable r on a symmetric interval $[-b, b]$ can be generated using a standard uniformly distributed random variable u as follows: $r = -b + 2bu$ Any standard generator of uniform random variables on [0,1] can be used to simulate u, like the function **rand** on MATLAB.

5. Frequency Estimation of Single-Tone in the Spectral Domain

The maximum likelihood (ML) frequency estimator can be given by the frequency where the peak of the Fourier transform occurs [8]

$$f_{ML} = \arg\big(\max|X(f)|\big) \tag{9}$$

where $X(f)$ is the Fourier transform of the single-tone signal $x(t)$, computed from the sampled version of the input signal $x(n)$ of length N by the DFT as:

$$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-\frac{2\pi kn}{N}}; \quad 0 \le k \le N \tag{10}$$

Due to the discretization, the actual frequency of the sinusoid may reside between DFT samples. Since the index of Fourier transform cannot be a non-integer, interpolation between points near the peak of the DFT can improve the estimation accuracy.

There are different interpolation methods, below are the mostly used ones.

Quadratic interpolation
In this method a quadratic fit of the form $y = a+bx+cx^2$ is sought within a neighborhood of the DFT peak $\max\{X(k)\}$ using three sample counts as follows [5]:

$$\begin{aligned} K-1;\quad u_1 &= |X_{K-1}| \\ K;\quad u_2 &= |X_K| \\ K+1;\quad u_3 &= |X_{K+1}| \end{aligned}$$

where $K = \arg\big(\max\{|X(k)|\}\big)$ is the count number at the absolute maximum of the magnitude of DFT. The actual frequency represented by DFT count K will be given by $F = (K \cdot f_s)/N$, with f_s representing the sampling frequency. The maximum point of the above quadratic is $y_m = -\frac{b}{2c}$, which gives the frequency interpolation:

$$\begin{aligned} u &= K+g \\ g &= \frac{u_3-u_1}{2*(2*u_2-u_1-u_3)} \end{aligned}$$

The frequency would be estimated as $F_0 = \frac{u \cdot f_s}{N}$.

Barycentric method
In this method we use the same form (i.e., $u = K + g$) with the same three points as above but with a different structure of interpolation as follows [23]:

$$g = \frac{u_3-u_1}{u_1-u_2+u_3}$$

The frequency would be estimated as before using the formula $F_0 = \frac{u \cdot f_s}{N}$.

Quinn's first estimator
It takes three DFT points as follows [24]:

$$\begin{aligned} K-1;\quad u_1 &= |X_{K-1}| = r_1 + i \cdot v_1 \quad (\text{with } i = \sqrt{-1}) \\ K;\quad u_2 &= |X_K| = r_2 + i \cdot v_2 \\ K+1;\quad u_3 &= |X_{K+1}| = r_3 + i \cdot v_3 \end{aligned}$$

The following quantities should be computed:

$$\begin{aligned} R &= r_2^2 + v_2^2; \\ S &= \frac{r_3 \cdot r_2 + v_3 \cdot v_2}{R}; \\ T &= \frac{-S}{1-S}; \\ W &= \frac{r_1 \cdot r_2 + v_1 \cdot v_2}{R}; \\ E &= \frac{W}{1-W}; \end{aligned}$$

Now in case $S > 0$ and $W > 0$ then, $g = S$; else, $g = W$; with the final interpolation $u = K + g$; and the frequency estimate of $F_0 = \dfrac{u \cdot f_s}{N}$.

Quinn's second estimator
Using the same three points and other parameters of the first estimator as above, the following steps are applied to get the interpolation [25]:

$$g = \frac{S+W}{2} + H(S^2) + H(W^2)$$

where

$$H(x) = \frac{\frac{1}{4}\ln(3x^2 + 6x + 1) - \frac{\sqrt{6}}{24}\ln(x + 1 - \sqrt{2/3})}{x + 1 + \sqrt{2/3}}$$

Then the interpolation is $u = K + g$; with the final frequency estimate of $F_0 = \dfrac{u \cdot f_s}{N}$.

6. Frequency Estimation Based on Autocorrelation

While the autocorrelation samples can be used to estimate the frequency from the phase of the available signal's autocorrelation using a fixed number of lags [9, 10].

The autocorrelation sequence $\{R_y(m)\}$ of the input signal samples $\{y(n)\}$ can be written as follows:

$$R_y(m) = \frac{1}{N-m} \sum_{n=0}^{N-1} y(n) \cdot y(n-m) \tag{11}$$

An estimate of the frequency f_o of a single-tone sinusoid can be obtained from the phase angle of the autocorrelation samples $\{R_y(m)\}$. By using the minimal order linear predictor [26], which is a special case of the Pisarenko harmonic decomposer frequency estimator [27], and avoiding the case of zero lag ($m = 0$) to get rid of the effect of noise, the frequency can be estimated as follows:

$$m \cdot \omega_o = \measuredangle[R_y(m)] \ \langle \text{mod } 2\pi \rangle \tag{12}$$

$$m \cdot \omega_o = \measuredangle[R_y(m)] + 2k\pi \tag{13}$$

with $0 \leq k < m$; k being an integer. Hence, substituting $m = 1$ (the first autocorrelation sample at non-zero lag) in Equation (13) and putting $l = 0$ will lead to an estimate of frequency as follows:

$$\omega_o = \measuredangle[R_y(1)] \tag{14}$$

which is called the minimum order linear predictor [10]. It is shown that the performance of this linear predictor can be improved by using different correlation lags [28]; also by choosing more than one correlation coefficient [23].

7. IF Estimation Based on Time-Frequency Analysis

For non-stationary signals with time-varying frequency content (like frequency modulated (FM) signals and biomedical signals), the Fourier transform (FT) cannot reveal the time-varying characteristics of the signal (like the IF law) due to the time-averaging process (time-integration) in the FT. In this case there is a need for time-frequency analysis. A time-frequency distributions (TFD) is a double transform from the time-domain into the time-frequency domain representing the Fourier transform of the instantaneous autocorrelation of the analytic associate of the signal. The simplest formula of time -frequency distribution (TFD) is windowed frequency distribution called the short-time Fourier transform (STFT) [7]:

$$\rho_s(t, f) = \int_{-\infty}^{+\infty} s(\lambda)h(\lambda - t)e^{-j2\pi f\lambda}d\lambda = \underset{\lambda\to f}{\mathcal{F}}\{s(\lambda)h(\lambda - t)\} \tag{15}$$

where $s(t)$ is the analytic signal, $h(t)$ is a time window. By solving the optimization problem below, the instantaneous frequency (IF) can be estimated as mentioned in [18, 20, 29, 30]:

$$\hat{f}_i(t) = f_{ML} = \arg\big(\max_f\{\rho_z(t, f)\}\big); \quad 0 \leq f \leq \frac{f_s}{2} \tag{16}$$

where $z(t)$ is analytic signal as in Equation (3). So the TDF is a revealing representation of the non-stationary signal because it shows the distribution of signal's energy over two-dimensional domain: the time-frequency space. In this work, the periodogram (which is $|\text{STFT}|^2$) is used to estimate the IF of linear and quadratic FM signals.

8. Simulation Results

DFT estimation of single-tone frequency The above algorithms using relevant signal models with additive white Gaussian noise (AWGN) and multiplicative noise (MN) using MATLAB. For single-tone sinusoid, the simulated signal has total time length $L = 70s$, the sampling interval is $T_s = 0.001s$, and the number of samples is given by $N = [\frac{L}{T_s}]$. The signal amplitude is $A = 1$ volt, ω_o is angler frequency $\omega_o = 2\pi f_o$, where $f_o = 23$ Hz. MN is modelled as zero-mean Gaussian, Rayleigh, and uniform processes. Monte Carlo simulations were performed with $M = 20$ realizations.

The signal-to-noise ratio (SNR) in the presence of both AWGN and MN is defined as follows:

$$SNR = \frac{p_{xr}}{p_n} \tag{17}$$

where $p_{xr} = (p_x + p_m)$, p_x being the signal power, p_m being the MN power, and p_n is the additive noise power. This is so because the multiplicative noise power is affecting both the amplitude and phase of the signal.

The relative squared-error under each SNR and MN power is calculated as follows:

$$e = \left| \frac{F_o - f_o}{f_o} \right|^2 \tag{18}$$

Figures (1)–(3) show the autocorrelation of Gaussian, uniform and Rayleigh processes respectively. It is clear that the autocorrelation function of Gaussian and uniform processes are weighted delta function in the lag domain, meaning that their samples (in the time domain) are uncorrelated with each other. On the other hand, the Rayleigh noise process is correlated with a maximum at $\tau = 0$. This fact will argue for the smaller error encountered under Rayleigh noise as shown in the Figures (4)–(15). Figures (4)–(6) show the estimated frequency versus SNR using interpolated FT peak for various multiplicative noise models with different powers. Note that under Rayleigh noise better IF estimate is obtained at lower SNR under the same multiplicative noise power.

Note that in cases where the method succeeds in lowering MSE as SNR increases, the curve of MSE goes to a specific asymptote representing a lower bound on error called the Cramer-Rao Bound (CRB) [4].

Figures (7)–(9) show the estimated frequency versus SNR using autocorrelation method for various multiplicative noise models and powers. It is clear that under high MN power this method fails, though better results are obtained under Rayleigh noise due to its inter-correlated structure.

Simulation results of TFD IF estimation of FM signals Linear frequency modulation (LFM) signal is simulated as follows:

$$y(t) = n(t) \cdot \sin\left\{2\pi \left(f_o t + \frac{\alpha}{2} t^2\right)\right\} + \epsilon(t) \tag{19}$$

where $n(t)$ is multiplicative zero-mean noise and $\epsilon(t)$ is additive white Gaussian noise (AWGN) with zero mean, $\alpha = 0.5$ is slope of IF law. Figures (10)–(12) show relative mean-squared error (MSE) of IF estimation at mid-time of the LFM signal versus different SNRs with MN power $= -30$ dB, 0 dB, and 10 dB, respectively. Quadratic frequency modulation (QFM) signal has also been simulated as follows:

$$y(t) = n(t) \cdot \sin\left\{2\pi \left(f_o t + \frac{\alpha}{2} t^2 + \frac{\beta}{3} t^3\right)\right\} + \epsilon(t) \tag{20}$$

where $n(t)$ is multiplicative zero-mean noise (MN), and $\epsilon(t)$ is additive white Gaussian zero-mean noise; with QFM parameters $\alpha = 3$ and $\beta = -0.5$. Monte Carlo simulations

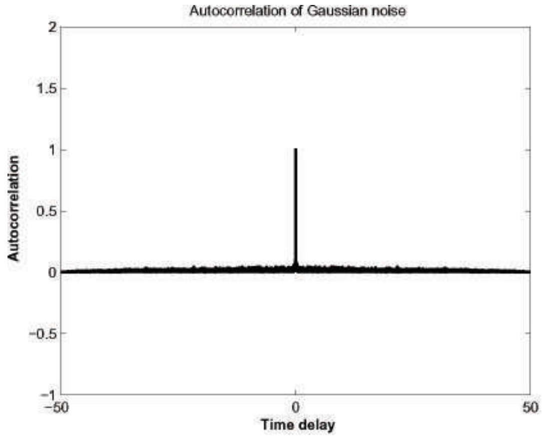

Figure 1: Autocorrelation function of Gaussian noise.

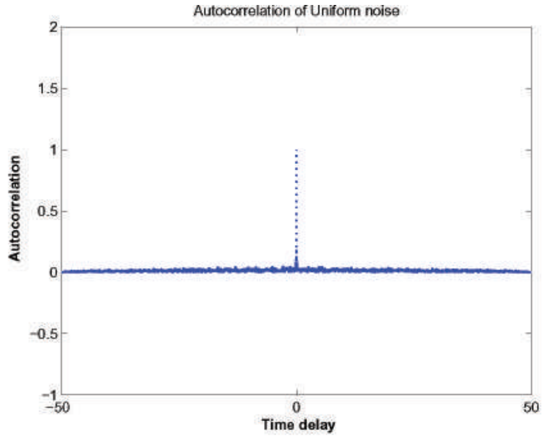

Figure 2: Autocorrelation function of uniform noise.

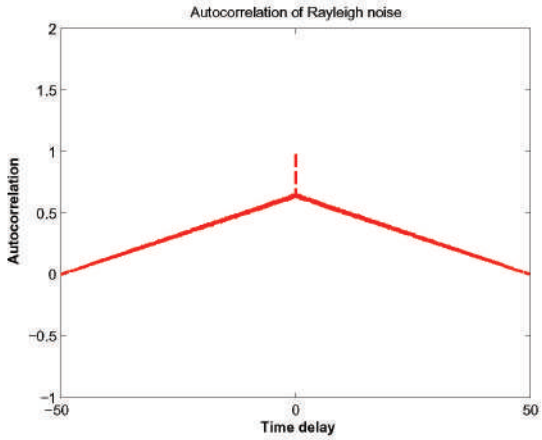

Figure 3: Autocorrelation function of Rayleigh noise.

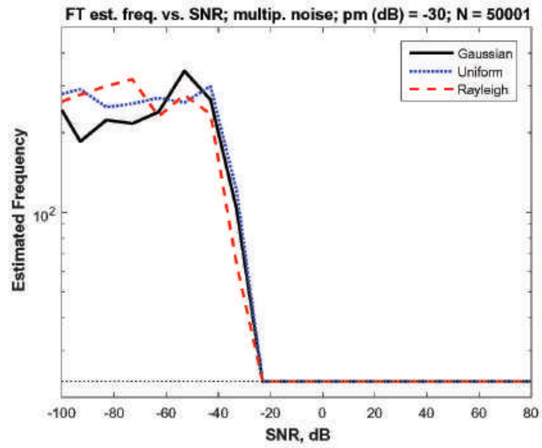

Figure 4: Interpolated FT peak method for various multiplicative noise models, with power = -30 dB; $f_o = 23$ Hz.

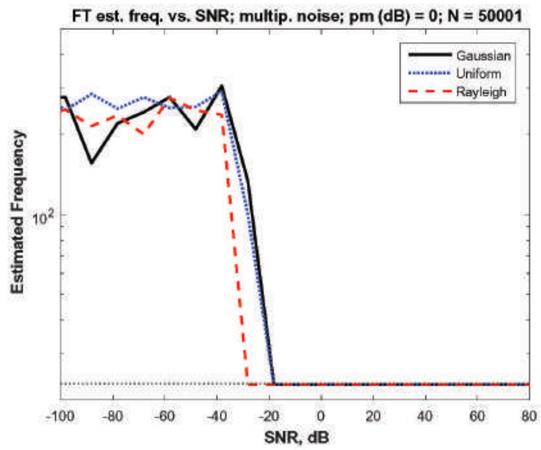

Figure 5: Interpolated FT peak method for various multiplicative noise models, with power = 0 dB; $f_o = 23$ Hz.

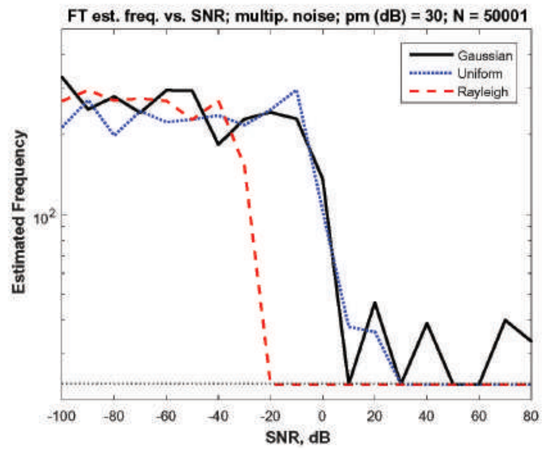

Figure 6: Interpolated FT peak method for various multiplicative noise models, with power = 30 dB; $f_o = 23$ Hz.

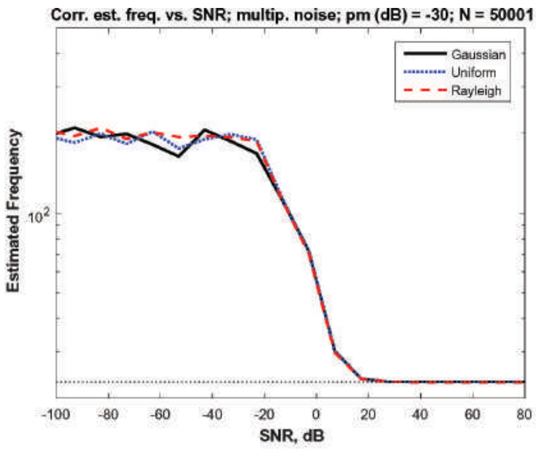

Figure 7: Autocorrelation method for frequency estimation under various multiplicative noise models, with power = -30 dB; $f_o = 23$ Hz.

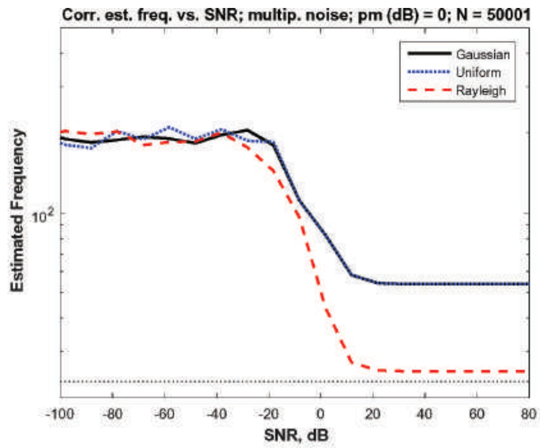

Figure 8: Autocorrelation method for frequency estimation under various multiplicative noise models, with power = 0 dB; $f_o = 23$ Hz.

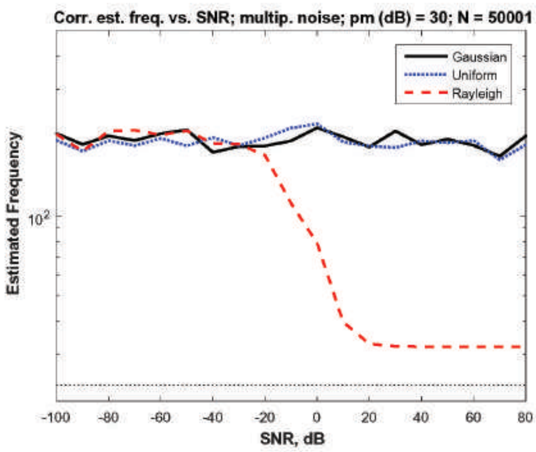

Figure 9: Autocorrelation method for frequency estimation under various multiplicative noise models, with power = 30 dB; $f_o = 23$ Hz.

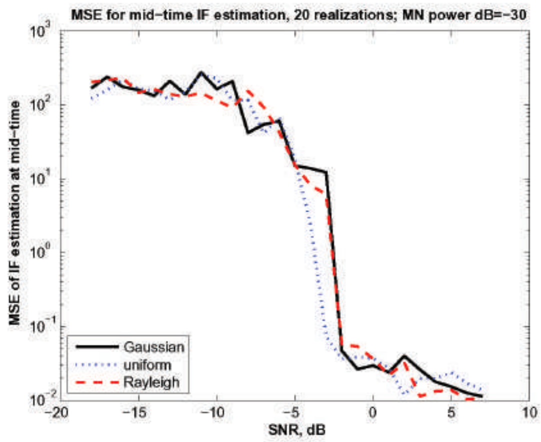

Figure 10: MSE of FFT IF estimation at mid-time of LFM signal vs. SNR with MN power = -30 dB.

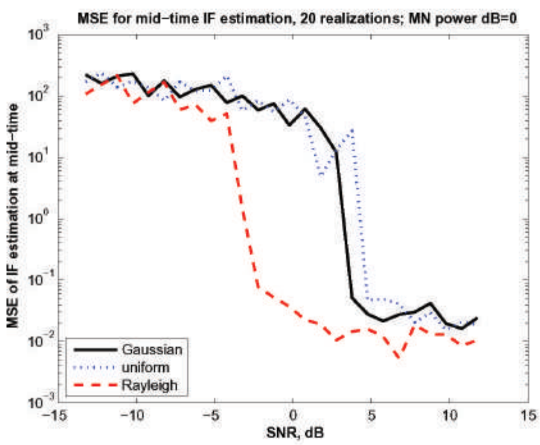

Figure 11: MSE of FFT IF estimation at mid-time of LFM signal vs. SNR with MN power = -30 dB.

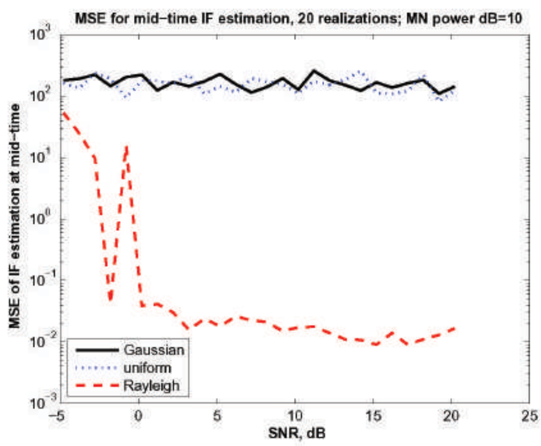

Figure 12: MSE of FFT IF estimation at mid-time of LFM signal vs. SNR with MN power = 10 dB.

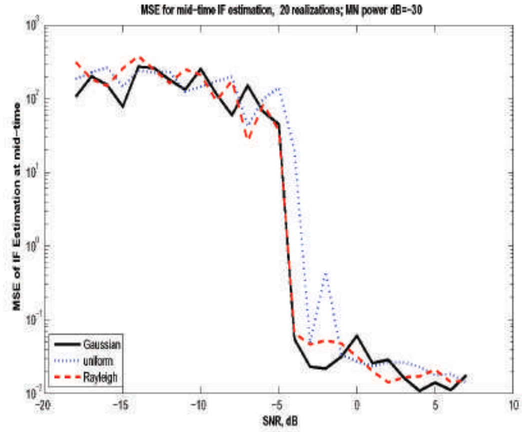

Figure 13: MSE of FFT IF estimation at mid-time of QFM signal vs. SNR with MN power = -30 dB.

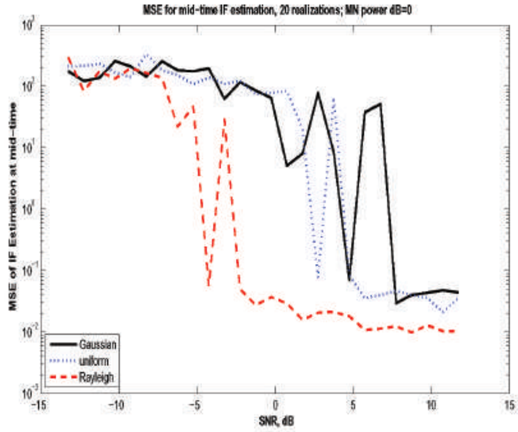

Figure 14: MSE of FFT IF estimation at mid-time of QFM signal vs. SNR with MN power = 0 dB.

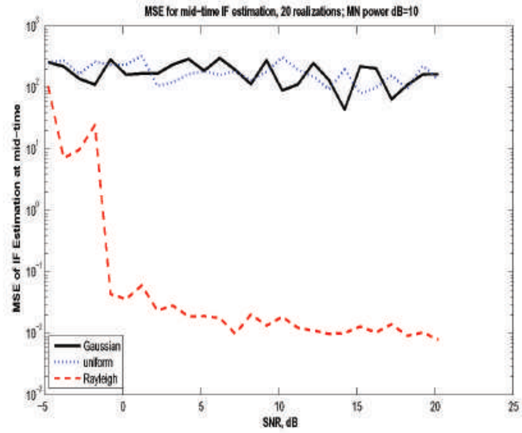

Figure 15: MSE of FFT IF estimation at mid-time of QFM signal vs. SNR with MN power = 10 dB.

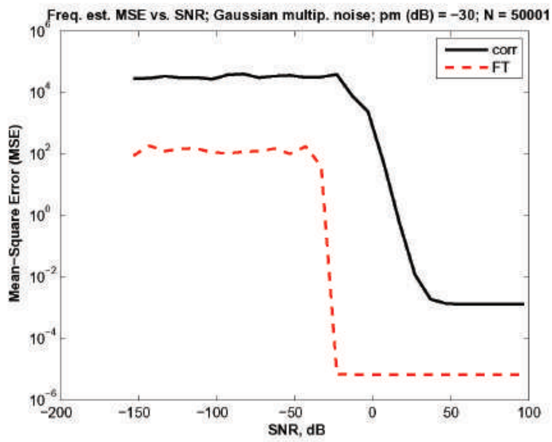

Figure 16: MSE vs. SNR in dB using interpolated FT peak and correlation method with Gaussian multiplicative noise power = -30 dB.

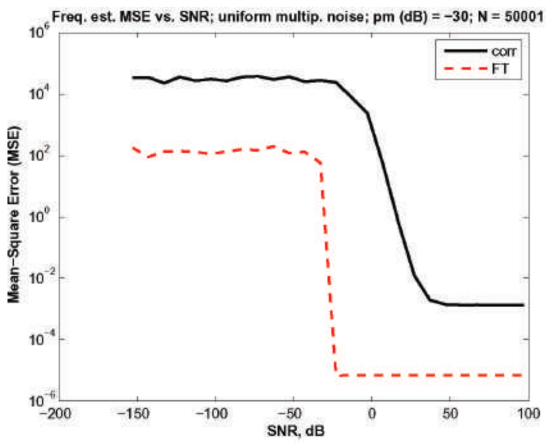

Figure 17: MSE vs. SNR in dB using interpolated FT peak and correlation method with uniform multiplicative noise power = -30 dB.

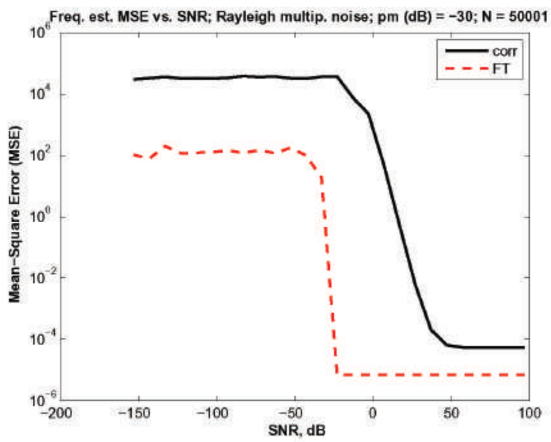

Figure 18: MSE vs. SNR in dB using interpolated FT peak and correlation method with Rayleigh multiplicative noise power = -30 dB.

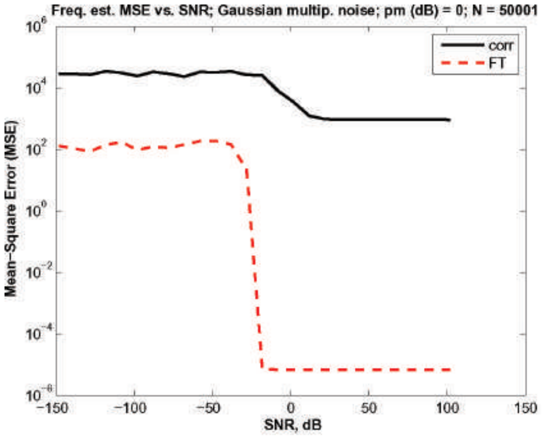

Figure 19: MSE vs. SNR in dB using interpolated FT peak and correlation method with Gaussian multiplicative noise power = 0 dB.

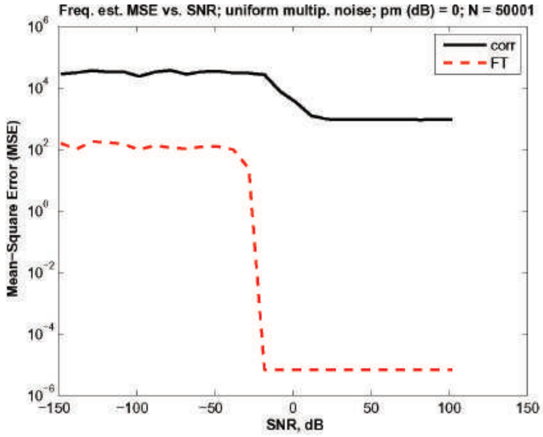

Figure 20: MSE vs. SNR in dB using interpolated FT peak and correlation method with uniform multiplicative noise power = 0 dB.

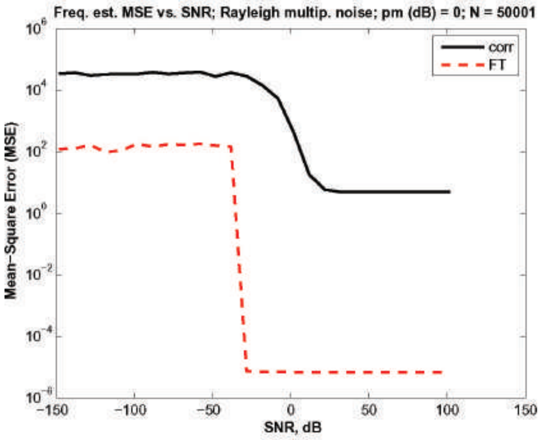

Figure 21: MSE vs. SNR in dB using interpolated FT peak and correlation method with Rayleigh multiplicative noise power = 0 dB.

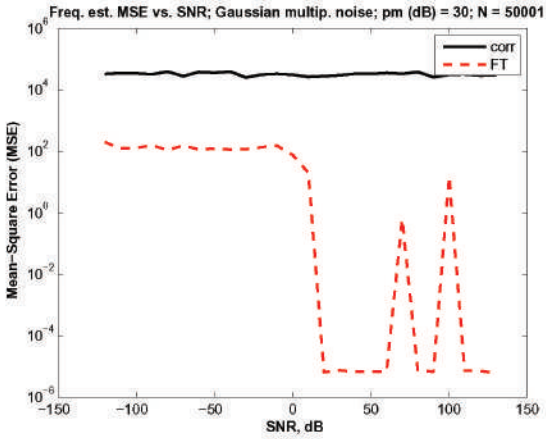

Figure 22: MSE vs. SNR in dB using interpolated FT peak and correlation method with Gaussian multiplicative noise power = 30 dB.

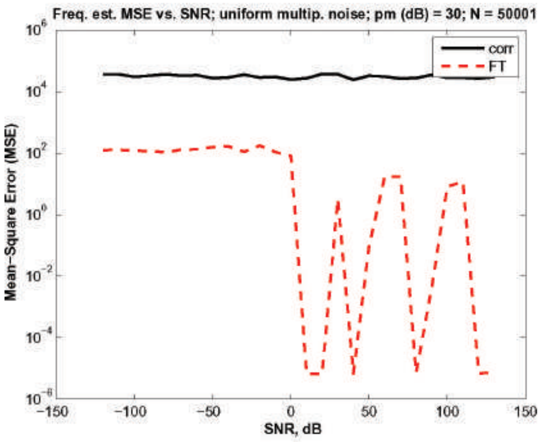

Figure 23: MSE vs. SNR in dB using interpolated FT peak and correlation method with uniform multiplicative noise power = 30 dB.

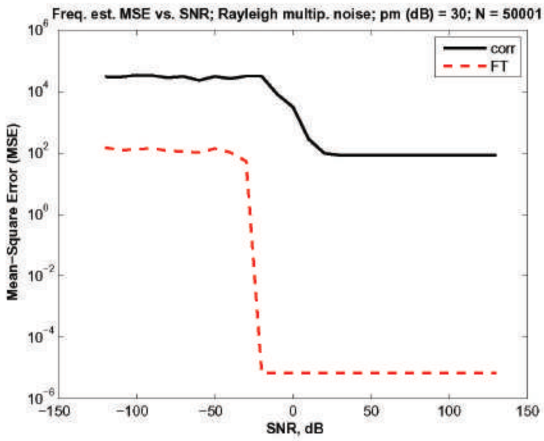

Figure 24: MSE vs. SNR in dB using interpolated FT peak and correlation method with Rayleigh multiplicative noise power = 30 dB.

were performed with M=20 realizations. Figures (13)–(15) show MSE of IF estimation at mid-time of QFM signal versus different SNRs with MN power = −30 dB, 0 dB, and 10 dB, respectively. From these figures it can be seen that IF estimation for LFM and QFM signals based on TFD gives reasonable results under MN up to MN power of 0 dB for the cases of Gaussian and uniform MN models, while it will hold for higher power values in case of Rayleigh model. Figures (16)–(18) show the MSE of estimated frequency versus different SNRs with MN power = −30 dB for three various multiplicative noise models (Gaussian, Uniform and Rayleigh) respectively, using interpolated FT peak frequency and autocorrelation estimation method. Figures (19)–(21) show the MSE of estimated frequency versus different SNRs with MN power =0 dB for three various multiplicative noise models (Gaussian, Uniform and Rayleigh) respectively, using interpolated FT peak frequency and autocorrelation estimation method. Figures (22)–(24) show the MSE of estimated frequency versus different SNRs with MN power=30 dB for three various multiplicative noise models (Gaussian, Uniform and Rayleigh) respectively, using interpolated FT peak frequency and autocorrelation estimation method. It is very clear that these methods with Gaussian and uniform multiplicative noise failed at high power of MN noise especially at $p_m = 30$ dB. Also it is shown that interpolated FT method is more accurate as compared with the autocorrelation method under all MN models.

9. Conclusions

This paper investigates the performance of instantaneous frequency (IF) estimation of mono-component sinusoidal signals under additive Gaussian noise while affected by multiplicative noise (MN). DFT and correlation methods are used for single-tone signals, while time-frequency analysis (specifically periodogram) has been used to estimate IF laws of linear frequency-modulated (LFM) and quadratic FM (QFM) signals. Three models for multiplicative noise have been considered: Gaussian, uniform, and Rayleigh. As Rayleigh process is correlated, performance under Rayleigh MN is better for all signals and methods. However, the situation is more severe in case of FM signals, where IF estimation using the periodogram fails at lower MN powers than those in single-tone signals. This is due to the principal role of phase in representing information held by FM signals, and the fact that MN has more damaging effect on phase than on magnitude.

Acknowledgements

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