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College of Computer science and information  
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**On a subclass of Univalent Analytic Functions  
Associated with the Generalized Differential  
Operator**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

(وقل رببي زدني علما )

صدق الله العلي العظيم

# ((الاهداء))

الى من ضحوا بدمائهم الطاهرة من اجل الاسلام ....  
اهل البيت (( عليهم السلام ))  
الى كل من ضحى من اجل الحق والعلم .....  
العلماء  
الى من اوحى الله بهما برا " واحسانا" .....  
ابي وامي  
الى الذي لم يبخل علي بعلمه وارشاده ....  
استاذي الفاضل  
الى من رافقتي سنوات الدراسة .....  
اصدقائي وزملائي  
والى من يقدر المجهود العلمي اهدي ثمرة جهودي.

## ﴿ شكر وتقدير ﴾

الحمد والشكر لله العلي القدير والصلاة والسلام على سيدنا محمد وآل بيته الطيبين الطاهرين  
الى يوم الدين . . . . . بعد ان اشكر الله سبحانه وتعالى في تيسير امرنا في اتمام بحثنا اتقدم  
بخالص شكري وتقديري الى استاذي الفاضل ((عباس كريم وناس))  
الذي قدم لي النصائح والمتابعة العلمية الصادقة التي كان لها الاثر الواضح في اتمام هذا البحث  
ادعو من الله عز وجل التوفيق والتقدم في مسيرته العلميـه . . .

# Abstract

In the present investigation , we introduce a subclass of univalent analytic functions in the open unit disk and obtain some properties , like , coefficient estimates , growth and distortion theorem , extreme points and closure theorems. Also , we discuss convolution and integral properties .

Mathematics Subject Classification : 30 c 45 .

**Keywords :** Analytic functions , growth and distortion , extreme points , differential operator .

# 1 Introduction

Let  $\mathcal{A}$  denote the class of functions  $f$  of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad (1)$$

which are analytic in the open unit disk  $U := \{z \in \mathbb{C} : |z| < 1\}$ .

A function  $f$  in the class  $\mathcal{A}$  is said to be in the class  $S^*(\alpha)$  of starlike functions of order  $\alpha$  in  $U$ , if it satisfy the inequality

$$\Re \frac{zf'(z)}{f(z)} > \alpha \quad (z \in U, 0 \leq \alpha < 1). \quad (2)$$

Note that  $S^*(0) = S^*$  is the class of starlike functions.

Denote by  $\mathcal{B}$  the subclass of  $\mathcal{A}$  consisting of functions  $f$  of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0). \quad (3)$$

This subclass was introduced and studied by Silverman [8].

Let  $f$  be a function in the class  $\mathcal{A}$ . We define the following differential operator:

$$\begin{aligned} D_{\lambda\mu}^0 f(z) &= f(z) \\ D_{\lambda\mu}^1 f(z) &= D_{\lambda\mu} f(z) = \lambda\mu z^2 f''(z) + (\lambda - \mu)zf'(z) + (1 - \lambda + \mu)f(z) \\ D_{\lambda\mu}^m f(z) &= D_{\lambda\mu}(D_{\lambda\mu}^{m-1} f(z)) \end{aligned} \quad (4)$$

where  $0 \leq \mu \leq \lambda \leq 1$  and  $m \in \mathbb{N} := \{1, 2, \dots\}$ .

If  $f$  is given by (1) then from the definition of the operator  $D_{\lambda\mu}^m f$  it is easy to see that

$$D_{\lambda\mu}^m f(z) = z + \sum_{n=2}^{\infty} [1 + (\lambda\mu n + \lambda - \mu)(n-1)]^m a_n z^n. \quad (5)$$

When  $\lambda = 1$  and  $\mu = 0$  we get the Salagean differential operator [7] and when  $\mu = 0$  we obtain the differential operator defined by Al-Oboudi [1].

If  $f \in \mathcal{B}$  is given by (3) then we have

$$D_{\lambda\mu}^m f(z) = z - \sum_{n=2}^{\infty} [1 + (\lambda\mu n + \lambda - \mu)(n-1)]^m a_n z^n. \quad (6)$$

**Definition 1.1** A function  $f \in \mathcal{A}$  is said to be in the class  $T(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  if it satisfy the following condition:

$$\operatorname{Re} \left\{ \alpha \frac{zG'(z)}{G(z)} - (\alpha + \beta - 1) \right\} > \alpha k \left| \frac{zG'(z)}{G(z)} - 1 \right| \quad (7)$$

Where  $0 \leq \gamma < 1$ ,  $\alpha \geq 1$ ,  $k \geq 0$ ,  $0 \leq \beta < 1$  and

$$G(z) = (1 - \gamma) D_{\lambda\mu}^m f(z) + \gamma z (D_{\lambda\mu}^m f(z))'. \quad (8)$$

Also, we define

$$S(\gamma, \alpha, k, \beta, m, \lambda, \mu) = T(\gamma, \alpha, k, \beta, m, \lambda, \mu) \cap \mathcal{B}$$

By suitably specializing the parameters involved, the classes  $T(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  and  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  can be reduced to new or to known much simpler classes of functions which were studied in earlier works (see [2], [3], [4], [5], [6]).

The object of this paper is to study various properties for functions belonging to the class  $T(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  and  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ , respectively.

In order to prove our results from this section we need the following lemma.

**Lemma 1.1** Let  $\beta$  be a real number and let  $w$  be a complex number. Then  $\operatorname{Re}(w) \geq \beta$  if and only if  $|w + (1 - \beta)| - |w - (1 + \beta)| \geq 0$ .

## 2 Coefficient estiamates

First we give a sufficient coefficient bound for functions in the class  $T(\gamma, \alpha, k, \beta, m, \lambda, \mu)$

**Theorem 2.1** Let  $f \in \mathcal{A}$  given by (1). If

$$\sum_{n=2}^{\infty} [1 - \beta + \alpha(n-1)(1+k)] A_n(\lambda, \mu, \gamma, m) |a_n| \leq 1 - \beta \quad (9)$$

Where

$$A_n(\lambda, \mu, \gamma, m) = [1 + \gamma(n-1)][1 + (\lambda\mu n + \lambda - \mu)(n-1)]^m \quad (10)$$

then  $f \in T(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

**Proof.** In virtue of Definition 1.1 and Lemma 1.1 it is sufficient to show that

$$\begin{aligned} & \left| \alpha \frac{zG'(z)}{G(z)} - (\alpha - 1) - k \left| \alpha \frac{zG'(z)}{G(z)} - \alpha \right| - (1 + \beta) \right| \\ & \leq \left| \alpha \frac{zG'(z)}{G(z)} - (\alpha - 1) - k \left| \alpha \frac{zG'(z)}{G(z)} - \alpha \right| + (1 - \beta) \right|. \end{aligned} \quad (11)$$

For the right-hand and left-hand side of (11) we may, respectively, write

$$\begin{aligned} R &:= \left| \alpha \frac{zG'(z)}{G(z)} - (\alpha - 1) - k \left| \alpha \frac{zG'(z)}{G(z)} - \alpha \right| + (1 - \beta) \right| \\ &= \frac{1}{|G(z)|} \left| \alpha zG'(z) - (\alpha - 1)G(z) - ke^{i\theta} |\alpha zG'(z) - \alpha G(z)| + (1 - \beta)G(z) \right| \\ &> \frac{|z|}{|G(z)|} \left[ 2 - \beta - \sum_{n=2}^{\infty} [2 - \beta + \alpha(n-1)(1+k)] A_n(\lambda, \mu, \gamma, m) |a_n| \right] \end{aligned}$$

and similarly

$$\begin{aligned} L &:= \left| \alpha \frac{zG'(z)}{G(z)} - (\alpha - 1) - k \left| \alpha \frac{zG'(z)}{G(z)} - \alpha \right| - (1 + \beta) \right| \\ &= \frac{1}{|G(z)|} \left| \alpha zG'(z) - (\alpha - 1)G(z) - ke^{i\theta} |\alpha zG'(z) - \alpha G(z)| - (1 + \beta)G(z) \right| \end{aligned}$$

$$< \frac{|z|}{|G(z)|} \left[ \beta + \sum_{n=2}^{\infty} [\alpha(n-1)(1+k) - \beta] A_n(\lambda, \mu, \gamma, m) |a_n| \right]$$

Since

$$R-L > \frac{|z|}{|G(z)|} \left[ 2(1-\beta) - 2 \sum_{n=2}^{\infty} [1-\beta + \alpha(n-1)(1+k)] A_n(\lambda, \mu, \gamma, m) |a_n| \right] \geq 0,$$

the required condition (11) is satisfied.

In the next theorem we obtain a necessary and sufficient condition for a function  $f \in \mathfrak{B}$  to be in the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

**Theorem 2.2** Let  $f \in \mathfrak{B}$  given by (3). Then  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  if and only if

$$\sum_{n=2}^{\infty} [1-\beta + \alpha(n-1)(1+k)] A_n(\lambda, \mu, \gamma, m) a_n \leq 1-\beta \quad (12)$$

where  $A_n(\lambda, \mu, \gamma, m)$  is defined by (10). The result is sharp.

**Proof.** Assume that inequality (12) holds true. In virtue of Theorem 2.1 and the definition of  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  we have that  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

Conversely, suppose  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ . Choosing the values of  $z$  on the positive real axis the inequality (7) reduces to:

$$\begin{aligned} & \frac{1 - \sum_{n=2}^{\infty} [1 + \alpha(n-1)] A_n(\lambda, \mu, \gamma, m) a_n z^{n-1}}{1 - \sum_{n=2}^{\infty} A_n(\lambda, \mu, \gamma, m) a_n z^{n-1}} - \beta \\ & > k \left| \frac{\sum_{n=2}^{\infty} \alpha(n-1) A_n(\lambda, \mu, \gamma, m) a_n z^{n-1}}{1 - \sum_{n=2}^{\infty} A_n(\lambda, \mu, \gamma, m) a_n z^{n-1}} \right|. \end{aligned} \quad (13)$$

Letting  $z \rightarrow 1^-$ , we obtain the desired inequality.

Finally, equality holds for the function  $f$  defined by

$$f(z) = z - \frac{1-\beta}{[1-\beta + \alpha(n-1)(1+k)] A_n(\lambda, \mu, \gamma, m)} z^n \quad (n \geq 2). \quad (14)$$

**Corollary 2.1** If  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ , then

$$a_n \leq \frac{1 - \beta}{[1 - \beta + \alpha(n - 1)(1 + k)]A_n(\lambda, \mu, \gamma, m)} \quad (n \geq 2). \quad (15)$$

Equality is obtained for the function  $f$  given by (14).

### 3 Growth and distortion theorem

**Theorem 3.1** Let  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ . Then, for  $|z| = r < 1$

$$r - \frac{1 - \beta}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} r^2 \leq |f(z)| \leq r + \frac{1 - \beta}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} r^2 \quad (16)$$

and

$$1 - \frac{2(1 - \beta)}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} r \leq |f'(z)| \leq 1 + \frac{2(1 - \beta)}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} r \quad (17)$$

where

$$B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) := [1 - \beta + \alpha(n - 1)(1 + k)]A_n(\lambda, \mu, \gamma, m) \quad (n \geq 2). \quad (18)$$

The inequalities (16) and (17) are sharp for the function  $f$  given by

$$f(z) = z - \frac{1 - \beta}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} z^2.$$

**Proof.** Since  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ , from Theorem 2.2 it follows

$$\sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) a_n \leq 1 - \beta$$

where  $B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta)$  is given by (18). We have

$$\begin{aligned} B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta) \sum_{n=2}^{\infty} a_n &= \sum_{n=2}^{\infty} B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta) a_n \\ &\leq \sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) a_n \leq 1 - \beta \end{aligned}$$

and therefore

$$\sum_{n=2}^{\infty} a_n \leq \frac{1 - \beta}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)}. \quad (19)$$

Since is  $f$  given by (3) we obtain

$$\begin{aligned} |f(z)| &\leq |z| + |z|^2 \sum_{n=2}^{\infty} a_n |z|^{n-2} \leq r + r^2 \sum_{n=2}^{\infty} a_n \\ &\leq r + \frac{1 - \beta}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} r^2 \end{aligned}$$

and

$$\begin{aligned} |f(z)| &\geq |z| - |z|^2 \sum_{n=2}^{\infty} a_n |z|^{n-2} \geq r - r^2 \sum_{n=2}^{\infty} a_n \\ &\geq r - \frac{1 - \beta}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} r^2. \end{aligned}$$

In virtue of Theorem 2.2, we also have

$$\begin{aligned} \frac{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)}{2} \sum_{n=2}^{\infty} n a_n &= \sum_{n=2}^{\infty} \frac{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)}{2} n a_n \\ &\leq \sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) a_n \leq 1 - \beta \end{aligned}$$

which yields

$$\sum_{n=2}^{\infty} n a_n \leq \frac{2(1 - \beta)}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)}.$$

Thus,

$$\begin{aligned} |f'(z)| &\leq 1 + \sum_{n=2}^{\infty} n a_n |z|^{n-1} \leq 1 + r \sum_{n=2}^{\infty} n a_n \\ &\leq 1 + \frac{2(1 - \beta)}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} r \end{aligned}$$

and

$$|f'(z)| \geq 1 - \sum_{n=2}^{\infty} n a_n |z|^{n-1} \geq 1 - r \sum_{n=2}^{\infty} n a_n$$

$$\geq 1 - \frac{2(1-\beta)}{B_2(\lambda, \mu, \gamma, m, \alpha, k, \beta)} r.$$

Now, the proof of our theorem is completed.

## 4 Extreme points

Next, we examine the extreme points for the function class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

**Theorem 4.1** Let the functions

$$f_1(z) = z \text{ and } f_n(z) = z - \frac{1-\beta}{B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta)} z^n \quad (20)$$

$(0 \leq \mu \leq \lambda \leq 1, 0 \leq \gamma \leq 1, m \in \mathbb{N}, \alpha \geq 1, k \geq 0, 0 \leq \beta < 1, n \geq 2)$ .

Then  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  if and only if

$$f(z) = \sum_{n=1}^{\infty} \lambda_n f_n(z) \quad (z \in \mathbb{U}) \quad (21)$$

where  $\lambda_n \geq 0$  ( $n \geq 1$ ) and  $\sum_{n=1}^{\infty} \lambda_n = 1$ .

**Proof.** Assume that  $f$  can be written as in (21). Then

$$\begin{aligned} f(z) &= \lambda_1 z + \sum_{n=2}^{\infty} \lambda_n \left[ z - \frac{1-\beta}{B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta)} z^n \right] \\ &= z - \sum_{n=2}^{\infty} \lambda_n \frac{1-\beta}{B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta)} z^n. \end{aligned}$$

Since

$$\sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) \lambda_n \frac{1 - \beta}{B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta)}$$

$$= (1 - \beta) \sum_{n=2}^{\infty} \lambda_n = (1 - \beta)(1 - \lambda_1) \leq 1 - \beta$$

it follows, in virtue of Theorem 2.2, that  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .  
Conversely, suppose  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  and consider

$$\lambda_n := \frac{B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta)}{1 - \beta} a_n, n \geq 2 \text{ and } \lambda_1 := 1 - \sum_{n=2}^{\infty} \lambda_n.$$

Then  $(z) = \sum_{n=1}^{\infty} \lambda_n f_n(z)$ . Thus, we completed the proof of the theorem.

**Corollary 4.1** The extreme points of the function class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  are the functions given by (20).

## 5 Closure theorems

Let the functions  $f_j \in \mathfrak{B}$  ( $j = 1, 2, \dots, p$ ) defined by

$$f_j(z) = z - \sum_{n=2}^{\infty} a_{n,j} z^n \quad (z \in \mathbb{U}). \quad (22)$$

We obtain the following results for the closure functions of the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

**Theorem 5.1** Let the functions  $f_j$  ( $j = 1, 2, \dots, p$ ) given by (22) be in the

class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  and let  $c_j \geq 0$  ( $j = 1, 2, \dots, p$ ) such that  $\sum_{j=1}^p c_j = 1$ .

Then the function  $h$  defined by

$$h(z) = \sum_{j=1}^p c_j f_j(z)$$

is also in the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

**Proof.** In virtue of the definition of  $h$ , we can write

$$h(z) = \sum_{j=1}^p c_j \left[ z - \sum_{n=2}^{\infty} a_{n,j} z^n \right] = z - \sum_{n=2}^{\infty} \left( \sum_{j=1}^p c_j a_{n,j} \right) z^n.$$

Since the functions  $f_j$  are in  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ , for every  $j = 1, 2, \dots, p$  we have

$$\sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) a_{n,j} \leq 1 - \beta.$$

Hence, we get

$$\sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) \left( \sum_{j=1}^p c_j a_{n,j} \right) \leq \sum_{j=1}^p c_j (1 - \beta) = 1 - \beta$$

which implies that  $h$  is in the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ . Thus, the proof of the theorem is completed.

**Corollary 5.1** The class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  is closed under convex linear combination.

**Proof.** Assume that the functions  $f_j$  ( $j = 1, 2$ ) given by (22) are in  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

It is sufficient to show that the function  $h$  defined by

$$h(z) = cf_1(z) + (1 - c)f_2(z) \quad (0 \leq c \leq 1)$$

is in the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ . By taking  $p = 1$ ,  $c_1 = c$  and  $c_2 = 1 - c$  in Theorem 5.1 we obtain the corollary.

## 6 Convolution and integral properties

In this section we shall prove that the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  is closed under convolution and integral operator.

**Theorem 6.1** Let  $g(z)$  of the form

$$g(z) = z - \sum_{n=2}^{\infty} b_n z^n \quad (0 \leq b_n \leq 1, n \geq 2)$$

be analytic in  $U$ . If the function  $f$  belongs to the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  then, the function  $f * g$  is also in the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

The symbol " $*$ " stands for Hadamard product (or convolution).

**Proof.** Since  $f \in S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  we have

$$\sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) a_n \leq 1 - \beta.$$

By making use of the last inequality and the fact that

$$(f * g)(z) = z - \sum_{n=2}^{\infty} a_n b_n z^n$$

we obtain

$$\sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) a_n b_n \leq \sum_{n=2}^{\infty} B_n(\lambda, \mu, \gamma, m, \alpha, k, \beta) a_n \leq 1 - \beta$$

and hence, in virtue of Theorem 2.2, the result follows.

Let  $I_c : T \rightarrow T$  be the integral operator defined by

$$F(z) = I_c(f)(z) = \frac{c+1}{z^c} \int_0^z t^{c-1} f(t) dt \quad (c > -1, z \in \mathbb{U}). \quad (23)$$

We note that if  $f \in T$  is given by (3) then

$$F(z) = z - \sum_{n=2}^{\infty} \frac{c+1}{c+n} a_n z^n. \quad (24)$$

**Theorem 6.2** If the function  $f$  is in the class  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$  then the function  $F$  given by (23) is also in  $S(\gamma, \alpha, k, \beta, m, \lambda, \mu)$ .

**Proof.** From (24) it results that  $F(z) = (f * g)(z) (z \in U)$ , where

$$g(z) = z - \sum_{n=2}^{\infty} \frac{c+1}{c+n} z^n \text{ and } 0 \leq \frac{c+1}{c+n} \leq 1 \ (n \geq 2).$$

By Theorem 5.1, the proof is trivial .

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## On a subclass of Univalent Analytic Functions Associated with the Generalized Differential Operator

### بحث تخرج

مقدم الى كلية علوم الحاسوب وتكنولوجيا المعلومات جامعة القادسية وهو جزء من  
متطلبات نيل درجة البكالوريوس في علوم الرياضيات .

من قبل الطالب  
عباس يوسف جهيد

بإشراف الاستاذ  
عباس كريم وناس