

**Ministry of Higher Education
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College of Computer Science
& Information Technology
Department of Mathematics**



Observing the shadow of modified gravity black hole

**A research Submitted to the Department of Mathematics, College of Computer
Science and Mathematics, University of Al-Qadisiyah as partial fulfillment the
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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

((وَعِنْدَهُ مَفَاتِحُ الْغَيْبِ لَا يَعْلَمُهَا إِلَّا هُوَ وَيَعْلَمُ مَا فِي
الْبَرِّ وَالْبَحْرِ وَمَا تَسْقُطُ مِنْ وَرَقَةٍ إِلَّا يَعْلَمُهَا وَلَا حَبَّةٍ
فِي ظُلُمَاتِ الْأَرْضِ وَلَا رَطْبٍ وَلَا يَابِسٍ إِلَّا فِي كِتَابٍ
مُبِين.))

صدق الله العلي العظيم

الأنعام (٥٩)

الاهداء

الى بلدي الحبيب

العراق عشقا

الى من ربياني صغيرا وطالما دعوا الله الي بالتوفيق
لكم مني وافر الشكر وعظيم الامتنان

امي وابي

الى من اشد بها ازري ... وأشركها في امري

اختي

الى استاذي الفاضل تقديرا وعرفانا بالجميل

مشرفي أ.م.د. وقاص غالب

والي كل من علمني حرفا ... وأضاء شمعة في دربي

اساتذتي

اهدي ثمره جهدي المتواضع

الباحث

الشكر والتقدير

بسم الله الرحمن الرحيم

((رَبِّ أَوْزِعْنِي أَنْ أَشْكُرَ نِعْمَتَكَ الَّتِي أَنْعَمْتَ عَلَيَّ وَعَلَى وَالِدَيَّ وَأَنْ أَعْمَلَ صَالِحًا تَرْضَاهُ وَأَدْخِلْنِي بِرَحْمَتِكَ فِي عِبَادِكَ الصَّالِحِينَ))

النمل أية (١٩)

أتوجه بخالص الشكر والتقدير الى الأستاذ المساعد الدكتور وقاص غالب عطشان لتفضله بالإشراف على هذا البحث واغنائه بالملاحظات والآراء القيمة، وواجب الوفاء ان أتقدم بشكري الى جميع الأساتذة الافاضل الذين أشرفوا على تدريسي واخص منهم بالشكر أ.م.د. قصي حاتم عكار و أ.د. نوري فرحان عذاب و أ.م.د. رعد عزيز حسين العبد الله و م.م. ضياء غازي صالح و م.م. عباس كريم وناس والى جميع أساتذة كلية علوم الحاسوب وتكنولوجيا المعلومات خصوصا أساتذة قسم الرياضيات والى من لم يسعني ذكرهم.

Certificate of Originality

I declare that the work considered in this paper entitled "Observing the shadow of modified gravity black hole" is my own work and was composed by myself during my study at University of Al-Qadisiyah except where otherwise acknowledged.

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Ahmed Alhamzawi

Author's Publication (impact factors)

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2. Alhamzawi, Ahmed (2016). Gravitational lensing by $f(R, T)$ gravity. *International Journal of Modern Physics D*. 25, 1650020
3. Alhamzawi, Ahmed (2016). Gravitational lensing in the strong field limit by modified gravity. *General Relativity and Gravitation*. 48, 167.

Abstract

A study of the shadow cast by rotating black holes in different models of modified gravity is presented. It is shown that the size of the shadow cast depends on the modified gravity model used. The distortions of the shadow cast by modified gravity black holes are investigated and the effects are compared with the distortions cast by Kerr black hole. The shadow of a rotating black hole in modified gravity is found to be similar to the shadow cast by Kerr black hole but with different sizes and distortion effects. The naked singularity by rotating modified gravity black hole is discussed. Furthermore, the area of the black hole shadow is used to study the energy emission rate. Finally, It is shown that some modified gravity models can present a considerable contribution to the size of black hole shadow.

PACS numbers 04.50.Kd, 04.20.-q, 04.25.Nx

Keywords: Modified Gravity, Rotating Black Hole, Naked Singularity, Energy Emission Rate

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Abstract A study of the shadow cast by rotating black holes in different models of modified gravity is presented. It is shown that the size of the shadow cast depends on the modified gravity model used. The distortions of the shadow cast by modified gravity black holes are investigated and the effects are compared with the distortions cast by Kerr black hole. The shadow of a rotating black hole in modified gravity is found to be similar to the shadow cast by Kerr black hole but with different sizes and distortion effects. The naked singularity by rotating modified gravity black hole is discussed. Furthermore, the area of the black hole shadow is used to study the energy emission rate. Finally, It is shown that some modified gravity models can present a considerable contribution to the size of black hole shadow.

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1 Introduction

While black holes are not visible objects, they may be observed indirectly from the shadow they cast when it is in front of a bright background. The observation of the rotation of galaxies and increasing evidence for the existence of a supermassive black hole at the center of many galaxies (Eisenhauer et al. 2005) has promoted several attempts in recent years to advance the theoretical investigations of the shadows of black holes as far as possible (Abdujabbarov et al. 2015; Shipley and Dolan 2016; Johannsen et al. 2016). While its unlikely to observe black holes at present time there is hope that in the near future some new methods will be available to make the observations of black holes possible (Lu et al. 2014; Psaltis et al. 2015; Cunha et al. 2015).

One of these methods is the Event Horizon Telescope that will be used to assemble data from several very-long-baseline interferometry (VLBI) stations (Lu et al. 2014) which can be used to explore the central black hole and the inner region of the center of our galaxy and may enable the imaging of the predicted black hole shadow.

In the past few years several authors have investigated the shadow cast by different types of rotating black holes in different environments. In (Takahashi 2004) the authors studied the shapes and positions of black hole shadows in accretion disks and measured the minimum interval between a black hole mass center and the bisector perpendicular to a line segment of the maximum width of the black hole shadow. In (Hioki and Maeda 2009) it was shown that defining some observables characterizing the radius and distortion of the apparent shape of the shadow can be used to determine the spin parameter and inclination angle by rotating black hole from observation. In (Wei and Liu 2013) the effects of dilaton parameter (b) on the shadow cast by Einstein-Maxwell-Dilaton-Axion black hole was studied where it was shown that distortion of the shadow increases monotonically with dilaton parameter. In (Perlick et al. 2015) an analytical calculation of the influence of plasma on the shadow of a black hole was presented and the general formulas for the spherically symmetric plasma density on an unspecified spherically symmetric and static spacetime was derived. In (Wei et al. 2015), the authors investigated the effects of the dimensionless noncommutative parameter on the shadow cast by rotating black hole inspired by noncommutative geometry where it was shown that this may offer a way to distinguish between noncommutative geometry inspired black hole and Kerr black hole using astronomical instruments. In (Abdujabbarov et al. 2016) the shadow cast by different types of rotating regular black

holes was studied and it was shown that the distortion of the shadow is related to the different parameters of regular black holes. In (Amir and Ghosh 2016) the authors presented qualitative analysis of the shapes of rotating nonsingular black hole shadows where it was shown that size of the shadow decreases due to the deviation parameter (k).

One type of black holes that has not been explored fully is the $f(R)$ modified gravity black holes and how this type of modification effects the shadow cast by rotating black hole. In this paper its our aim to extend the discussion of black hole shadows for rotating black hole in $f(R)$ modified gravity. We will consider specific types of models of modified gravity and show how each model effects the size and the distortions of the shadow. Then, we make comparison between the shadows of modified gravity black holes and Kerr black hole.

The paper will be organized as follows. In Sec. 2, we will discuss some modified gravity models. In Sec. 3, we will calculate the null geodesics and the effective potential for rotating black hole in modified gravity. In Sec. 4, we will obtain the corresponding observables and investigate the apparent shape of the shadow cast. In Sec. 5, the energy emission rate is examined and comparison is made with Kerr black hole. In Sec. 6. The shadow for a naked singularity is discussed. Finally, the summary and remarks follow in Sec. 7. We will adopt the geometrical units ($c = G = 1$).

2 Some modified gravity models

In the past several models were proposed to describe some of the phenomena that general relativity fails to explain. In this paper we will consider specific type of modified gravity models that assume solution in Boyer-Lindquist coordinates (Erbin and Heurtier 2015; Newman and Janis 1965; Demiański 1972) in the form

$$ds^2 = -A(r)dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2d\theta^2 + 2a(A(r) - H(r))\sin^2\theta dtd\phi + (a^2(2H(r) - A(r))\sin^2\theta + \rho^2)\sin^2\theta d\phi^2 \quad (1)$$

$$\text{where } d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2,$$

$$A(r) = 1 - \frac{r_s r}{\rho^2} + \epsilon \mathcal{A}(r), \quad (2)$$

$$B(r) = 1 - \frac{r_s r}{\rho^2} + \epsilon \mathcal{B}(r), \quad (3)$$

$\rho^2 = r^2 + a^2 \cos^2\theta$, $\Delta = B(r)\rho^2 + a^2 \sin^2(\theta)$, $H(r) = \sqrt{\frac{A(r)}{B(r)}}$, $\epsilon > 0$, r_s schwarzschild radius, a is spin parameter and $\mathcal{A}$ and $\mathcal{B}$ are some functions of r . One can see that in the limit $\epsilon \rightarrow 0$, (1) reduces to Kerr metric (Hioki and Maeda 2009)

$$ds^2 = -\left(1 - \frac{r_s r}{\rho^2}\right)dt^2 + \rho^2d\theta^2 - \frac{2ar_s r \sin^2\theta dtd\phi}{\rho^2} + \frac{\rho^2 dr^2}{\Delta_0} + \left(r^2 + a^2 + \frac{r_s r a^2 \sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2 \quad (4)$$

where $\Delta_0 = r^2 - r_s r + a^2$. Some models that assume solution of this form are

I Model $f_1(R) = R + \frac{\mu^4}{R}$

Due the discovery of expanding universe (Perlmutter et al. 1999; Padmanabhan 2003), several theories where proposed to explain this phenomena. One of these theories is the $f(R) = R - \mu^4/R$ model of gravity (Carroll et al. 2004). Shortly after this model was proposed it was realized that this model suffers from matter instability (Dolgov and Kawasaki 2003) and the instability of cosmological perturbations (Bazeia et al. 2007). Furthermore, it was shown that this model may not satisfy the local gravity constraints (Olmo 2005; Faraoni 2006). In (Faraoni 2006), Faraoni showed in his study of models of the type $f(R) = R + \epsilon\phi(R)$ that this instability is caused by an instability condition such that a model is unstable if $\frac{d^2 f}{dR^2} < 0$, where the instability is no longer present for $F(R) = R + \frac{\mu^4}{R}$, where $\mu^4 > 0$. An investigation in (Amendola et al. 2007) for models of type $f(R) = R + \alpha R^{-n}$, $n > 0$ found that this model is inconsistent with observation when non-relativistic matter is present and cannot produce a standard era of matter domination. However, An investigation in (Evans et al. 2008) of models of the type $F(R) = R + \sigma \frac{\mu^4}{R}$, where $\sigma = \pm 1$ showed that the standard epoch of matter domination can be produced given that there is a sufficiently long epoch to satisfy observations. The solution for this model in the strong field limit is given by (Saaidi et al. 2012)

$$\mathcal{A}(r) = -\left(\frac{9r_s^4}{128}\right)^{\frac{1}{3}} \left(\frac{r}{r_s} - 1\right)^{\frac{2}{3}}, \quad (5)$$

$$\mathcal{B}(r) = +\left(\frac{r_s^4}{384}\right)^{\frac{1}{3}} \left(\frac{r}{r_s} - 1\right)^{\frac{2}{3}}, \quad (6)$$

$$\text{with } \epsilon = -\mu^{\frac{4}{3}}.$$

II Model $f_2(R) = R + \epsilon \ln R$

This model was considered in (Saffari and Rahvar (2008)) to seek an explanation for the anomalous in the acceleration of the Pioneer spacecraft and flat rotation curve of spiral galaxies. The pioneer data and Supernova Type Ia gold sample was used to put constraints on the parameters of the model. It was shown that vacuum solution of this model results in a positive late time acceleration for the universe. The solution of this model is given by (Saaidi et al. 2012)

$$\mathcal{A}(r) = -\frac{r}{\sqrt{6}} \quad (7)$$

$$\mathcal{B}(r) = -\frac{r}{\sqrt{6}} \quad (8)$$

III Model $f_3(R) = R^{1+\epsilon}$

Another model that we are going to consider was considered (Clifton and Barrow 2005) where the authors studied the weak-field properties of modified gravity by extending general relativity by a lagrangian proportional to $R^{1+\epsilon}$. The predictions of this theory were compared to observational data to impose several constraints on the value of ϵ . An important constraint was made from the study of the perihelion precession of Mercury where it was found that ϵ should be constrained to lie within the range of $0 \leq \epsilon < 7.2 \times 10^{-19}$. The solution of this model is given by (Clifton and Barrow 2005)

$$\mathcal{A}(r) = \frac{3c \log r^2}{2r} + \log r^2 \quad (9)$$

and

$$\mathcal{B}(r) = \frac{c \log r^2}{2r} + \frac{2c}{r} + 2 \quad (10)$$

One important thing to notice is that for this model the values of $\mathcal{A}$ and $\mathcal{B}$ are positive for $r > r_s$. In the next sections we will show that this difference in sign will result in different effects on shape and size of the shadow than the other models.

IV Model $f_4(R) = R - \lambda R_0 \left[1 - \left(\frac{R_0^2}{R_0^2 + R^2} \right)^n \right] - \frac{R^2}{6m^2}$

There several models that have been proposed to overcome the problems that arises in the model $f(R) = R + \mu^4/R$ while at the same time still explain the late-time accelerating expansion. Unfortunately, few models

were successful in explaining the recent observational data, one of these models is

$$f(R) = R - \lambda R_0 \left[1 - \left(1 + \frac{R^2}{R_0^2} \right)^{-n} \right] - \frac{R^2}{6m^2}, \quad (11)$$

where $n, \lambda > 0$, and $|R_0|$ is of the order of the presently observed effective cosmological constant.

The second term was proposed in (Starobinsky (2007)) after several unsuccessful previous attempts to find viable dark energy model in $f(R)$ gravity using divergent or non-analytic function at $R = 0$. While the third term $-\frac{R^2}{6m^2}$ is added as result of quantum corrections to matter loops in curved space-time (Starobinsky (1979)). The effects of the additional term in dark energy models in the infrared-modified $f(R)$ gravity were thoroughly considered in (Bamba et al. (2008); Nojiri and Odintsov (2003); Motohashi and Nishizawa (2012); Dolgov et al. (2012)). While this model is still just a toy model it has offered some promise in passing several experimental constraints (Sotiriou et al. 2010; Jain and Zhang 2008). The field equations can be writing as (Alhamzawi 2016)

$$\epsilon \left(b \frac{d}{dx} + \frac{2b}{x+1} + \frac{\dot{a} + \dot{b}}{2} + \frac{(x+1)(b-a)}{2(x+\epsilon a(x+1))} \right) \left(\epsilon \dot{a} + \frac{1}{(x+1)^2} \right) \frac{dF'}{dx} = \frac{\epsilon}{3\phi} - \frac{1}{3} \left(\frac{\epsilon}{\phi} - \frac{2F}{F'} \right) F' - \left(\frac{x}{x+1} \frac{d^2}{dx^2} + \frac{2x+1}{(x+1)^2} \frac{d}{dx} \right) F' \quad (12)$$

$$\left(\frac{x}{x+1} \frac{d}{dx} + \frac{1}{2(x+1)^2} + \epsilon \left(b \frac{d}{dx} + \frac{\dot{b}}{2} \right) \right) \frac{dF'}{dx} = \frac{\epsilon}{3\phi} + \epsilon \left(\frac{b}{(x+1)^2} + \frac{\dot{a}}{x+1} + \frac{b-a}{x+\epsilon a(x+1)} \right) \left(\epsilon \dot{a} + \frac{1}{(x+1)^2} \right) (1+F') + \frac{1}{6} \left(\frac{\epsilon}{\phi} + \frac{F}{F'} \right) F' \quad (13)$$

$$\frac{1}{2} \left(1 + \epsilon \frac{(x+1)(b-a)}{x+\epsilon a(x+1)} \right) \left(\epsilon \dot{a} + \frac{1}{(x+1)^2} \right) \frac{dF'}{dx} = \frac{\epsilon}{3\phi} + \epsilon \left(\frac{\dot{b}}{x+1} + \frac{b}{(x+1)^2} \right) (1+F') + \frac{1}{6} \left(\frac{\epsilon}{\phi} + \frac{F}{F'} \right) F' \quad (14)$$

where $F = f - R$, $\phi = \epsilon/R$, $x = r - r_s$, $(\cdot)$ and $(\cdot)'$ stand for differentiation with respect to x and R , respectively. While it is difficult to find an exact metric solution for this model due to the complexity of the

second term in (11), a solution may be obtained near the region of strong gravitational field. The solution for the above equations for $x \ll 1$ was given in (Alhamzawi 2016) by

$$\mathcal{A}(r) = -\frac{n+1}{3n+2} \left(\frac{6n^3 + 7n^2 + 2n}{n^2 + 2n + 1} \right)^{\frac{1}{2n+2}} \left(\frac{x}{\sqrt{6}} \right)^{\frac{2n+1}{n+1}} \quad (15)$$

$$\mathcal{B}(r) = -\frac{n+1}{9n+6} \left(\frac{36n^3 + 42n^2 + 12n}{n^2 + 2n + 1} \right)^{\frac{1}{2n+2}} x^{\frac{2n+1}{n+1}} \quad (16)$$

In the next section we will find the effective potential for rotating black hole and determine the turning point corresponding to an unstable circular orbit. Then we will calculate the important quantities used to determine the shape of the shadow.

3 Rotating modified gravity black hole

To obtain the apparent shape of black hole we must first analyze the geodesic equations of the black hole. It is difficult to find these geodesic equations due to the complexity of the functions A and B in (1). However, it was argued in (Abdujabbarov et al. 2016) in their consideration of Ayon-Beato-Garica black hole that this problem may be solved if an approximation in θ is considered such that $\theta \approx \pi/2 + \delta$ where δ is small angle. This approximation allows photons to arrive near the equatorial plane for a person far away from black hole (Abdujabbarov et al. 2016). A rotating black hole is characterized by the energy E , the component of the angular momentum L_z and the Carter constant $\mathcal{K}$, where E and L_z are given by

$$\frac{\partial \mathcal{L}}{\partial \dot{t}} = g_{tt} \dot{t}^2 + g_{t\phi} \dot{\phi}^2 = -E, \quad (17)$$

and

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = g_{\phi\phi} \dot{\phi}^2 + g_{t\phi} \dot{t}^2 = L_z. \quad (18)$$

By solving the above equation and using the approximation $\theta \approx \pi/2 + \delta$, one can easily obtain geodesic equations

$$\frac{dt}{d\sigma} = \frac{aA(L_z - aE) + aH(2aE - L_z) + Er^2}{aH^2 + r^2A} \quad (19)$$

and

$$\frac{d\phi}{d\sigma} = \frac{A(L_z - aE) + aEH}{a^2H^2 + r^2A} \quad (20)$$

The remaining geodesic equations can be determined from the Hamilton-Jacobi equation

$$\frac{\partial S}{\partial \sigma} = -\frac{1}{2} g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} \quad (21)$$

where σ is an affine parameter along the geodesic and S is the Jacobi action. Now, we can seek a separable solution for S of the form

$$S = -\frac{1}{2} m_0^2 \sigma - Eu + \Phi L_z + S_\delta(\delta) + S_r(r) \quad (22)$$

where m_0 is the rest mass of the particle and $S_\delta(\delta)$ and $S_r(r)$ are functions of δ and r , respectively. Setting $m_0 = 0$ then inserting (22) in (21) and separating the coefficients of r and δ equal to the Carter constant, we obtain the geodesic equations

$$r^2 \frac{dr}{d\sigma} = \pm \sqrt{R} \quad (23)$$

and

$$r^2 \frac{d\delta}{d\sigma} = \pm \sqrt{\Theta} \quad (24)$$

where

$$\mathcal{R} = \left(aL_z - E \left(a^2 + \frac{r^2}{H} \right) \right)^2 - \Delta(\mathcal{K} + (\Phi - aE)^2) \quad (25)$$

and

$$\Theta = \mathcal{K}. \quad (26)$$

Now, to discuss circular orbits we must first define the effective potential. To achieve this we will introduce the dimensionless variables $\xi = L_z/E$ and $\eta = \mathcal{K}/E^2$ such that $\mathcal{R}$ now reads

$$\mathcal{R} = E^2 \left(a\xi - \left(a^2 + \frac{r^2}{H} \right) \right)^2 - E^2 \Delta(\eta + (a - \xi)^2) \quad (27)$$

To calculate the effective potential we can use the equation (Wei and Liu 2013; Amir and Ghosh 2016)

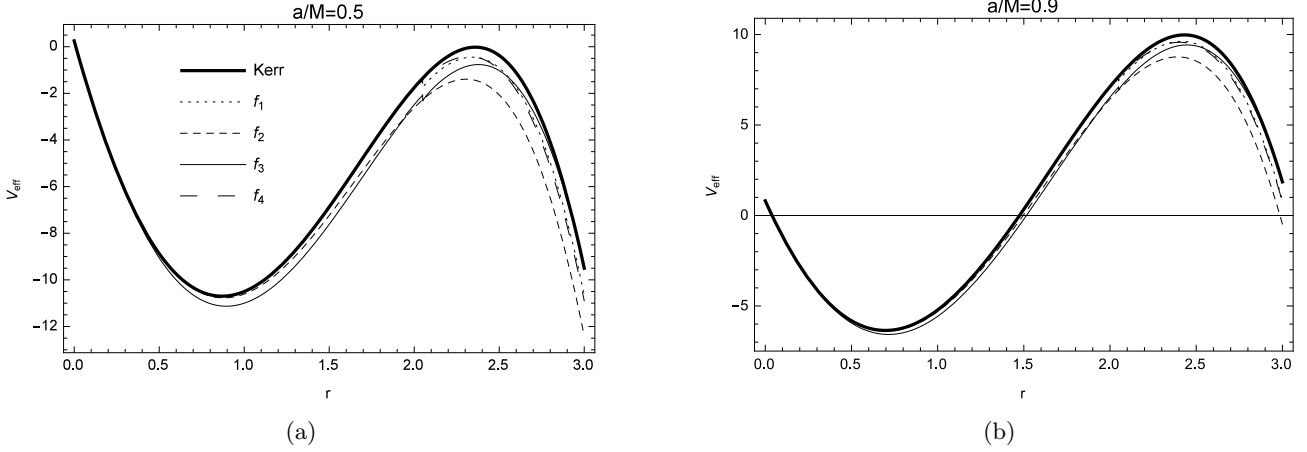


Fig. 1 : Plot showing the behavior of the effective potential V_{eff} versus r for $\xi = 4$ and $\eta = 1$

$$\left(r^2 \frac{dr}{d\sigma}\right)^2 + V_{eff} = 0 \quad (28)$$

such that the effective potential is given by

$$V_{eff} = E^2 \Delta (\eta + (a - \xi)^2) - E^2 \left(a\xi - \left(a^2 + \frac{r^2}{H} \right) \right)^2. \quad (29)$$

The various bounds the effective potential for particular values of ξ, η and a are demonstrated in Fig. 1. A photon coming from infinity reaches a turning point with zero radial velocity. The radius of the unstable circular orbits corresponding to this turning point must satisfy the conditions $V_{eff} = 0$ and $\frac{dV_{eff}}{dr} = 0$ (Wei and Liu 2013). Using these conditions and solving for ξ and η in (29) gives

$$\xi = \frac{1}{aH(rB' + 2B)} \left[2r(a^2 + r^2B) \frac{H'}{H} + ((a^2rH + r^3)B' - 2B(r^2 - a^2H) - 4a^2) \right] \quad (30)$$

and

$$\eta = \frac{r^3}{a^2H^2(rB' + 2B)^2} \left[8a^2B' - r(rB' - 2B)^2 + \frac{4(a^2 + r^2B)}{H^2} (H'H(2B - rB') - rBH'^2) \right]. \quad (31)$$

The conserved quantities ξ and η can be used to determine the contour of the shadow in the (ξ, η) plane. If we substitute $\epsilon = 0$ in the above equation, one easily

obtain the ξ and η parameters for the Kerr metric (Amir and Ghosh 2016)

$$\xi = \frac{2a^2r + a^2r_s + 2r^3 - 3r^2r_s}{a(r_s - 2r)}, \quad (32)$$

$$\eta = \frac{r^3(8a^2r_s - 4r^3 + 12r^2r_s - 9rr_s^2)}{a^2(r_s - 2r)^2} \quad (33)$$

Now, we are ready to discuss the shadows cast by the modified gravity models considered in Section 2. In the next section we will determine how modified gravity effects the apparent shape of black holes and how that compares with the apparent shape of Kerr black hole.

4 Shadows of rotating modified gravity black hole

When a black hole is in front of distant bright object it casts a shadow, the study of this shadow offers an interesting way to describe the nature of black hole. The apparent shape of the shadow can be demonstrated by the celestial coordinates

$$\alpha = \lim_{r_0 \rightarrow \infty} \left(-r_0^2 \sin \theta_0 \frac{d\phi}{dr} \right) \quad (34)$$

$$\beta = \lim_{r_0 \rightarrow \infty} \left(r_0^2 \frac{d\delta}{dr} \right) \quad (35)$$

where the coordinates α and β describe the apparent perpendicular distance of the shadow from the axis of symmetry and from its projection on the equatorial

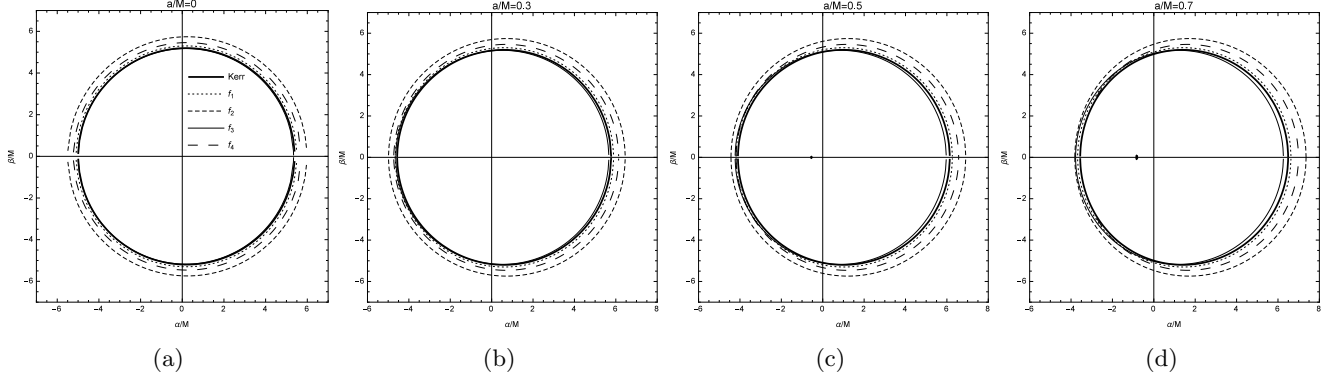


Fig. 2 : Plot showing the shapes of the shadow cast by rotating black hole in modified gravity in comparison with Kerr black hole for some particular values of a

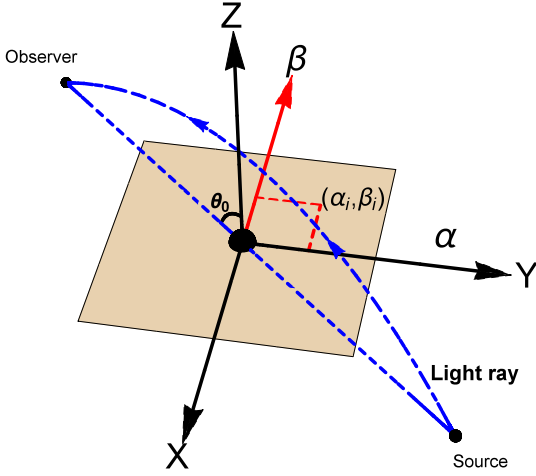


Fig. 3 : A geometrical representation of the celestial coordinates α and β for an observer situated far away from the black hole.

plane, respectively. From Eqs. (19), (20), (23) and (24) we have

$$\alpha = -\xi \quad (36)$$

and

$$\beta = \pm\sqrt{\eta} \quad (37)$$

Since we are considering an observer that is placed in equatorial plane ($\theta = \pi/2$), then from Eqs. (36) and (37) we obtain the relation

$$\alpha^2 + \beta^2 = \frac{1}{H^4 (rB' + 2B)^2} \left[(rB' + 2B) \times (a^2 H^2 - 2H(4a^2 - r^3 B' + 2r^2 B)) + 4(a^2 + r^2 B)(r(rB' + 2B)H' + 4) + (a^2 + r^2 B) \left(\frac{4r^2 H'^2}{H^2} - \frac{16rH'}{H} \right) \right] \quad (38)$$

Plot in Fig. 2 shows the contour of the apparent shadow of rotating black hole in modified gravity in comparison with Kerr black hole for particular values of spin parameter.

I Nonrotating case

In the case for nonrotating black hole ($a = 0$) the shadow is characterized by the standard circle

$$\alpha^2 + \beta^2 = \frac{4r^3 BH'}{H^3 (rB' + 2B)^2} \left[\frac{rH'}{H} + H(rB' + 2B) + \frac{H^2(r^2 B'^2 - 4B^2) + 8B(H - rH')}{2rBH'} \right] \quad (39)$$

In the limit $\epsilon \rightarrow 0$ we obtain the relation for Kerr black hole (Hioki and Maeda 2009)

$$\alpha^2 + \beta^2 = \frac{2r^2(4r^2 - 3r_s^2)}{(2r - r_s)^2} \quad (40)$$

II Rotating case

If we consider nonzero values of spin parameter then the shadow becomes deformed circle. In the general relativity limit ($\epsilon \rightarrow 0$), Eqs. (38) reduces to the relation for Kerr black hole

$$\alpha^2 + \beta^2 = \frac{8r^4 + a^2(2r + r_s)^2 - 6r^2 r_s^2}{(r_s - 2r)^2} \quad (41)$$

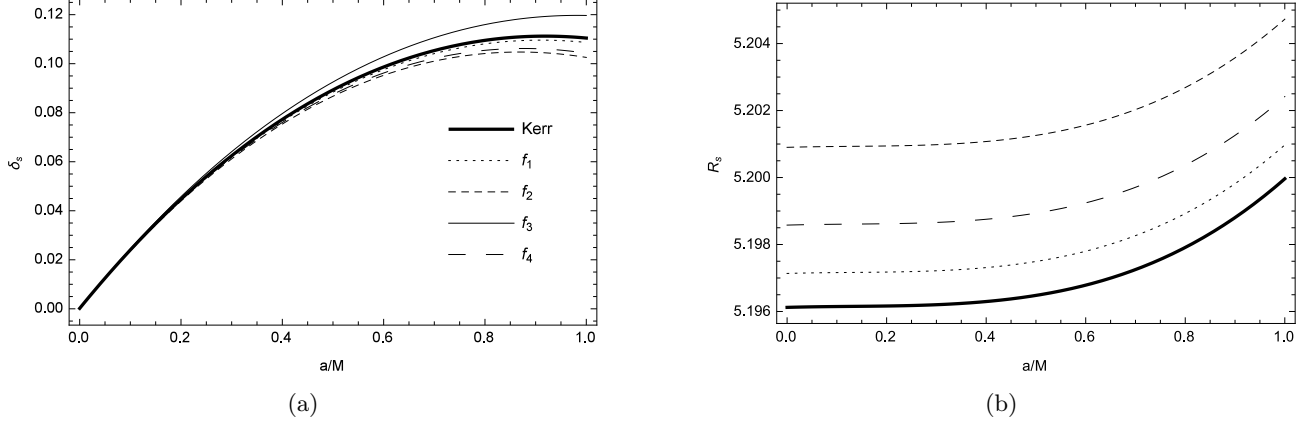


Fig. 4 : (a) Plot showing the behavior of the observable δ_s of the shadow verses a . (b) Plot showing the behavior of the observable R_s of the shadow verses a .

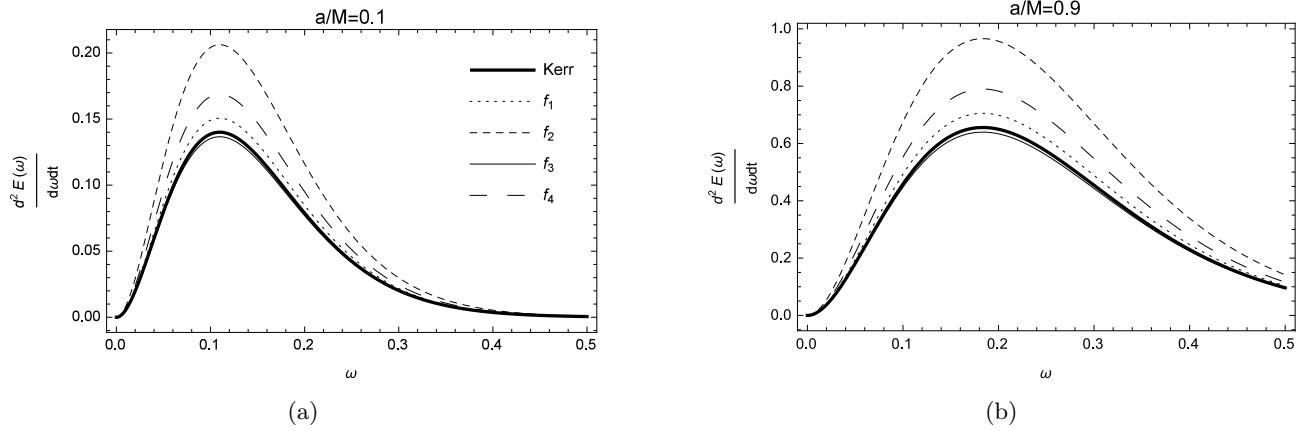


Fig. 5 : Behaviors of the energy emission rate $\frac{d^2 E(\omega)}{d\omega dt}$ versus frequency ω for different values of a

To extract information from the shape of the shadow we must introduce some new observables (Wei and Liu 2013; Amir and Ghosh 2016). If we define the three points: the top position (α_t, β_t) of the shadow, the bottom position $(\alpha_t, -\beta_t)$ of the shadow, and the most right end position $(\alpha_r, 0)$ of the shadow, then we can define the observable R_s as the radius of the circle that passes the three points. The second observables we are going to introduce is the distortion observable $\delta_s = D_{cs}/R_s$, where D_{cs} is the distance between the left most point of the circle and the shadow. These observables can be written in terms of α_t , α_r and β_t such that R_s and δ_s are given by (Amir and Ghosh 2016)

$$R_s = \frac{(\alpha_l - \alpha_r)^2 + \beta_l^2}{2(\alpha_r - \alpha_l)}, \quad (42)$$

$$\delta_s = \frac{\alpha_p - \tilde{\alpha}_p}{R_s}, \quad (43)$$

where the points $(\alpha_p, 0)$ and $(\tilde{\alpha}_p, 0)$ represent the points that cut the horizon at opposite sides of $(\alpha_r, 0)$. These observables can be used to measure the size of the shadow and its deformation from the reference circle. The behavior of δ_s and R_s is shown in Fig. 4. For small values of a the value of δ approaches zero which means the shape of the shadow approaches perfect circle. However as the value of a increases the apparent shape of the black hole deforms and eventually a naked singularity occurs. One interesting thing to notice from Fig. 4 is that the model $f(R) = R^{1+\epsilon}$ produces larger distortions than the other models. Furthermore, one can see that for all models the effects of modified gravity on the distortions of the shadows increases as the spin of the black hole increases. However, for slowly spinning black hole the effects of modified gravity are no longer apparent and eventually vanish for $a = 0$.

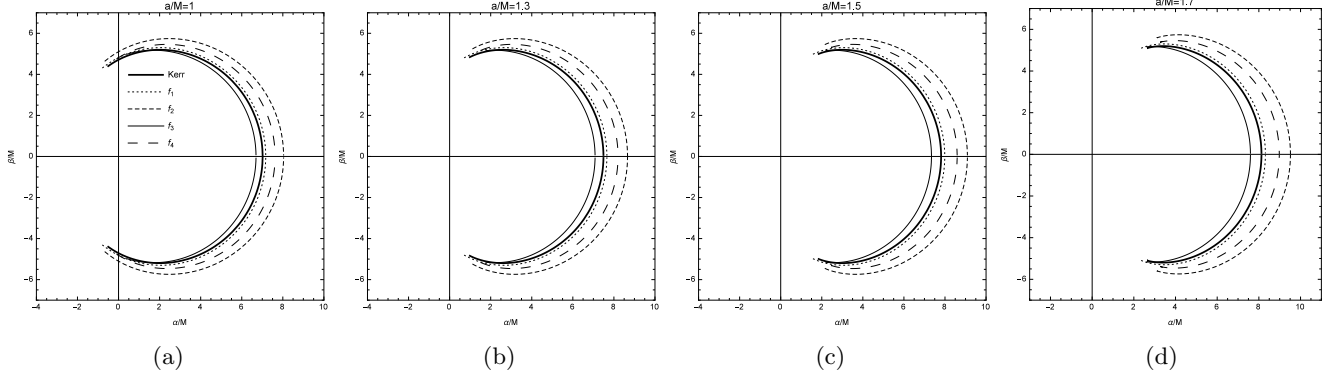


Fig. 6 : Plot of shadows of naked singular for rotating black hole in modified gravity in comparison with Kerr black hole for some combination of values of a

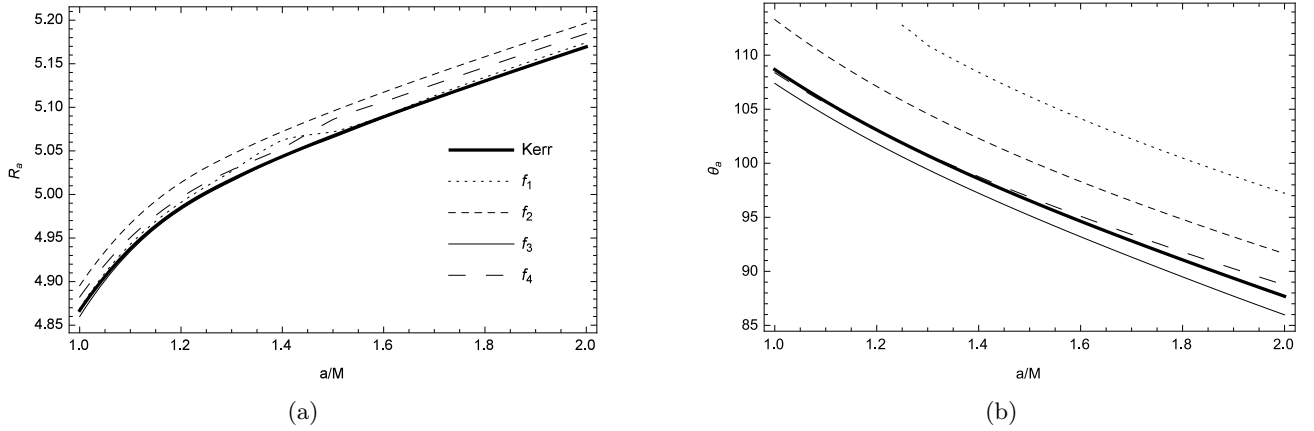


Fig. 7 : (a) Plot showing the behavior of the radius of the reference circle R_a of the shadow verses a . (b) Plot showing the behavior of the observable θ_a of the shadow verses a .

5 Energy Emission Rate

The study of the energy emission can be used to extract information from black hole. The limiting constant value (σ_{lim}) of the absorption cross section for spherically symmetric black hole was approximated in (Wei and Liu 2013) by

$$\sigma_{\text{lim}} \approx \pi R_s^2. \quad (44)$$

Hence the energy emission rate reads

$$\frac{d^2 E(\omega)}{d\omega dt} = \frac{2\pi^3 R_s^2}{e^{\omega/T} - 1} \omega^3 \quad (45)$$

where

$$T \approx \frac{1}{4\pi r_+(a^2 + r_+^2)} \sqrt{\left[\left((r_+^2 - a^2) (1 + \epsilon \mathcal{B}(r_+)) \right. \right. \\ \left. \left. + \epsilon r_+ (a^2 + r_+^2) \mathcal{A}'(r_+) \right) \left(\epsilon r_+ (a^2 + r_+^2) \mathcal{B}'(r_+) \right. \right. \\ \left. \left. + (r_+^2 - a^2) (1 + \epsilon \mathcal{B}(r_+)) \right) \right]} \quad (46)$$

One can see that for $\epsilon = 0$, the above expression reduces to the Hawking temperature for Kerr black hole such that

$$T = \frac{r_+^2 - a^2}{4\pi r_+(r_+^2 + a^2)} \quad (47)$$

Fig. 5 shows the behavior of the energy emission rate against the frequency ω for specific values of the spin parameter. One can see that $f(R) = R^{1+\epsilon}$ produces smaller peak than the other models, this can be

attributed to that fact that the area of the shadow produced by $f(R) = R^{1+\epsilon}$ is smaller than the area produced by the other models. This is clearly demonstrated in Fig. 2. One can see that modified gravity black hole behaviors similar to the Kerr black hole such that as the rotation of the black hole increases the value of θ_a decreases producing a naked singularity with smaller arc.

6 Naked Singularity Shadow

While the idea of naked singularities has not been confirmed, their study is interesting because their existence means that it is possible to observe the collapse of an object with infinite density. The shadow of a naked singularity consists of a dark spot caused by the non existence of the horizon so photons will travel back to the observer and an arc caused by the instability of the photon sphere with positive radius (Wei and Liu 2013). For an observer situated in the equatorial plane the dark spot is not very visible and only the arc can be clearly observed. In Fig. 6 the naked singularity for particular values of the spin parameter is shown. Since rotating naked singularity has different shadow than rotating black hole, then the observables defined in (42) and (43) are no longer valid to describe the shape of the shadow. Hence it is appropriate at this time to define some new observables. We can define the radius of the circle passing the middle point $(\alpha_r, 0)$ and the end points of the arc $(\alpha_l, \pm\beta_l)$ given by (Wei and Liu 2013)

$$R_a = \frac{(\alpha_l - \alpha_r)^2 + \beta_l^2}{2(\alpha_r - \alpha_l)}. \quad (48)$$

The second observables we are going to define is the angle that the arc makes with center of the reference circle

$$\theta_a = \arccos\left(\frac{2R_a^2 - d^2}{2R_a^2}\right) \quad (49)$$

where d is the distance between the end point (α_l, β_l) and the middle point $(\alpha_r, 0)$. The behavior of R_a is shown in Fig. 7 (a) while the behavior of θ_a is shown in Fig. 7 (b). One can see that as the value of the spin parameter increases the the size of the angle decreases producing smaller arc.

7 Conclusion

The investigation into the observables of the black hole shadow is important because these observables can help

to measure the spin parameter and inclination angle and determine if a black hole is a naked singularity. In recent years several authors have studied the effects of different gravitational models on the apparent shape of the shadow of black hole. While no clear observations of the predicted shadow of the black hole has been made at the present time it is interesting to study how different gravitational theories effect the size and distortion of the shadow. In previous works the effects of some types of modified gravity theories on black hole shadows were discussed such as Scalar-Tensor-Vector-Gravity (STVG) (Moffat 2015) and Chern-Simons modified gravity (Amarilla et al. 2010).

In this paper we have investigated the effects of some $f(R)$ modified gravity models on the shape of the apparent shadow of the black hole. The celestial coordinates α and β were used to plot the shadow of the black hole. Then, the observables R_s and δ_s were introduced to describe the radius and the distortions of the black hole shadow for several values of the spin parameter a . We compared the radius of the shadow R_s and the deformation of the shadow δ_s with that of the Kerr black hole. Furthermore, the parameter R_s was used to define the area of the shadow and the energy emission rate was obtained. The energy emission rate was studied for different values of the spin parameter a . We showed how the different models produce different peaks for different values of spin parameter. Finally, we studied the naked singularities produced by each model and found that similar to the Kerr black hole the shadow cast has two parts, an arc and a dark region.

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المستخلص

في هذا البحث تم تقديم دراسة للظل المدلى من خلال الثقوب السوداء الدورية في نماذج مختلفة من الجاذبية المعدلة. وتم اثبات أن حجم الظل يعتمد على نموذج الجاذبية المعدلة. حيث تم حساب تشوهات الظل التي تلقىها الثقوب السوداء في الجاذبية المعدلة ومقارنة الآثار مع التشوهات التي يلقيها ثقب كير الأسود. ووجد ان ظل الثقب الأسود الدوار في الجاذبية المعدلة يكون مشابه لظل ثقب كير الأسود ولكن مع الاختلاف في الحجم واثار التشويه. وتم مناقشة التفردية المجردة للثقب الأسود في الجاذبية المعدلة. وبالإضافة الى ذلك، تم استخدام مساحة الظل لحساب معدل انبعاث الطاقة. وأخيرا، ظهر أن بعض نماذج الجاذبية المعدلة تقدم مساهمة كبيرة في حجم ظل الثقب الأسود.

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مفاتيح الكلمات: الجاذبية المعدلة، الثقب الأسود الدوار، التفردية المجردة، معدل انبعاث الطاقة



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قسم الرياضيات

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بإشراف

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