Studying some of the nuclear properties for ^{170} Er Isotope by the Interacting Boson Model-1

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Abstract:

In this research Er^{170} isotope has been studied some nuclear properties by using interaction boson model -1 to determine the low energy levels and using the program (IBMT) was used for evaluating the reduced electric transition probability $B(E_2)$. Also using the program (IBMP) was used for evaluating the potential energy surface for determine the deformation of nuclei .In this research has been calculated all the square of rotational energy and the moment of inertia for Er^{170} isotope .when the comparative of these properties with, its found experimentally from literature survey were found a good agreements .the attestations refer to this isotope belong to the rotational limit SU(3).

الخلاصة

تم في هذا البحث دراسة بعض الخواص النووية لنظير ^{170}Er باستعمال نموذج البوزونات المتفاعلة الأول لتحديد مستويات الطاقة الواطئة . واستعمل البرنامج (IBMT) لحساب احتمالية الانتقالات الكهربائية المختزلة B(E2) .وكذلك استعمال البرنامج (IBMP) لحساب طاقة جهد السطح لتحديد تشوه النواة. تم في هذا البحث حساب كل من مربع الطاقة الدورانية وعزم والقصور الذاتي لنظير ^{170} Er .

لقد وجد تطابق جيد عند مقارنة هذه الخصائص مع ما موجود عمليا من الدراسات السابقة. وتشير الدلائل إن هذا النظير ينتمي إلى التحديد ألدوراني SU(3) .

Introduction:

A study of nuclear physics centers around two main problems .First , one hopes to understand the properties of the force which holed the nucleus to gather . Second one attempts to describe the behavior of systems of many particles such as nuclei are [1].

Physicists can discuss many particle system only within certain approximations which are determined by the particular experimental fact they wish to explain . the approximation description are called "models". Each one suited only for a limited range of experimental situation .In this researcher studies the nuclear structure by using interaction boson model.

Arima and Iachell [2] In (1974) were suggested nuclear model called the IBM ,its a model for the structure of even-even collective nuclei which assumes that the monopole and quadruple degrees of freedom are the most important[3,4]. It also assumes that all excitations are bosons because of the existence of pairing interactions which are dominant at low energies. The (IBM-1) model developed after input of degrees of freedom where its discovered anew nuclear properties for the nuclei. Therefore this model called (IBM-2)[5,6]. Some scientists studied the nuclear structure of Erbium isotopes, therefore the following some the researchers about this subject .

In (1998)[7] L.M.Chen studied the negative , high – spin state of the Even - odd Er nucleus with mass number ($159<A<165$) by using the interacting boson – fermion

model. Its was found that the energy spectra up to (I=57\2$^-$) of the Even - odd Er isotopes could be reproduced quite well. The observed B (E2) values were calculated and compared with the experimental data.

In (2000)[8] R.S.Guo and L.M.Chen studied the positive , high – spin state of the odd $Er^{155\text{-}165}$ isotopes by using the interacting boson – fermion model. Its was found that the calculated positive high – spin state energy spectra of the odd Er isotopes agree quilt well with the experimental data. The B (E2) values for $Er^{155\text{-}165}$ are calculated and compared with the experimental data.

In (2004)[9] Yazar and Uluer studied the energy levels and transition energy in $Er^{166\text{-}168}$ isotopes by using the proton – neutron interacting boson approximation (IBM-2). The results obtain for $Er^{166\text{-}168}$ are reasonably in good agreement with the previous experimental and theoretical values.

The dynamic symmetry of Rotational limits Su(3):

This dynamic symmetry based on the boson energy (ε) is smaller than the reaction potential (V) ,(V>>ε) . Both the reaction electric quadruple moment (Q.Q) and the reaction angular moment (L.L) are control on the Rotational limit SU(3) .therefore the general Hamilton formula for this limit is[10] :

$$H^1 = a_1 L^2 + a_2\ Q^2 \text{.....(1)}$$

and the equation of eigen value to Hamilton is given by [11] :

$$E\left|N,(\lambda,\mu),K,L,M\right\rangle = \frac{a_2}{2}(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)+(a_1-\frac{3a_2}{8}).L(L-1)\text{.....(2)}$$

where :

{(λ,μ),K,L,M} the quantum numbers, but (λ,μ) determined the Rotational limit SU(3) state.

The transition operator $T_m^{(E_2)}$ for this limits were given by following formula [10]:

$$T_m^{(E_2)} = \alpha_2^2\left[\left(d^\dagger s + s^\dagger d\right)_m^{(2)} - \frac{\sqrt{7}}{2}\left(d^\dagger d\right)_m^{(2)}\right]\text{.....(3)}$$

The selection rules [12,13] are ($\Delta\lambda = 0, \Delta\mu = 0$) , the Equation which using for calculation evaluate the reduced transition probability B(E2) is [14] :

$$B(E2\ ,L+2\to L) = \alpha_2^2\frac{3}{4}\frac{(L+2)(L+1)}{(2L+3)(2L+5)}(2N-L)(2N+L+3)\text{.....(4)}$$ or

$$B(E2\ ,2_1^+\to 0_1^+) = \alpha_2^2\frac{N}{5}(2N+3)\text{.....(5)}$$

The formal for determining the electric quadruple moment (Q) to this limits equal :[13]

$$QL = -\alpha_2\sqrt{\frac{16\pi}{40}}\frac{L}{2L+3}(4N+3)\text{.....(6)}$$ or

$$Q_{2_1^+} = -\alpha_2\sqrt{\frac{16\pi}{40}}\frac{2}{7}(4N+3)\text{.....(7)}$$

The relation between (α_2) and (β_2) for this limits are :[12]

$$\beta_2 = -\frac{\sqrt{7}}{2}\alpha_2 \text{.....(8)}$$

Where ((α_2) and (β_2)) parameters using for determining the reduced transition probability.

Potential energy surface

The potential energy surface function of total number of bosons and deformation factors of (β,γ) . it was calculated from the equation (9).[11,12]

$$V(N,\beta,\gamma) = \frac{\langle N,\beta,\gamma \left| \overset{\Lambda}{H} \right| N,\beta,\gamma\rangle}{\langle N,\beta,\gamma | N,\beta,\gamma\rangle} \text{.....(9)}$$

By derivative the equation (9) respect with (γ,β) get on :[11,12]

$$V(N,\beta,\gamma) = \frac{N}{1+\beta^2}\left(\varepsilon_s + \varepsilon_d\beta^2\right) + \frac{N(N-1)}{\left(1+\beta^2\right)^2}\left(A_1\beta^4 + A_2\beta^3\cos 3\gamma + A_3\beta^2 + A_4\right) \text{.....(10)}$$

N : total number of bosons.

β : Magnitude of Nuclear Deformation ,take the value (0 to 2.4).

γ : Asymmetry Angle , take the value (0^o to 60^o).

A_1,A_2,A_3,A_4 : Parameters of potential surface .

The Deformed Nuclei depended on (β,γ) factors .when $\beta=0$ the Nuclei is spherical and when $\beta>0$ the Nuclei is Deformed , otherwise when $\gamma=0^o$ the Deformations nuclei are spherical of Prelate Shape and when $\gamma=60^o$ the Deformations nuclei are spherical of Oblate Shape.

Rotational Motion Nucleus and Moment of Inertia:

The formulas for calculating all the square of rotational energy and the moment of inertia are: [15]

$$\hbar^2\omega^2 = (L^2 - L + 1)\left[\frac{E(L \to L-2)}{2L-1}\right]^2 \text{.....(11)}$$

$$\frac{2\upsilon}{\hbar^2} = \frac{4L-2}{E(L \to L-2)} \text{.....(12)}$$

Calculations

When study the nuclear properties of Er^{170} isotope it was found that this isotope determine for rotational limit, depended on the results which calculated by using program interacting boson model -1 (IBM-1) and comparison with the Experimental result, in addition to the shape of potential energy surface.

The properties of nuclear which studied in this research are energy levels, the reduced transition probability, the potential energy surface, the square of rotational energy and the moment of inertia.

Calculation energy levels

The experimental values of the energy levels show the rotation natural , means the ^{170}Er isotope determined to rotational limit SU(3). The parameter was given best coincidence with experimental value [16, 17, 18, 19] show in the table (1).

Table (1) parameters using in this program for calculation energy levels ^{170}Er in Mev

isotopes	Eps	P.P	L.L	Q.Q	T3.T3	T4.T4	CH_1
^{170}Er	0	0	0.0096	-0.010	0	0	-1.00

Either the theoretical of the energy levels were calculated by using (IBM-1) model and comparison with the Experimental of the energy levels shown in table (4).

Calculation the reduced transition probability B(E_2)

The values of reduced transition probability B(E_2) were calculated by using program (IBMT-1) , depends on the value of parameters (β_2, α_2) which it calculated from equations(5,8). In this study determination these parameters depends on the experimental value for transition $B(E_2, 2_1^+ \rightarrow 0_2^+)$ .The parameters which used in the (IBMT-1) program (E_2SD & E_2DD) can be calculated as follow: [13]

$$E2SD = \alpha_2 \quad \ldots\ldots(9)$$

$$E2DD = \sqrt{5}\beta_2 \quad \ldots\ldots(10)$$

In addition to ,The value of (α_2)can be found after calculated transition probability B(E2) from the following equation .[20]

$$B(E2)\downarrow = \frac{56.57}{E_\gamma^5 t_{\frac{1}{2}} [1+\alpha_{tot}]} (e^2 b^2) \quad \ldots\ldots(11)$$

Where:

E_γ : gamma ray transition energy in (KeV)

$t_{1/2}$: half life for the level (2_1^+).

α_{tot} : Total internal conversion coefficient which can be found from the table [21].

The table (2) show the parameters (E2SD & E2DD) using in (IBMT-1) program for calculation reduced transition probability B(E_2)of ^{170}Er .

Isotopes	E2SD	E2DD
^{170}Er	0.09561	-0.28282

table (3) show the values of theoretical forB(E2) in ^{170}Er by using (IBMT)Cood compared with the experiential values of B(E2)

i→f	$B(E_2)\downarrow e^2b^2$ The.	$B(E2)\downarrow e^2b^2$ exp[15,20]
$2_1^+ \rightarrow 0_1^+$	1.1630280	1.164
$2_1^+ \rightarrow 0_2^+$	0.0469830	----------
$2_2^+ \rightarrow 0_1^+$	0.1007575	----------
$2_2^+ \rightarrow 0_2^+$	0.0023627	----------
$2_2^+ \rightarrow 0_3^+$	0.0258483	----------
$2_3^+ \rightarrow 0_2^+$	1.0304850	----------
$2_3^+ \rightarrow 0_3^+$	0.0052185	----------
$2_4^+ \rightarrow 0_2^+$	0.0005503	----------
$2_4^+ \rightarrow 0_3^+$	0.6563919	----------
$2_1^+ \rightarrow 2_3^+$	0.0610162	----------
$2_2^+ \rightarrow 2_1^+$	0.1564144	----------
$2_2^+ \rightarrow 2_3^+$	0.0011347	----------
$4_1^+ \rightarrow 2_1^+$	1.5759380	----------
$4_1^+ \rightarrow 2_2^+$	0.0015161	----------
$4_1^+ \rightarrow 2_3^+$	0.0750297	----------
$4_2^+ \rightarrow 2_1^+$	0.0527230	----------
$4_2^+ \rightarrow 2_2^+$	0.4884615	----------
$4_2^+ \rightarrow 2_3^+$	0.0035351	----------
$4_3^+ \rightarrow 2_1^+$	0.0006046	----------
$4_3^+ \rightarrow 2_2^+$	0.0000883	----------
$4_3^+ \rightarrow 2_3^+$	1.3816890	----------

Table (4) the energy levels in ^{170}Er found experimentally compared with IBM-1 for positive parity calculation :

J^+	Exp.(MeV)[22]	The.(MeV)
2^+	0.07859	0.07940
	0.9598	0.85674
	1.0105	0.96900
	—	1.59107
	—	1.64346
	—	1.78041
	—	2.21975
	—	2.226391
	—	2.36678
	—	2.50901
4^+	0.2601	0.26464
	(1.123)	1.04315
	—	1.15465
	—	1.72361
	—	1.77891
	—	1.83110
	—	1.96655
	—	2.40584
	—	2.41090
	—	2.45530
6^+	0.5411	0.55569
		1.33593
	—	1.44638
	—	2.01900
	—	2.07358
	—	2.12615
	—	2.25903
	—	2.60048
	—	2.69961
	—	2.71262
0^+	0.000	0.000
	0.8909	0.88943
	0.932	1.51048
	—	1.70063
	—	2.18205
	—	2.42889
	—	2.69275
	—	2.83903
	—	3.07284
	—	3.11047

Calculation potential energy surface

The potential energy surface was calculated after determining the parameters of Hamilton function operator that specialized for ^{170}Er . Table (5) shows the parameters which are used in (IBMP) program for calculating potential energy surface V (β,γ) Table (5) parameters used in this program for calculating potential energy surface for ^{170}Er in MeV

Isotope	A_1	A_2	A_3	A_4	ES	ED
^{170}Er	-0.050	0.038	-0.003	-0.021	-0.040	0.000

The potential energy surface gives the last shape of nucleus .Its agreed with Hamilton function [21] of two parameters (β,γ).Fig (1) shows ^{170}Er isotope that belong to the rotational limit SU(3) if comparative of the papers [11].

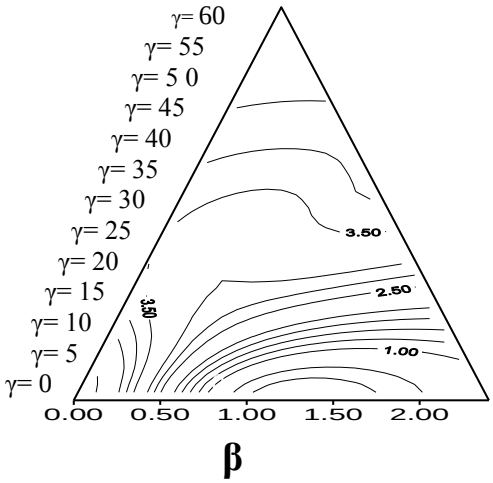

Fig (1) Contours diagram for potential energy surface of ^{170}Er isotope

Calculation the square of rotational energy and the moment of inertia.

The square of rotational energy and the moment of inertia can be calculated from equation (10, 11) after found the energy levels by using (IBM-1) program and angular moment to all energy levels. The fig (2) and (3) show comparative between the theoretical and experimental values [15] for all the square of rotational energy and the moment of inertia.

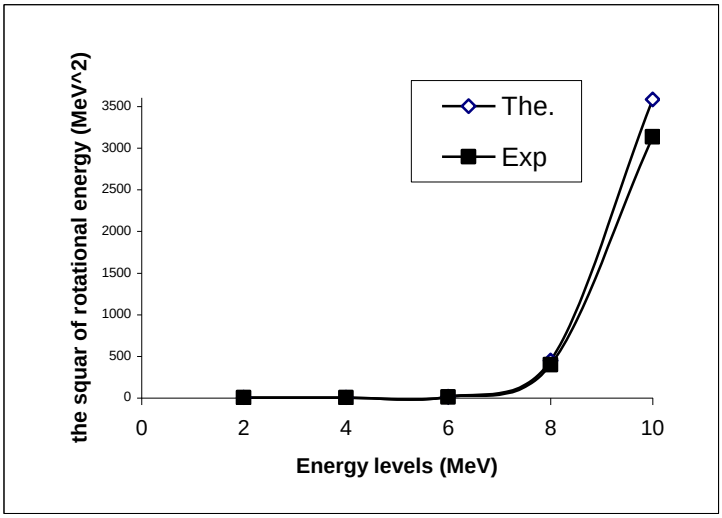

Fig (2) Shows the comparison between the theoretical and experimental values for the square of rotational energy.

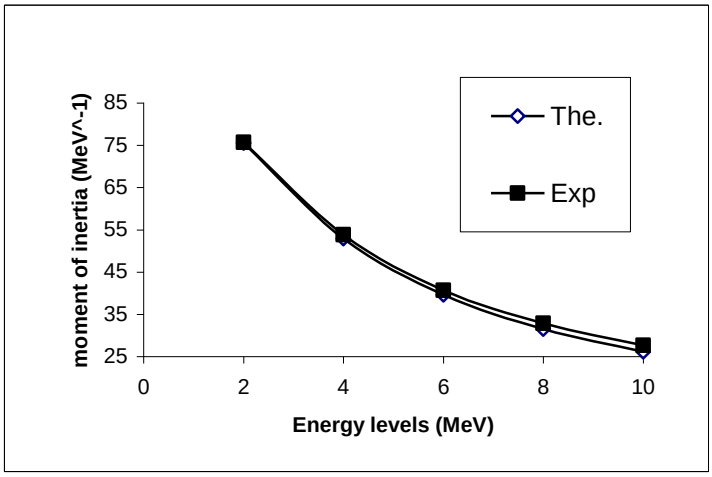

Fig (3) Shows the comparison between the theoretical and experimental values for the moment of inertia.

Result and Dissection

The nuclei of ^{170}Er consists of (68) protons and (102) neutrons .therefore the total number of bosons are (17) bosons. The number of protons and neutrons are hit nearly of half closed shell between (50-82) and (82-126) on succession. this happen refers to ^{170}Er that rotational limits and this property doesn't found in vibration limits .the table (1) shows parameters using in this program for calculation energy levels .it was found the (Q.Q) and (L.L) dominate on other parameters .also there are three indicia refers to ^{170}Er SU (3) The energy levels , Reduced electric transition probability and potential energy surface as follow :

1) Energy levels: the ratio of energy levels $\frac{E0_2^+}{E2_1^+}, \frac{E4_1^+}{E2_1^+}, \frac{E8_1^+}{E2_1^+}, \frac{E6_1^+}{E2_1^+}$

Refers to a good nearly to SU (3) after comparative with exemplary value [12,23].

2) Reduced electric transition probability: the levels decay 2_1^+ to 0_1^+ and 4_1^+ to 2_1^+ in one band and don't decay between bands[24].

3) Potential energy surface: The cantors shape in fig (1) shows this isotope Su(3) after comparative with exemplary figure of SU(3) [11].

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