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Differential Subordination Results Defined by New Class for Higher-Order Derivatives of Multivalent Analytic Functions

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Dedicated to my best friend and my colleague **Firas Hussean Maghool** on the occasion of her getting scientific upgrade

ABSTRACT: By making use of the principle of differential subordination, we introduce and study a new class for higher-order derivatives of multivalent analytic functions in the open unit disk U. We obtain some interesting results of this class. Also we derive some convolution properties in geometric function Theory.

KEY WORDS: Multivalent function, Subordination, Convex univalent, Hadamard product, Higher-order derivatives.

I. INTRODUCTION

Let $R(p,m)$ denote the class of functions f of the form:

$$f(z)=z^p+\sum_{n=m}^{\infty}a_{n+p}z^{n+p} \quad (p,m\in\mathbb{N}=\{1,2,\dots\}), \tag{1}$$

which are analytic in the open unit disk $U=\{z\in\mathbb{C}:|z|<1\}$. Upon differentiating both sides of (1) q-times with respect to z, we obtain (see [1])

$$f^{(q)}(z)=\delta(p,q)z^{p-q}+\sum_{k=m}^{\infty}\delta(n+p,q)a_{n+p}z^{n+p-q}, \qquad (q\in\mathbb{N}_0=\mathbb{N}\cup\{0\}, p>q),$$

where

$$\delta(i,j)=\frac{i!}{(i-j)!}=\begin{cases}1 & (j=0)\\ i(i-1)\dots(i-j+1) & (j\neq 0)\end{cases}.$$

For two functions f and g analytic in U, we say that the function f is subordinate to g, written $f\prec g$ or $f(z)\prec g(z)$ $(z\in U)$, if there exists a Schwarz function w, analytic in U with $w(0)=0$ and $|w(z)|<1$ $(z\in U)$ such that $f(z)=g(w(z))$, $(z\in U)$.In particular, if the function g is univalent in U, then $f\prec g$ if and only if $f(0)=g(0)$ and $f(U)\subset g(U)$.

If $f\in R(p,m)$ is given by (1) and $g\in R(p,m)$ given by

$$g(z)=z^p+\sum_{n=m}^{\infty}b_{n+p}z^{n+p} \quad (p,m\in\mathbb{N}),$$

then the Hadamard product (or convolution) $f*g$ of the functions f and g is defined by

$$(f*g)(z)=z^p+\sum_{n=m}^{\infty}a_{n+p}b_{n+p}z^{n+p}=(g*f)(z).$$

A function $f\in R(1,m)$ is said to be starlike of order α in U if and only if

$$Re\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha, (0\le\alpha<1, z\in U).$$

Denote the class of all starlike functions of order α in U by $S^*(\alpha)$.

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A function $f \in R(1,m)$ is said to be prestarlike of order α in U if

$$\frac{z}{(1-z)^{2(1-\alpha)}} * f(z) \in S^*(\alpha), (\alpha < 1).$$

Denote the class of all prestarlike functions of order α in U by $\Re(\alpha)$.

Clearly a function $f \in R(1,m)$ is in the class $\Re(0)$ if and only if f is convex univalent in U and $\Re\left(\frac{1}{2}\right) = S^*\left(\frac{1}{2}\right)$.

Let H be the class of functions h with $h(0) = 1$, which are analytic and convex univalent in U.

Recently, many authors have introduced and studied some new subclasses of analytic functions defined by various linear operators, like, Dziok and Srivastava [2,3], Srivastava et al. [11,12], Patel et al. [8,9], Liu et al. [4,5,6], Wang et al. [13] and Yang et al. [14]. Now we introduce the following subclass of $R(p,m)$ for higher-order derivatives.

Definition 1:- A function $f \in R(p,m)$ is said to be in the class $M(\gamma,\eta,p,q,m;h)$ if it satisfies the subordination condition:

$$\frac{(1-\gamma)(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}} - \eta\right) + \frac{\gamma(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}} - \eta\right) \prec h(z), \tag{2}$$

Where $\gamma \in C, p \in \mathbb{N}, q \in \mathbb{N}_0, p > q, 0 \le \eta < p$ and $h \in H$.

We need the following Lemmas in order to derive our main results for the class $M(\gamma,\eta,p,q,m;h)$.

Lemma 1:- [7] Let g be analytic in U and let h be analytic and convex univalent in U with $h(0) = g(0)$. If

$$g(z) + \frac{1}{\mu} z g'(z) \prec h(z), \tag{3}$$

where $Re(\mu) \ge 0$ and $\mu \ne 0$, then

$$g(z) \prec \tilde{h}(z) = \mu z^{-\mu} \int_0^z t^{\mu-1} h(t)dt \prec h(z)$$

and $\tilde{h}$ is the best dominant of (3).

Lemma 2:- [10]. Let $\alpha < 1, f \in S^*(\alpha)$and $g \in \Re(\alpha)$. Then, for any analytic function F in U.

$$\frac{g*(fF)}{g*F}(U) \subset \overline{co}\big(F(U)\big),$$

where $\overline{co}\big(F(U)\big)$ denotes the closed convex hull of $F(U)$.

II. MAIN RESULTS

Theorem 1:- Let $0 \le \gamma_1 < \gamma_2$. Then

$$M(\gamma_2,\eta,p,q,m;h) \subset M(\gamma_1,\eta,p,q,m;h).$$

Proof: Let $0 \le \gamma_1 < \gamma_2$ and $f \in M(\gamma_2,\eta,p,q,m;h)$.

Suppose that

$$g(z) = \frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}} - \eta\right). \tag{4}$$

Then the function g is analytic in U with $g(0) = 1$.

Since $f \in M(\gamma_2,\eta,p,q,m;h)$, then we have

$$\frac{(1-\gamma_2)(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}} - \eta\right) + \frac{\gamma_2(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}} - \eta\right) \prec h(z), \tag{5}$$

Differentiating both sides of (4) with respect to z and using (5), we have

$$\frac{(1-\gamma_2)(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}} - \eta\right) + \frac{\gamma_2(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}} - \eta\right) = g(z) + \frac{\gamma_2}{p-q} z g'(z) \prec h(z).$$

Hence, an application of Lemma 1 with $\mu = \frac{p-q}{\gamma_2}$, yields

$$g(z) \prec h(z). \tag{6}$$

Noting that $0 \le \frac{\gamma_1}{\gamma_2} < 1$ and that h is convex univalent in U, it follows from (4), (5) and (6) that

$$\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}} - \eta\right)$$

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$$= \frac{\gamma_1}{\gamma_2}\left[\frac{(1-\gamma_2)(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)+\frac{\gamma_2(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}}-\eta\right)\right]+\left(1-\frac{\gamma_1}{\gamma_2}\right)g(z) \prec h(z).$$

Therefore, $f \in M(\gamma_1, \eta, p, q, m; h)$ and the proof of Theorem 1 is complete. □

Theorem 2:- Let $f \in M(\gamma, \eta, p, q, m; h), g \in R(p, m)$ and

$$Re\left\{\frac{g(z)}{z^p}\right\} > \frac{1}{2}, \quad (7)$$

then $f * g \in M(\gamma, \eta, p, q, m; h)$.

Proof: Let $f \in M(\gamma, \eta, p, q, m; h)$ and $g \in R(p, m)$. Then, we have

$$\frac{(1-\gamma)(p-q)!}{p!-\eta(p-q)!}\left(\frac{\left((f*g)(z)\right)^{(q)}}{z^{p-q}}-\eta\right)+\frac{\gamma(p-q)!}{p!-\eta(p-q)!}\left(\frac{\left((f*g)(z)\right)^{(q+1)}}{(p-q)z^{p-q-1}}-\eta\right)$$

$$= \frac{(1-\gamma)(p-q)!}{p!-\eta(p-q)!}\left(\frac{g(z)}{z^p}\right)*\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)+\frac{\gamma(p-q)!}{p!-\eta(p-q)!}\left(\frac{g(z)}{z^p}\right)*\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}}-\eta\right)=\left(\frac{g(z)}{z^p}\right)*\psi(z), \quad (8)$$

where

$$\psi(z) = \frac{(1-\gamma)(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)+\frac{\gamma(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}}-\eta\right) \prec h(z), \quad (9)$$

From (7) note that the function $\frac{g(z)}{z^p}$ has the Herglotz representation

$$\frac{g(z)}{z} = \int_{|x|=1}\frac{d\mu(x)}{1-xz} \quad (z \in U), \quad (10)$$

where $\mu(x)$ is a probability measure defined on the unit circle $|x| = 1$ and

$$\int_{|x|=1} d\mu(x) = 1.$$

Since h is convex univalent in U, it follows from (8), (9) and (10) that

$$\frac{(1-\gamma)(p-q)!}{p!-\eta(p-q)!}\left(\frac{\left((f*g)(z)\right)^{(q)}}{z^{p-q}}-\eta\right)+\frac{\gamma(p-q)!}{p!-\eta(p-q)!}\left(\frac{\left((f*g)(z)\right)^{(q+1)}}{(p-q)z^{p-q-1}}-\eta\right)=\int_{|x|=1}\psi(xz)\,d\mu(x) \prec h(z).$$

This shows that $f * g \in M(\gamma, \eta, p, q, m; h)$.□

Corollary 1:- Let $f \in M(\gamma, \eta, p, q, m; h)$be defined as in (1) and

$$Re\left\{1+\sum_{n=m}^{\infty}\frac{c+p}{c+p+n}z^n\right\} > \frac{1}{2}.$$

Then

$$k(z) = \frac{c+1}{z^c}\int_0^z t^{c-1}f(t)dt, \quad (c > -p)$$

is also in the class $M(\gamma, \eta, p, q, m; h)$.

Proof: Let $f \in M(\gamma, \eta, p, q, m; h)$ be defined as in (1). Then

$$k(z) = \frac{c+1}{z^c}\int_0^z t^{c-1}f(t)dt = z^p+\sum_{n=m}^{\infty}\frac{c+p}{c+p+n}a_{n+p}z^{n+p} = \left(z^p+\sum_{n=m}^{\infty}a_{n+p}z^{n+p}\right)*\left(z^p+\sum_{n=m}^{\infty}\frac{c+p}{c+p+n}z^{n+p}\right)$$

$$= (f*G)(z), \quad (11)$$

where

$$f(z) = z^p+\sum_{n=m}^{\infty}a_{n+p}z^{n+p} \in M(\gamma, \eta, p, q, m; h)$$

$$G(z) = z^p+\sum_{n=m}^{\infty}\frac{c+p}{c+p+n}z^{n+p} \in R(p, m).$$

Note that

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$$Re\left\{\frac{G(z)}{z^p}\right\} = Re\left\{1+\sum_{n=m}^{\infty}\frac{c+p}{c+p+n}z^n\right\} > \frac{1}{2}. \tag{12}$$

From (11) and (12) and by using Theorem 2, we get $k(z) \in M(\gamma,\eta,p,q,m;h)$.□

Theorem 3. Let $f \in M(\gamma,\eta,p,q,m;h), g \in R(p,m)$ and $z^{1-p}g(z) \in \Re(\alpha), (\alpha < 1)$. Then

$$f * g \in M(\gamma,\eta,p,q,m;h).$$

Proof: For $f \in M(\gamma,\eta,p,q,m;h)$ and $g \in R(p,m)$, from (8) (used in the proof of Theorem 2), we can write

$$\frac{(1-\gamma)(p-q)!}{p!-\eta(p-q)!}\left(\frac{\left((f*g)(z)\right)^{(q)}}{z^{p-q}}-\eta\right)+\frac{\gamma(p-q)!}{p!-\eta(p-q)!}\left(\frac{\left((f*g)(z)\right)^{(q+1)}}{(p-q)z^{p-q-1}}-\eta\right)=\frac{\left(z^{1-p}g(z)\right)*\left(z\psi(z)\right)}{\left(z^{1-p}g(z)\right)*z}, \tag{13}$$

where $\psi(z)$ is defined as in (9).

Since h is convex univalent in U, $\psi(z) \prec h(z)$, $g(z) \in \Re(\alpha)$ and $z \in S^*(\alpha)$, $(\alpha < 1)$, it follows from (13) and Lemma 2, we obtain the result.□

Theorem 4:- Let $f \in M\left(\gamma,\eta,p,q,m;\frac{1+Az}{1+Bz}\right)$,with $\gamma > 0$ and $-1 \le B < A \le 1$. Then

$$\frac{p-q}{\gamma}\int_0^1 \frac{1-Au}{1-Bu}u^{\frac{p-q}{\gamma}-1}du < Re\left\{\frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)\right\} < \frac{p-q}{\gamma}\int_0^1 \frac{1+Au}{1+Bu}u^{\frac{p-q}{\gamma}-1}du. \tag{14}$$

Proof: Let g be defined as in (4). Then the function g is analytic with $g(0) = 1$.

After a short calculation and considering that $f \in M\left(\gamma,\eta,p,q,m;\frac{1+Az}{1+Bz}\right)$, we can conclude that

$$g(z)+\frac{\gamma}{p-q}zg'(z) \prec \frac{1+Az}{1+Bz}.$$

An application of Lemma 1, yields

$$\frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right) \prec \tilde{h}(z) = \frac{p-q}{\gamma}z^{-\frac{(p-q)}{\gamma}}\int_0^z \frac{1+At}{1+Bt}t^{\frac{p-q}{\gamma}-1}dt$$

$$= \frac{p-q}{\gamma}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{p-q}{\gamma}-1}du \prec \frac{1+Az}{1+Bz} \quad (z \in U)$$

and $\tilde{h}$ is the best dominant.

Now

$$Re\left\{\frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)\right\} < \sup_{z\in U} Re\left\{\frac{p-q}{\gamma}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{p-q}{\gamma}-1}du\right\}$$

$$\le \frac{p-q}{\gamma}\int_0^1 \sup_{z\in U} Re\left(\frac{1+Azu}{1+Bzu}\right)u^{\frac{p-q}{\gamma}-1}du$$

$$< \frac{p-q}{\gamma}\int_0^1 \frac{1+Au}{1+Bu}u^{\frac{p-q}{\gamma}-1}du, \tag{15}$$

and

$$Re\left\{\frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)\right\} > \inf_{z\in U} Re\left\{\frac{p-q}{\gamma}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{p-q}{\gamma}-1}du\right\}$$

$$\ge \frac{p-q}{\gamma}\int_0^1 \inf_{z\in U} Re\left(\frac{1+Azu}{1+Bzu}\right)u^{\frac{p-q}{\gamma}-1}du$$

$$> \frac{p-q}{\gamma}\int_0^1 \frac{1-Au}{1-Bu}u^{\frac{p-q}{\gamma}-1}du. \tag{16}$$

Combining (15) and (16), we get (14) and the proof is complete.□

Theorem 5:- Let $\gamma > 0, \lambda > 0$ and $f \in M(\gamma,\eta,p,q,m;\lambda h+1-\lambda)$. If $\lambda \le \lambda_0$, where

$$\lambda_0 = \frac{1}{2}\left(1-\frac{p-q}{\gamma}\int_0^1 \frac{u^{\frac{p-q}{\gamma}-1}}{1+u}du\right)^{-1}, \tag{17}$$

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then $f \in M(0,\eta,p,q,m;h)$. The bound λ_0 is the sharp when $h(z)=\frac{1}{1-z}$.
Proof: Suppose that

$$g(z)=\frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right). \tag{18}$$

Let $f \in M(\gamma,\eta,p,q,m;\lambda h+1-\lambda)$ with $\gamma>0$ and $\lambda>0$. Then, we have

$$g(z)+\frac{\gamma}{p-q}zg'(z)=\frac{(1-\gamma)(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)+\frac{\gamma(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}}-\eta\right)\prec\lambda h(z)+1-\lambda.$$

By using Lemma 1, we have

$$g(z)\prec\frac{\lambda(p-q)}{\gamma}z^{-\frac{(p-q)}{\gamma}}\int_0^z t^{\frac{p-q}{\gamma}-1}h(t)dt+1-\lambda=(h*\phi)(z), \tag{19}$$

where

$$\phi(z)=\frac{\lambda(p-q)}{\gamma}z^{-\frac{(p-q)}{\gamma}}\int_0^z \frac{t^{\frac{p-q}{\gamma}-1}}{1-t}dt+1-\lambda. \tag{20}$$

If $0<\lambda\le\lambda_0$, where $\lambda_0>1$ is given by (17), then it follows from (20) that

$$Re\big(\phi(z)\big)=\frac{\lambda(p-q)}{\gamma}\int_0^1 u^{\frac{p-q}{\gamma}-1}Re\left(\frac{1}{1-uz}\right)du+1-\lambda>\frac{\lambda(p-q)}{\gamma}\int_0^1\frac{u^{\frac{p-q}{\gamma}-1}}{1+u}du+1-\lambda\ge\frac{1}{2}.$$

Now, by using the Herglotz representation for $\phi(z)$, from (18) and (19), we get

$$\frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)\prec(h*\phi)(z)\prec h(z).$$

Since h is convex univalent in U,then $f \in M(0,\eta,p,q,m;h)$.
For $h(z)=\frac{1}{1-z}$ and $f \in R(p,m)$ defined by

$$\frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)=\frac{\lambda(p-q)}{\gamma}z^{-\frac{(p-q)}{\gamma}}\int_0^z\frac{t^{\frac{p-q}{\gamma}-1}}{1-t}dt+1-\lambda,$$

we have

$$\frac{(1-\gamma)(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)+\frac{\gamma(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q+1)}(z)}{(p-q)z^{p-q-1}}-\eta\right)=\lambda h(z)+1-\lambda.$$

Thus, $f \in M(\gamma,\eta,p,q,m;\lambda h+1-\lambda)$. Also, for $\lambda>\lambda_0$, we have

$$Re\left\{\frac{(p-q)!}{p!-\eta(p-q)!}\left(\frac{f^{(q)}(z)}{z^{p-q}}-\eta\right)\right\}\longrightarrow\frac{\lambda(p-q)}{\gamma}\int_0^1\frac{u^{\frac{p-q}{\gamma}-1}}{1+u}du+1-\lambda<\frac{1}{2}\quad(z\to1),$$

which implies that $f \notin M(0,\eta,p,q,m;h)$. Therefore, the bound λ_0 cannot be increased when $h(z)=\frac{1}{1-z}$ and this completes the proof of the theorem.□

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