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On cocompact open set

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Abstract

In this paper, we have studied coc-continuous, coc-closed, coc-open functions and coc-compact space. We shall provided some properties of these concepts and it will explain the relationship among them and some results on this subjects are proved Throughout this work, some important and new concept have been illustrated including coc-compact, K-space.

Mathematics Subject Classification: 54XX

1. Introduction

In [1] S. Al Ghour and S. Samarah introduced coc-open sets in topological spaces. In [2] Sharma J.N. introduced definition of continuous function, in section two we introduce new definition coc-continuous and coc$'$-continuous function and some properties of these concepts . In [3] N. Bourbaki introduced closed and open function, through this paper we introduce new definitions of coc-open, coc'-open, coc-closed and coc'-closed functions. N. Bourbaki [3] introduced the concept of compact space , R. Engleking [4] and Willard S. [5] presented the concepts ofcompactly closed set and K-space, In [1] S. Al Ghour and S. Samarah introduced CC space, In section four introduces the definition of coc-compact space,compactly coc-closed, coc- K-space, CC' space, CC'' space and CC''' space and give useful characterizations of this concepts.

Definition (1.1) [1]:

A subset A of a space (X,τ) is called cocompact open set (notation : coc-open set) if for every $x \in A$ there exists an open set $U \subseteq$ X and a compact subset K $\in$ C(X,τ) such that $x \in U -$ K $\subseteq$ A.The complement of coc-open set is called coc-closed set.
The family of all coc-open subsets of a space (X, τ) is denoted by τ^k.

Theorem (1.2) [1]:

Let (X, τ) be a space Then the collection τ^k forms a topology on X.

Theorem(1.3) [1]:

Let X be a space. Then $\tau \subseteq \tau^k$.

Definition(1.4)[1]:

A space X is called CC if every compact set in X is closed.

Theorem(1.5) [1]:

Let X be a space. Then the following statements are equivalent:
i. X is CC.
ii. $\tau = \tau^k$.

Theorem(1.6) [1]:

If X is a hereditarily compact space, then τ^k is discrete topology.

Definition(1.7)[1]:

Let X be a space and A $\subseteq X$. The intersection of all coc-closed sets of X containing A called coc- closure of A and is denoted by $\overline{A}^{coc}$ or coc- $Cl_\tau(A)$.

Remark(1.8):

$\overline{A}^{coc}$ smallest coc-closed set containing A.

Proposition(1.9):

Let X be a space and $A \subseteq B \subseteq$ X. Then

i. . $\overline{A}^{coc}$ is an coc-closed set .

ii. A is an coc-closed set if and only if $A = \overline{A}^{coc}$

iii. $\overline{A}^{coc} \subseteq \overline{A}$

iv. $\overline{A}^{coc} = \overline{\overline{A}^{coc}}^{\text{coc}}$

v. if $A \subseteq B$ then. $\overline{A}^{coc} \subseteq \overline{B}^{\text{coc}}$

Proof:

(i) Follows from definition (1.7)

ii. Let $\overline{A}^{coc}$ is an coc-closed set in X , since $A \subseteq \overline{A}^{coc}$ and A coc-closed set such that $A \subseteq A$, $\overline{A}^{coc} \subseteq A$ (since $\overline{A}^{coc}$ smallest coc-closed set containing A)then $A = \overline{A}^{coc}$

Conversely let $A = \overline{A}^{coc}$, since $\overline{A}^{coc}$ is an coc-closed set in X .Then A is an coc-closed set

iii. see [1]

iv. Thus from (i) and (ii)

v. $B \subseteq \overline{B}^{\text{coc}}$ and since $A \subseteq B$ then $A \subseteq \overline{B}^{\text{coc}}$, $\overline{B}^{\text{coc}}$ is an coc-closed set in X containing A, then $\overline{A}^{coc} \subseteq \overline{B}^{\text{coc}}$ (since $\overline{A}^{coc}$ smallest coc-closed set containing A)

Definition(1.9):

Let X be a space and $A \subseteq X$. The union of all coc-open sets of X contained in A is called coc- Interior of A and denoted by $A^{\circ\text{coc}}$ or coc-$In_\tau(A)$.

Remark(1.10):

$A^{\circ\text{coc}}$ largest coc- open set contained in A.

Proposition(1.11):

Let X be a space and $A \subseteq$ X. Then $x \in A^{\circ\text{coc}}$ iff there is an coc-open set U containing x such that $\in U \subseteq A$.

Proof:

Let $x \in A^{\circ\text{coc}}$,Then $x \in \cup \{V: V\ is \text{ coc} - \text{open in X and V} \subseteq \text{A}\}$, Thus $\exists$ U coc-open set $\ni x \in U \subseteq A$.

Conversely, let $x \in U \subseteq A \ni U$ coc-open set $x \in \cup \{V: V\ is \text{ coc} - \text{open in X and V} \subseteq \text{A}\}$,Then $x \in A^{\circ\text{coc}}$.

Proposition(1.12):

Let X be a space and $A \subseteq B \subseteq X$. Then:

i. $A^{\circ coc}$ is an coc-open set .

ii. A is an coc-open set if and only if $A = A^{\circ coc}$

iii. $A^{\circ} \subseteq A^{\circ coc}$

iv. $A^{\circ coc} = (A^{\circ coc})^{\circ coc}$

v. if $A \subseteq B$ then. $A^{\circ coc} \subseteq B^{\circ coc}$

Proof:

(i) and (ii) Follows from definition (1.10) and theorem(1.2)

iii. Let $\in A^{\circ}$, there is an open set U containing x such that $x \in U \subseteq A$, since every open set is coc-open set, then by proposition (1.11) $x \in A^{\circ coc}$.

iv. Thus from (i) and (ii)

v. $A^{\circ coc}$ is an coc-open set contained in A and since $A \subseteq B$ then $A^{\circ coc}$ is an coc-open set contained in B. But $B^{\circ coc}$ is the largest coc-open set contained in B Thus $A^{\circ coc} \subseteq B^{\circ coc}$.

Proposition(1.13) [1]:

Let X be a space and Y any nonempty closed in X .If B is an coc-open set in X then $B \cap Y$ is an coc-open set in Y.

Definition(1.14):

A space X is called cocT_2-space (coc-Hausdorff space) iff for each $x \neq y$ in X there exists disjoint an coc-open setsU, V such that $x \in U, y \in V$.

Remark(1.15):

It is clear that every Hausdorff space is coc-Hausdorff space. But the converse is not true in general as the following example shows :

Let X be a finite set contain more than one point and T be indiscrete topology on X then (X, T) is not T_2-space, but(X, T) is cocT_2-space since T^k is discrete topology on X.

2.1 Certain types of coc-continuous functions

In this section , we review the definition of coc-continuous , remarks and propositions about this concept.

Definition(2.1):

Let $f: X \to Y$ be a function of a space X into a space Y then f is called an coc-continuous function if $f^{-1}(A)$ is an coc-open set in X for every open set A in Y.

Theorem(2.2)[1]:

A function$f: (X, \tau) \to (Y, \tau')$ is an coc-continuous if and only if $f: (X, \tau^k) \to (Y, \tau')$ is a continuous.

Example(2.3):

Let $f: X \to Y$ be a function of a space X into a space Y then

i. The constant function is an coc-continuous function.

ii. If X is discrete then f is an coc-continuous function.

iii. If X is a finite set and τ any topology on X then is f an coc-continuous function.

Remark(2.4):

It is clear that every continuous function is an coc-continuous function, but the converse not true in general as the following example shows:

Let $X = \{a, b\}$ and $Y = \{c, d\}$, τ be indiscrete topology on X and $\tau' = \{\phi, Y, \{c\}\}$ be a topology on Y. Let $f: X \to Y$ be a function defined by $f(a) = c, f(b) = d$ then f is an coc-continuous ,but is not continuous.

Proposition(2.5):

Let $f: X \to Y$ be a function of a space X into a space Y. Then the following statements are equivalent:

i. f is an coc-continuous function.

ii. $f^{-1}(A^\circ) \subseteq (f^{-1}(A))^{\circ coc}$ for every set A of Y.

iii. $f^{-1}(A)$ is a coc-closed set in X for every closed set A in Y.

iv. $f(\overline{A}^{coc}) \subseteq \overline{f(A)}$ for every set A of X.

v. $\overline{f^{-1}(A)}^{coc} \subseteq f^{-1}(\overline{A})$for every set A of Y.

Proof:

(i)→(ii) Let $A \subseteq Y$, since $A^\circ$is an open set in Y, then$f^{-1}(A^\circ)$ is an coc-open set in X thus $f^{-1}(A^\circ) = (f^{-1}(A^\circ))^{\circ coc} = (f^{-1}(A))^{\circ coc}$.

(ii)→(iii) Let A be a closed subset of Y then A^c is an open set in Y .Thus $f^{-1}(A^c) \subseteq (f^{-1}(A^c))^{\circ coc}$ and hence $(f^{-1}(A))^c \subseteq ((f^{-1}(A))^c)^{\circ coc}$ and therefore$(f^{-1}(A))^c = ((f^{-1}(A))^c)^{\circ coc}$ hence $(f^{-1}(A))^c$ is an coc-open set in X and $f^{-1}(A)$ is coc-closed set in X.

(iii)→(iv) Let $A \subseteq Y$.Then $\overline{f(A)}$ is closed set in Y then by (iii) $f^{-1}(\overline{f(A)})$ is an coc-closed set in X containing A.Thus $\overline{A}^{coc} \subseteq f^{-1}(\overline{f(A)})$ and hence $f(\overline{A}^{coc}) \subseteq \overline{f(A)}$.

(iv)→(v) Let $A \subseteq Y$.Then by (iv)$f(\overline{(f^{-1}(A)}^{coc}) \subseteq \overline{f(f^{-1}(A))}$ .Thus $\overline{(f^{-1}(A))}^{coc} \subseteq f^{-1}(\overline{A})$.

(v)→(i) Let Fbe an open set in Y then$F^c = \overline{F^c}$ by hypothesis $\overline{(f^{-1}(F^c))}^{coc} \subseteq f^{-1}(\overline{F^c})$.Hence $\overline{(f^{-1}(F^c))}^{coc} \subseteq f^{-1}(F)$.Therefore $f^{-1}(F)$ is an coc-open set in X.Thus f is an coc-continuous function.

Remark(2.6):

As a consequence of proposition(2.5) we have f is an coc- continuous if and only if the inverse image of every closed set in Y is an coc-closed set in X.

Definition(2.7):

Let $f: X \to Y$ be a function of a space X into a space Y then f is called an coc-irresolute (coc'-continuous for brief) function if $f^{-1}(A)$ is an coc-open set in X for every coc-open set A in Y.

Note that a function$f: (X, \tau) \to (Y, \tau')$ is an coc'-continuous if and only if $f: (X, \tau^K) \to (Y, \tau'^{K})$ is a continuous.

Example(2.8):

i. The constant function is an coc-continuous function.

ii. Let $X$and Y are finite sets and $f: X \to Y$ be a function of a space X into a space Y then coc'-continuous.

Remark(2.9):

Every an coc'continuous function is an coc-continuous function, but the converse not true in general as the following example shows:

Example(2.10):

Let U be usual topology on $\mathbb{R}$ and τ be indiscrete topology on $Y = \{1,2\}$.Let $f: \mathbb{R} \to Y$ be a function defined by $f(c) = \begin{cases} 1 & if x \in \mathbb{Q} \\ 2 & if x \in \mathbb{Q}^c \end{cases}$

Then f is coc- continuous but is not coc'-continuous.

Proposition(2.11):

Let $f: X \to Y$ be a function of a space X into a space Y then f is an coc'-continuous function if and only if the inverse image of every coc-closed in Y is an coc-closed set in X.

Proof:

Let f is an coc'-continuous and let B coc-closed set in Y, then B^c coc-open in Y, since f is an coc'-continuous ,then $f^{-1}(B^c)$ coc-open in X. $f^{-1}(B^c) = f^{-1}(Y - B) = f^{-1}(Y) - f^{-1}(B) = X - f^{-1}(B) = (f^{-1}(B))^c$. Then $(f^{-1}(B))^c$ coc-open in X therefore $f^{-1}(B)$ coc-closed set in X.

Conversely, let A coc-open in Y, then A^c coc-closed in Y ,then $f^{-1}(A^c)$ coc-closed in X $f^{-1}(A^c) = f^{-1}(Y - A) = f^{-1}(Y) - f^{-1}(A) = X - f^{-1}(A) = (f^{-1}(A))^c$. Then $(f^{-1}(B))^c$ coc-closed in X therefore $f^{-1}(A)$ coc-open set in X. Then f is an coc'-continuous.

Proposition(2.12):

Let $f: X \to Y$ be a function of a space X into a space Y. Then the following statements are equivalent:

i. f is an coc'-continuous function.

ii. $f(\overline{A}^{coc}) \subseteq \overline{f(A)}^{coc}$ for every set $A \subseteq X$.

iii. $\overline{f^{-1}(B)}^{coc} \subseteq f^{-1}(\overline{B}^{coc})$ for every set $B \subseteq Y$.

Proof:

(i)→(ii) Let $A \subseteq X$,then $f(A) \subseteq Y$, $\overline{f(A)}^{coc}$ coc-closed set in Y, since f is an coc'-continuous. Then $f^{-1}(\overline{f(A)}^{coc})$ coc-closed set in X, since $f(A) \subseteq \overline{f(A)}^{coc}$ then $f^{-1}(f(A)) \subseteq f^{-1}(\overline{f(A)}^{coc})$, since $A \subseteq f^{-1}(f(A))$,then $A \subseteq f^{-1}(\overline{f(A)}^{coc})$. Since $f^{-1}\left(\overline{f(A)}^{coc}\right)$ coc-closed ,then $\overline{A}^{coc} = f^{-1}(\overline{f(A)}^{coc})$ then $f\left(\overline{A}^{coc}\right) = f(f^{-1}\left(\overline{f(A)}^{coc}\right)) \subseteq \overline{f(A)}^{coc}$.

(ii)→(iii) Let $f(\overline{A}^{coc}) \subseteq \overline{f(A)}^{coc}$ $\forall A \subseteq X$ and let $B \subseteq Y$ then $f^{-1}(B) \subseteq X$, then $f(\overline{f^{-1}(B)}^{coc}) \subseteq \overline{f(f^{-1}(B))}^{coc}$, Since $f(f^{-1}(B)) \subseteq B$, then $\overline{f(f^{-1}(B))}^{coc} \subseteq \overline{B}^{coc}$, $f^{-1}(\overline{f(f^{-1}(B))}^{coc}) \subseteq f^{-1}(\overline{B}^{coc})$. Then $\overline{f^{-1}(B)}^{coc} \subseteq f^{-1}(\overline{B}^{coc})$

(iii)→(i) Let B coc-closed set in Y ,then $B = \overline{B}^{coc}$.Since $\overline{f^{-1}(B)}^{coc} \subseteq f^{-1}(\overline{B}^{coc})$ then $\overline{f^{-1}(B)}^{coc} \subseteq f^{-1}(B)$. Since $f^{-1}(B) \subseteq \overline{f^{-1}(B)}^{coc}$ Then $f^{-1}(B) = \overline{f^{-1}(B)}^{coc}$. Then $f^{-1}(B)$ coc-closed set in X. Therefore f is an coc'-continuous function.

Proposition(2.13):

Let $f: X \to Y$ be a function and A is a nonempty closed set in X

i If f coc- continuous then $f|_A: A \to Y$ is coc- continuous.

ii. If f coc'-continuous then $f|_A: A \to Y$ is coc'-continuous.

Proof:

i. see[1]

ii. Let B be an coc-open set in Y, since f is coc'- continuous then $f^{-1}(B)$ is coc-open set in X. $f^{-1}(B) \cap A$ is coc-open set in A .But $(f|_A(B))^{-1} = f^{-1}(B) \cap A$. Hence $(f|_A(B))^{-1}$ is coc-open set in A.Thus $f|_A$ is coc'- continuous

Remark(2.14):

A composition of two coc- continuous functions not necessary be an coc- continuous function as the following example shows:

Example(2.15):

Let $X = \mathbb{R}$ the set of Real numbers, $Y = \{0,1,2\}$,$W = \{a, b\}$,$\tau = \{X\} \cup \{U \subseteq X: 1 \notin U\}$,the compact set are $\{K \subseteq X: 1 \in K\} \cup \{K \subseteq X: 1 \notin K \ is\ finite\}$ hence $\tau^k = \tau \cup \{U \subseteq X: 1 \in U\ and\ X - U\ is\ finite\}$, $\tau' = \{\phi, Y, \{0\}, \{0,1\}\}$, $\tau'' = \{\phi, W, \{a\}\}$ be topology on Y and W respectively .If $f: X \to Y$ is function defined by

$f(x) = \begin{cases} 1 & if x \in \{0,1\} \\ 2 & otherwise \end{cases}$ and $g: Y \to W$ is a function defined by $g(0) = g(2) = a$ and $g(1) = b$.Then f, g are coc- continuous functions .But $f \circ g$ is not an coc- continuous since $(f \circ g\ )^{-1}(\{a\}) = \{0,1\}$ is not coc-open set in X.

Proposition(2.16) [1]:

Let X, Y and Z are spaces and $f: X \to Y$ is coc-continuous if $g: Y \to Z$ is continuous then $g \circ f: X \to Z$ is coc-continuous.

Proposition(2.17):

Let X, Y and Z are spaces and $f: X \to Y$, $g: Y \to Z$ are functions then if $f$is ancoc'-continuous and g is an coc-continuous then $g \circ f: X \to Z$ is coc-continuous.

Proof:

Let A be an open set in Z then $g^{-1}(A)$ is an coc-open set in Y.Since $f$is an coc'-continuous then$f^{-1}(g^{-1}(A)\ ) = (g \circ f)^{-1}(A)$ is an coc-open set in X .Hence $g \circ f$ is coc-continuous.

Proposition(2.18):

Let $f: X \to Y$ and $g: Y \to Z$ are coc'-continuous then $g \circ f: X \to Z$ is coc'continuous.

Proof: clear.

Theorem(2.19) [1]:

Let $f: X \to Y$ be a function for which X is CC then the following statements are equivalent:

i. f is continuous.

ii. f is coc-continuous.

Theorem(2.20):

Let $f: X \to Y$ be a function for which X is CC then the following statements are equivalent:

i. f is continuous.

ii. f is coc'-continuous.

Proof: clear

3. coc-closed and coc-open functions

In this section , we review the definition of coc-closed and coc-open functions and propositions about this subject.

Definition(3.1):

Let $f: X \to Y$ be a function of a space X into a space Y then:

i. f is called an coc-closed function if $f(A)$ is an coc-closed set in Y for every closed set A in X.

ii. f is called an coc-open function if $f(A)$ is an coc-open set in Y for every open set A in X.

Example(3.2):

i. The constant function is an coc-closed function.
ii. Let $f: X \to Y$ be a function of a space X into a space Y such that Y be a finite set then f is an coc-open function.

Remark(3.3):

i. A function$f: (X, \tau) \to (Y, \tau')$ is an coc-open if and only if $f: (X, \tau^k) \to (Y, \tau')$ is an open function.
ii. Every closed (open) function is an coc-closed (an coc-open) function, but the converse not true in general as the following example shows:

Example(3.4):

Let $X = \{1,2,3\}$,$Y = \{4,5\}$, $\tau = \{\phi, \mathrm{X}, \{3\}\}$ be a topology on X and τ' be indiscrete topology on Y. Let $f: X \to Y$ be a function defined by $(1) = f(2) = 4$, $f(3) = 5$ then f is an coc-closed (an coc-open) function but is not a closed (an open) function.

Proposition(3.5):

A function$f: X \to Y$ is an coc-closed if and only if $\overline{f(A)}^{coc} \subseteq f(\overline{A})$ for all $A \subseteq X$.
Proof:
Suppose that $f: X \to Y$ is an coc-closed function, let $A \subseteq X$, since $\overline{A}$ is closed set in X. Then $f(\overline{A})$ is coc-closed set in Y, since $A \subseteq \overline{A}$ then $f(A) \subseteq f(\overline{A})$, hence $\overline{f(A)}^{coc} \subseteq \overline{f(\overline{A})}^{coc}$ .But $\overline{f(\overline{A})}^{coc} = f(\overline{A})$.There for $\overline{f(A)}^{coc} \subseteq f(\overline{A})$.
Conversely, let F be a closed set of X, then $F = \overline{F}$ by hypothesis $\overline{f(F)}^{coc} \subseteq f(\overline{F})$, hence$\overline{f(F)}^{coc} \subseteq f(F)$, thus $f(F)$ is an coc-closed set in Y.There for $f: X \to Y$ is an coc-closed function.

Proposition(3.6):

Let $f: X \to Y$ be a function and $f(\overline{A}) = \overline{f(A)}^{coc}$ for each set A of X. Then f is coc-closed, continuous function.
Proof:
By proposition(3.5) f is an coc-closed function.
Now to prove that f is continuous, let $F \subseteq X$ then $f(\overline{F}) = \overline{f(F)}^{coc}$ .Since $\overline{f(F)}^{coc} \subseteq \overline{f(F)}$.Hence $f(\overline{F}) \subseteq \overline{f(F)}$,then f is continuous.

Proposition(3.7):

Let $f: X \to Y$ and $g: Y \to Z$ is an coc-closed function then if f is a closed and g is an coc-closed then $g \circ f$ is a coc-closed function.
Proof: clear

Proposition(3.8):

A function$f: X \to Y$ is coc-open if and only if $f(A^\circ) \subseteq (f(A))^{\circ coc}$ for all $A \subseteq X$.
Proof:
Suppose that $f: X \to Y$ is an coc-open function, let $A \subseteq X$, since A° open in X. Then $f(A^\circ)$ coc-open in Y,since $A^\circ \subseteq A$ then$f(A^\circ) \subseteq f(A)$ hence $(f(A^\circ))^{\circ coc} = (f(A))^{\circ coc}$ but $(f(A^\circ))^{\circ coc} \subseteq f(A^\circ)$ .Then$f(A^\circ) \subseteq (f(A))^{\circ coc}$.
Conversely, let A be open in X,then $A^\circ = A$ since $f(A^\circ) \subseteq (f(A))^{\circ coc}$,then $f(A) \subseteq (f(A))^{\circ coc}$,then $f(A) = (f(A))^{\circ coc}$. Hence $f(A)$ coc-open in Y. There for $f: X \to Y$ is an coc'-open function.

Proposition(3.9):

Let $f: X \to Y$ be a coc-closed function Then the restriction of f to a closed subset F of X is an coc-closed of F into Y.

Proof:

Since F is a closed subset in X. Then the inclusion function $i|_F: F \to X$ is a closed function. Since $f: X \to Y$ is an coc-closed function then by Proposition(3.7) $f \circ i|_F: F \to Y$ is an is an coc-closed function. But $f \circ i|_F = f|_F$ is an coc-closed function.

Proposition(3.10):

A bijective function $f: X \to Y$ is an coc-closed function if and only if f is an coc-open function.

Proof:

Let f be bijective, coc-closed function and U be an open subset of X. Thus U^c is a closed. Since f is an coc-closed, then $f(U^c)$ is an coc-closed set in Y.Thus$(f(U^c))^c$ is an coc-open. Since f is bijective then $(f(U^c))^c = f(U)$.Hence $f(U)$ is an coc-open set in Y.
In similar way we can prove that only if part.

Proposition(3.11):

Let $f: X \to Y$ be bijective function from a space X into a space Y then:
i. f is an coc-open function if and only if f^{-1} is an coc-continuous.
ii. f is an coc-closed function if and only if f^{-1} is an coc-continuous.

Proof:

i. Let $f: X \to Y$ be bijective function, then$(f^{-1})^{-1}(A) = f(A)\ \forall\ A \subseteq X$. Let A open set in X , sincef^{-1} is an coc-continuous then$(f^{-1})^{-1}(A)$ is an coc-open set in Y,since f is bijective then $f(A)$ is an coc-open set in Y. Hence f is an coc-open function.
Conversely, let f coc-open function, U be an open subset of X ,then $f(U)$ coc-open set in Y, $(f^{-1})^{-1}(A) = f(A)$ coc-open set in Y.Thenf^{-1} is an coc-continuous function. In similar way we can prove that (ii).

Definition(3.12):

Let X and Y are spaces then a function $f: X \to Y$ is called an coc-homeomorphism if:
i. f is bijective.
ii. $f$is an coc-continuous.
iii. f is an coc-closed (coc-open) .

It is clear that every homeomorphism is an coc-homeomorphism.

Definition(3.13):

Let $f: X \to Y$ be a function of a space X into a space Y then:
i. f is called an coc'-closed function if $f(A)$ is an coc-closed set in Y for every coc-closed set A in X.
ii. f is called an coc'-open function if $f(A)$ is an coc-open set in Y for every coc-open set A in X.

Example(3.14):

i. The constant function is an coc'-closed function.
ii. Let X and Y be finite sets and f be function from a space X into a space Y then f is an coc'-open function.

Proposition(3.15):

A function$f: X \to Y$ is coc'-closed if and only if $\overline{f(A)}^{coc} \subseteq f(\overline{A}^{coc})$ for all $A \subseteq X$.
Proof:
Suppose that $f: X \to Y$ is an coc'-closed function, let $A \subseteq X$, since $\overline{A}^{coc}$ is coc-closed set in X. Then $f(\overline{A}^{coc})$ is coc-closed set in Y, since $f(A) \subseteq f\left(\overline{A}^{coc}\right)$then $\overline{f(A)}^{coc} \subseteq f\left(\overline{A}^{coc}\right)$
Conversely, let A be a coc-closed set of X, then $A = \overline{A}^{coc}$ by hypothesis $\overline{f(A)}^{coc} \subseteq f\left(\overline{A}^{coc}\right)$, hence$\overline{f(F)}^{coc} \subseteq f(A)$, thus $f(A)$ is an coc-closed set in Y.There for $f: X \to Y$ is an coc'-closed function.

Proposition(3.16):

Let X, Y and Z are spaces and $f: X \to Y$, $g: Y \to Z$ be a function then:
i. If f and g are coc'-closed function then $g \circ f$ is coc'-closed function.
ii. If $g \circ f$ is coc'-closed function, f is coc'-continuous and onto then g is coc'-closed.
iii. If $g \circ f$ iscoc'-closed function, g is coc'-continuous and onto then f is coc'-closed.
Proof:
i. let F be a coc-closed set in X, then $f(F)$ is an coc-closed set in Y, thus $g(f(F))$ is an coc-closed set in Z. But $(g \circ f)(F) = g(f(F))$. Hence $g \circ f$ is coc'-closed function.
ii. let F be a coc-closed set in Y, then by Proposition(2.11) $f^{-1}(F)$ is coc-closed set in X. Thus $g \circ f(f^{-1}(F)\ )$ is coc-closed set in Z. Since f is onto Then $g \circ f(f^{-1}(F)\ ) = \ g(F)$, hence $g(F)$ is coc-closed set in Z. Thus g is coc'-closed.
iii. let F be a coc-closed set in X,then $g \circ f(F)$ is coc-closed set in Z. , then by Proposition (2.11) $g^{-1}(\ g \circ f(F))$ is coc-closed set in Y. Since g is onto, then $g^{-1}\big(\ g \circ f(F)\big) = f(F)$, hence $f(F)$ is coc-closed set in Y. Then g is coc'-closed.

Proposition(3.17):

Let X, Y and Z are spaces and $f: X \to Y$, $g: Y \to Z$ be a function then:
i. If f and g are coc'-open function then $g \circ f$ is coc'-open function.
ii. If $g \circ f$ is coc'- open function, f is coc'-continuous and onto then g is coc'- open.
iii. If $g \circ f$ iscoc'- open function, g is coc'-continuous and onto then f is coc'- open.
Proof:
i. let F be a coc- open set in X, then $f(F)$ is an coc- open set in Y, thus $g(f(F))$ is an coc-open set in Z. But $(g \circ f)(F) = g(f(F))$. Hence $g \circ f$ is coc'- open function.
ii. Let F be a coc-open set in Y, then $f^{-1}(F)$ is coc-open set in X. Thus $g \circ f(f^{-1}(F)\ )$ is coc-open set in Z. Since f is onto Then $g \circ f(f^{-1}(F)\ ) = \ g(F)$, hence $g(F)$ is coc- open set in Z. Thus g is coc'- open.
iii. Let F be a coc-open set in X,then $g \circ f(F)$ is coc-open set in Z. Then $g^{-1}(\ g \circ f(F))$ is coc-open set in Y. Since g is onto, then $g^{-1}\big(\ g \circ f(F)\big) = f(F)$, hence $f(F)$ is coc-open set in Y. Then g is coc'- open.

Proposition(3.18):

A function$f: X \to Y$ is coc'-open if and only if $f(A^{\circ coc}) \subseteq (f(A))^{\circ coc}$ for all $A \subseteq X$.

Proof:

Suppose that $f: X \to Y$ is an coc'-open function, let $A \subseteq X$, since $A^{\circ coc}$ coc-open in X. Then $f(A^{\circ coc})$ coc-open in Y, hence $f(A^{\circ coc}) = (f(A^{\circ coc}))^{\circ coc} \subseteq (f(A))^{\circ coc}$.
Conversely, let A coc-open in X,since $f(A^{\circ coc}) \subseteq (f(A))^{\circ coc}$,then $f(A) \subseteq (f(A))^{\circ coc}$,then $f(A) = (f(A))^{\circ coc}$. Hence $f(A)$ coc-open in Y
There for $f: X \to Y$ is an coc'-open function.

Proposition(3.19):

A bijective function $f: X \to Y$ is an coc'-closed function if and only if f is an coc'-open function.

Proof:

Let f be bijective, coc'-closed function and U be an coc-open subset of X. Thus U^c is a coc-closed. Since f is an coc'-closed, then $f(U^c)$ is an coc-closed set in Y.Thus$(f(U^c))^c$ is an coc-open. Since f is bijective then $(f(U^c))^c = f(U)$.Hence $f(U)$ is an coc-open set in Y.
There for f is an coc'-open function.
In similar way we can prove that only if part.

Proposition(3.20):

Let $f: X \to Y$ be bijective function from a space X into a space Y then:
i. f is an coc'-open function if and only if f^{-1} is an coc'-continuous.
ii.f is an coc'-closed function if and only if f^{-1} is an coc'-continuous.
Proof:
i. Let A open set in X , sincef^{-1} is an coc'-continuous then $(f^{-1})^{-1}(A)$ is an coc-open set in Y,since f is bijective$(f^{-1})^{-1}(A) = f(A)$ then $f(A)$ is an coc-open set in Y. Hence f is an coc'-open function.
Conversely, let f coc'-open function, U be an open subset of X ,then $f(U)$ coc-open set in Y, $(f^{-1})^{-1}(A) = f(A)$ coc-open set in Y.Then f^{-1} is an coc'-continuous function. In similar (ii).

Theorem(2.21):

Let $f: X \to Y$ be a function for which (X, τ) is CC then the following statements are equivalent:
i. f is open.
ii. f is coc-open.
iii. f is coc'-open.
Proof: clear

Theorem(2.22):

Let $f: X \to Y$ be a function for which (X, τ) is CC then the following statements are equivalent:
i. f is closed.
ii. f is coc-closed.
iii. f is coc'-closed.
Proof: clear

Definition(3.23):

Let X and Y are spaces then a function $f: X \to Y$ is called an coc'-homeomorphism if:
i. f is bijective.
ii. $f$is an coc'-continuous.
iii. f is an coc'-closed (coc'-open) .
It is clear that every coc'-homeomorphism is an coc-homeomorphism.

Proposition(3.24):

Let $f: (X, \tau) \to (Y, \tau')$ be bijective function then the following statements are equivalent:
i. f is coc'-homeomorphism.
ii. f is coc'-continuous and coc'-open.
iii. f is coc'-continuous and coc'-closed.
iv. $f\left(\overline{A}^{coc}\right) = \overline{f(A)}^{coc}$ $\forall A \subseteq X$

Proof:
(i)→(ii) obvious
(ii)→(iii) Let f iscoc'-continuous and coc'-open, then f is coc'-closed by Proposition(3.19).
(iii)→(iv) obvious
(iv)→(i) Since $f\left(\overline{A}^{coc}\right) \subseteq \overline{f(A)}^{coc}$ Then f is coc'-continuous and since $\overline{f(A)}^{coc} \subseteq f(\overline{A}^{coc})$ then f is coc'-closed and since f be bijective then f is coc'-homeomorphism.

4. Coc-compact space

We recall the concept of coc-compact space and give some important generalization on this concept.

Definition (4.1):

Let Xbe a space. A family F of subset of X is called an coc-open cover of X if F covers X and F is sub family of τ^k.

Definition (4.2):

A space X is said to be coc-compact if every coc-open cover of X has finite sub cover.

Example (4.3):

i. Every finite subset of a space X is an coc-compact.
ii. The indiscrete space is an coc-compact space.

Remark (4.4):

It is clear that every coc-compact space is compact but the converse is not true in general as the following example shows:

Example (4.5):

Let $X = \mathbb{R}$ the set of real numbers with τ is indiscrete topology, the coc-open set is $\{A: A \subseteq X\}$. Then X is compact space but not coc-copmpact.
It is clear that , a space (X, τ) is coc-compact iff the space (X, τ^k) is compact.

Proposition (4.6):

Every coc-compact subset of an coc-Hausdorff space is an coc-closed.
Proof:
Let Y be an coc-compact subset of the coc-Hausdorff space$X$. To prove $\overline{Y}^{coc} \subseteq Y$. Let x_o be a point such that $x_o \notin Y$. We show that there is an coc-open set contains x_o and disjoint from Y. For each $y \in Y$ choose disjoint an coc-open set U_{x_o} and V_y contains x_o and y (respectively) using an coc-Hausdorff condition. The collection $\{V_y : y \in Y\}$ is cover of Y by coc-open sets in X, hence there exists finitely many of them $V_{y_1}, \dots, V_{y_n}$ cover of Y. The coc-open set $V = \bigcup_{i=1}^{n} V_{y_i}$ contains Y and disjoint from coc-open set $U = \bigcap_{i=1}^{n} U_{x_{oi}}$ from by taking the intersection of coc-open sets contains x_o. Since if z is point of V then $z \in V_{y_i}$ for some i hence $z \notin U_{x_{oi}}$ and $z \notin U$. U is an coc-open set contain x_o disjoint from Y as desired.

Theorem (4.7):

In any space X the intersection of an coc-closed set with a coc-compact set is an coc-compact.
Proof:
Let A be an coc-closed set of X and let B be an coc-compact subset of X. Thus A is closed in (X, τ^k) , then $A \cap B$ is compact set in(X, τ^k) hence $A \cap B$ is compact set in X.

Theorem (4.8):

Let $f : X \rightarrow Y$ be an onto coc-continuous function. If X is coc-compact then Y is compact.
Proof:
Let $\{V_\alpha : \alpha \in \Lambda\}$ be an open cover of Y then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is an coc-open cover of X, since X is coc-compact .Then X has finite sub cover say $\{f^{-1}(V_{\alpha i}) : i = 1,2, \dots, n\}$ and $V_{\alpha i} \in \{V_\alpha : \alpha \in \Lambda\}$. Hence $\{V_{\alpha i} : i = 1,2, \dots, n\}$ is a finite sub cover of Y. Then Y is compact.

Theorem (4.9):

Let $f : X \rightarrow Y$ be an onto coc'-continuous function. If X is coc-compact then Y is coc-compact.
Proof:
Let $\{V_\alpha : \alpha \in \Lambda\}$ be an coc-open cover of Y then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is an coc-open cover of X, since X is coc-compact .Then X has finite sub cover say $\{f^{-1}(V_{\alpha i}) : i = 1,2, \dots, n\}$ and $V_{\alpha i} \in \{V_\alpha : \alpha \in \Lambda\}$. Hence $\{V_{\alpha i} : i = 1,2, \dots, n\}$ is a finite sub cover of Y. Then Y is coc-compact.

Proposition (4.10):

For any space X the following statement are equivalent:
i. X is coc-compact
ii. Every family of coc-closed sets $\{F_\alpha : \alpha \in \Lambda\}$ of X such that $\bigcap_{\alpha \in \Lambda} F_\alpha = \phi$, then there exist a finite subset $\Lambda_o \subseteq \Lambda$ such that $\bigcap_{\alpha \in \Lambda} F_\alpha = \phi$.
Proof:
(i)→(ii) Assume that X is coc-compact, let $\{F_\alpha : \alpha \in \Lambda\}$ be a family of coc-closed subset of X such that $\bigcap_{\alpha \in \Lambda} F_\alpha = \phi$. Then the family $\{X - F_\alpha : \alpha \in \Lambda\}$ is coc-open cover of the coc-compact (X, τ) there exist a finite subset Λ_o of Λ such that $X = \cup \{X - F_\alpha : \alpha \in \Lambda_o\}$ therefore $\phi = X - \cup \{X - F_\alpha : \alpha \in \Lambda_o\} = \bigcap \{X - (X - F_\alpha) : \alpha \in \Lambda_o\} = \cap \{X - F_\alpha : \alpha \in \Lambda_o\}$

(ii)→(i) Let $U = \{U_\alpha : \alpha \in \Lambda\}$ be an coc-open cover of the space (X, τ). Then $X - \{U_\alpha : \alpha \in \Lambda\}$ is a family of coc-closed subset of (X, τ) with $\cap \{X - U_\alpha : \alpha \in \Lambda\} = \phi$ by assumption there exists a finite subset Λ_o of Λ such that$\cap \{X - U_\alpha : \alpha \in \Lambda_o\} = \phi$ so $X = X - \cap \{X - U_\alpha : \alpha \in \Lambda_o\} = \cup \{U_\alpha : \alpha \in \Lambda_o\}$. Hence X is coc-compact.

Definition (4.11):

A subset B of a space X is said to be coc-compact relative to X if for every cover of B by coc-open sets of X has finite sub cover of B. The sub set B is coc-compact iff it is coc-compact as a sub space.

Proposition (4.12):

If X is a space such that every coc-open subset of X is coc-compact relative to X, then every subset is coc-compact relative to X.

Proof:

Let B be an arbitrary subset of X and let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of B by coc-open sets of X. Then the family $\{U_\alpha : \alpha \in \Lambda\}$ is a coc-open cover of the coc-open set $\cup \{U_\alpha : \alpha \in \Lambda\}$ Hence by hypothesis there is a finite subfamily $\{U_{\alpha i} : i = 1,2, \dots, n\}$ which covers $\cup \{U_\alpha : \alpha \in \Lambda\}$. .This the subfamily is also a cover of the set B.

Theorem (4.13):

Every coc-closed subset of coc-compact space is coc-compact relative.

Proof:

Let A be an coc-closed subset of X. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of A by coc-open subset of X. Now for each $x \in X - A$,there is a coc-open set V_x such that $V_x \cap A$ is a finite .Since X is coc-compact and the collection $\{U_\alpha : \alpha \in \Lambda\} \cup \{V_x : x \in X - A\}$ is a coc-open cover of X , there exists a finite sub cover $\{U_{\alpha i} : i = 1, \dots, n\} \cup \{V_{xi} : i = 1, \dots, n\}$ .Since $\bigcup_{i=1}^{n} (V_{xi} \cap A)$ is finite, so for each $x_j \in (V_{xi} \cap A)$, there is $U_{\alpha(x_j)} \in \{U_\alpha : \alpha \in \Lambda\}$ such that$x_j \in U_{\alpha(x_j)}$ and $j = 1, \dots, n$ .Hence $\{U_{\alpha i} : i = 1, \dots, n\} \cup \{U_{\alpha(x_j)} : j = 1, \dots, n\}$ is a finite sub cover of $\{U_\alpha : \alpha \in \Lambda\}$ and it covers A .Therefore , A is coc-compact relative to X.

Definition(4.14):

i. A space X is called CC' if every coc-compact set in X is coc-closed.
ii. A space X is called CC'' if every coc-compact set in X is closed.
iii. A space X is called CC''' if every compact set in X is coc-closed.

Theorem(4.15)[3]:

For any space (X, τ), then (X, τ^k) is CC.

Remark(4.16):

It is clear that every CC space is CC' space but the converse is not true in general as the following example shows:

Example(4.17):

Let $X = \{1,2,3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$, the coc-open sets are discrete. $\{1\}$ is compact set but not closed then X is not CC space.

Remark(4.18):

i. every CC space is CC'' space .
ii. every CC'' space is space CC' .

the converse of (ii) is not true in general as the following example shows:

Example(4.19):

Let $X = \{1,2,3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$, the coc-open sets all subsets of X. $\{1\}$ is coc-compact set but not closed then X is not CC'' space.

Remark(4.20):

i. every CC space is CC''' space .
ii. every CC''' space is space CC' .

the converse of (i) is not true in general as the following example shows:

Example(4.21):

Let $X = \{1,2,3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$, the coc-open sets are all subsets of X. $\{1\}$ is compact set but not closed then X is not CC space.

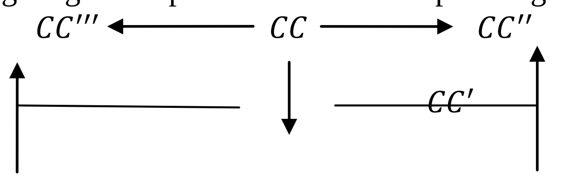
Remark(4.22):

If X is CC'' space , then it need not be CC''' space as the following example

Example(4.23):

Let $X = \{1,2,3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$, the coc-open sets are discrete. $\{1\}$ is coc-compact set but not closed then X is not CC space.

The following diagram explains the relationship among these types of CC spaces



Definition (4.24):

Let $f: X \to Y$ be a function of a space X into a space Y then f is called a coc-compact function if $f^{-1}(A)$ is compact set in X for every coc-compact set A in Y.

Remark (4.25):

Every coc-compact function is compact function.

Proposition (4.26):

Let X, Y and Z be spaces and $: X \to Y$, $g: Y \to Z$ be a continuous functions then:
i. If f is a compact function and g is an coc-compact function, then $g \circ f$ is an coc-compact function.
ii. If $g \circ f$ is coc-compact function f is onto, then g is coc-compact function.
iii. If $g \circ f$ is coc-compact function, g is coc'-continuous and bijective function then f is coc-compact function.
Proof:
i. Let K be a coc-compact in Z then $g^{-1}(K)$ is compact set in Y thus $f^{-1}(g^{-1}(K)) = (g \circ f)^{-1}(K)$ is compact set in X. Hence $g \circ f: X \to Z$ is an coc-compact function.
ii. Let K be a coc-compact in Z then $(g \circ f)^{-1}(K)$ is compact set in X and then $f((g \circ f)^{-1}(K))$ is compact set in Y. Now since f is onto, then $f((g \circ f)^{-1}(K)) = g^{-1}(K)$. Hence $g^{-1}(K)$ is compact set in Y then g is coc-compact function.
iii. Let K be a coc-compact in Y, then by Theorem (4.9) $g(K)$ is coc-compact set in Z thus$(g \circ f)^{-1}(g(K))$ is a compact set in X since g is one to one then $(g \circ f)^{-1}\big(g(K)\big) = f^{-1}(K)$. Hence $f^{-1}(K)$ is a compact set in X ,thus f is coc-compact function .

Definition (4.27):

Let $f: X \to Y$ be a function of a space X into a space Y then f is called a coc'-compact function if $f^{-1}(A)$ is coc-compact set in X for every coc-compact set A in Y.

Example (4.28):

Every function from a finite space into any space is coc'-compact function.

Remark (4.29):

Every coc'-compact function is coc-compact function.

Proposition (4.30):

Let X, Y and Z be spaces and $: X \to Y$, $g: Y \to Z$ be a continuous functions then:
i. If f and g are coc'-compact function, then $g \circ f$ is an coc'-compact function.
ii. If $g \circ f$ is coc'-compact function, g is coc'-continuous and bijective function then f is coc'-compact function.
Proof:
i. Let K be a coc-compact in Z then $g^{-1}(K)$ is coc-compact set in Y thus $f^{-1}(g^{-1}(K)) = (g \circ f)^{-1}(K)$ is coc-compact set in X. Hence $g \circ f: X \to Z$ is an coc'-compact function.
ii. Let K be a coc-compact in Y, then by Theorem (4.9) $g(K)$ is coc-compact set in Z thus$(g \circ f)^{-1}(g(K))$ is a coc-compact set in X since g is one to one then $(g \circ f)^{-1}\big(g(K)\big) = f^{-1}(K)$. Hence $f^{-1}(K)$ is a coc-compact set in X ,thus f is coc'-compact function .

Definition(4.31):

Let X be space. A subset W of X is called compactly coc-closed if for every coc-compact set K in X, $W \cap K$ is coc-compact.

Example (4.32):

i. Every finite subset of a space X is compactly coc-closed .
ii. Every subset of indiscrete space is compactly coc-closed.

Proposition(4.33):

Every coc-closed subset of a space X is compactly coc-closed set.

Poof:

Let A be an coc-closed subset of a space X and let K be an coc-compact set in X.Then $A \cap K$ is an coc-compact, thus A is compactly coc-closed set.

Proposition(4.34):

Let $f: X \to Y$ be an coc'-continuous,coc'-compact, bijective function, then A is compactly coc-closed set in X if and only if $f(A)$ is compactly coc-closed set in Y.

Proof:

Let A be compactly coc-closed set in X and let K be an coc-compact in . since $f$is acoc'-compact function, then $f^{-1}(K)$ is coc-compact in X, so $A \cap f^{-1}(K)$ is an coc-compact set. Then $(A \cap f^{-1}(K))$. But $f\left(A \cap f^{-1}(K)\right) = f(A) \cap K$. Then $f(A) \cap K$ an coc-compact set . Hence $f(A)$ is compactly coc-closed set.

Conversely, Let $f(A)$ be compactly coc-closed set in Y and let K be an coc-compact in . since $f$is acoc'-continuous function, then $f(K)$ is coc-compact in Y, so $f(A) \cap f(K)$ an coc-compact set, since $f$is a coc'-compact function then $f^{-1}\left(f(A) \cap f(K)\right) = f^{-1}(f(A)) \cap f^{-1}(f(K))$ is coc-compact set in X , since f is one to one function then $A = f^{-1}(f(A))$ and $K = f^{-1}(f(K))$, then $A \cap K = f^{-1}(f(A)) \cap f^{-1}(f(K))$. Hence $f(A) \cap K$ an coc-compact set . Therefore $f(A)$ is compactly coc-closed set in X.

Definition(4.35):

Let X be space. Then a subset A of X is called compactly coc- K-closed if for every coc-compact set K in X, $A \cap K$ iscoc-closed .

Example(4.36):

Every subset of a discrete space is compactly coc- K-closed set.

Proposition(4.37):

Every compactly coc- K-closed subset of a space X is compactly coc-closed

Proof:

Let A be compactly coc- K-closed subset of X and let K be an coc-compact in X, then $A \cap K$ is coc-closed set since $A \cap K \subseteq K$ and K is coc-compact set , then $A \cap K$ is coc-compact set. Therefore A is compactly coc-closed.

Theorem (4.38):

Let X be cocT_2-space and A is subset of X. Then A is compactly coc-closed iff A is compactly coc- K-closed set .

Proof:

Let A be compactly coc-closed subset of X and let K be an coc-compact in X, then $A \cap K$ is coc-compact set, since X is τ_2-space, then by proposition (4.6) $A \cap K$ is coc-closed. Hence A be compactly coc- K-closed set.

Conversely, by proposition(4.37)

Definition (4.39):

A space X is called coc- K-space if for every compactly coc-closed is coc-closed.

Proposition(4.40):

Let X space and Y is coc- K-space then every coc-compact, continuous onto function $f: X \to Y$ is an coc-closed function.

Proof :

Let F be a closed set in X. To prove $f(F)$ is an coc-closed set in Y. Let K is coc-compact in Y, since f coc-compact function then$f^{-1}(K)$ is compact set in X, $F \cap f^{-1}(K)$ is compact set and since f continuous function then $f(F \cap f^{-1}(K))$ is compact set in Y. But $f\left(F \cap f^{-1}(K)\right) = f(F) \cap K$, thus $f(F) \cap K$ is compact set in Y,since Y is coc- K-space. Then $f(F)$ is compactly coc-closed set in Y. Hence f is an coc-closed function.

Proposition(4.41):

Let X space and Y is coc- K-space then every coc'-compact, coc'-continuous onto function $f: X \to Y$ is an coc'-closed function.

Proof :

Let F be a coc-closed set in X. To prove $f(F)$ is an coc-closed set in Y. Let K is coc-compact in Y, since f coc'-compact function then $f^{-1}(K)$ is coc-compact set in X, by theorem (4.7) $F \cap f^{-1}(K)$ is coc-compact set and since f coc'-continuous function then $f(F \cap f^{-1}(K))$ is coc-compact set in Y. But $f\left(F \cap f^{-1}(K)\right) = f(F) \cap K$, thus $f(F) \cap K$ is coc-compact set in Y,since Y is coc- K-space. Then $f(F)$ is compactly coc-closed set in Y. Hence f is an coc'-closed function.

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الخلاصة

تناولنا خلال البحث الدوال المغلقة من النمط -coc و المفتوحة من النمط coc و المستمرة من النمط coc . سنبرهن بعض خصائص هذه الدوال و نبين العلاقة بينها. ظهرت خلال البحث مفاهيم مهمة وجديدة منها التي تم شرحها وتتضمن الفضاءات المتراص من النمط -coc - و فضاء K من النمط -coc والمجموعه مغلقة ضئيلة تراصياً من النمط coc بالإضافة إلى ذلك درسنا سلوك هذه الصفات تحت تأثير انواع معينه من الدوال.