# A new kind of Fuzzy Topological Vector Spaces


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**Abstract**. In this paper, we introduce a new kind of fuzzy topological vector spaces that is a locally affine fuzzy topological vector space, finological space, $S-space$. Finally, we evidence that $S-space$ has a base at zero.



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## 1. Introduction

Fuzzy topological vector space defined and studied by Katsaras [2, 3]. In [3] the author was give a characteristic to a base at zero for a fuzzy vector topology in fuzzy topological vector spaces. Moreover, he was introduced the concept of bounded sets in a fuzzy topological vector space.

In this paper, we introduce a new kind of fuzzy topological vector spaces that is a locally affine fuzzy topological vector space by dependence on a new concept that is affine fuzzy sets. After that we introduce a finological space by dependence on a concept of bounded sets and we study a special kind of finological spaces that is $S-$space and we evidence that $S-$space has a base at zero.

## 2. Preliminaries

Let $X$ be a non-empty set. A fuzzy set in $X$ is the element of the set $I^X$ of all functions from $X$ into the unit interval $I$ =[0,1]. If $C_\alpha : X \to I$ is a function defined by $C_\alpha(x) = \alpha$ for all $x \in X$ , $\alpha \in I$ , then $C_\alpha$ is called the constant fuzzy set.

Let $X$ be a vector space over a field $F$, where $F$ is the space of either the real or the complex numbers. If $A_1, A_2$, …, $A_n$ are fuzzy sets in $X$, then the sum $A_1 + A_2 + \cdots + A_n$ (see [2] ) is the fuzzy set $A$ in $X$ defined by $A(x) = \sup_{x_1+x_2+\cdots+x_n=x} \min\{A_1(x_1), A_2(x_2),..., A_n(x_n)\}$. Also, if $A$ is a fuzzy set in $X$ and $\alpha \in F$, then $\alpha A$ is a fuzzy set defined by $\alpha A(x) = \begin{cases} A(x/\alpha) & if\ \alpha \neq 0\ forall\, x \in X \\ 0 & if\ \alpha = 0, x \neq 0 \\ \sup_{y \in X} A(y) & if\ \alpha = 0, x = 0 \end{cases}$.

If $A$ is a fuzzy set in $X$ and $x \in X$, then $x + A$ is a fuzzy set in $X$ and defined by $(x + A)(y) = A(y - x)$. A fuzzy set $A$ in $X$ is called a balanced fuzzy set if $\alpha A \subset A$ or $A(\alpha x) \geq A(x)$ for all $\alpha$ with $|\alpha| \leq 1$. For the definition of a fuzzy topology, we will use the one given by Lowen [1] that is a fuzzy topology on a set $X$ we will mean a subset $\gamma$ of $I^X$ satisfying the following conditions :

(i) $\gamma$ contains every constant fuzzy set in $X$;

(ii) If $A_1, A_2 \in \gamma$, then $A_1 \cap A_2 \in \gamma$;

(iii) If $A_i \in \gamma$ for all $i \in \Lambda$ ( $\Lambda$ any index), then $\bigcup_{i \in \Lambda} A_i \in \gamma$.

The pair $(X, \gamma)$ is called a fuzzy topological space . If $A \in \gamma$, then $A$ is called an open fuzzy set and $A$ is called a neighborhood of $x \in X$ if there exists an open fuzzy set $B$ with $B \subseteq A$ and $B(x) = A(x) > 0$. Moreover, $A$ is open fuzzy set in a fuzzy topological space $X$ if and only if $A$ is a neighborhood of $x$ for each $x \in X$ with $A(x) > 0$. A fuzzy vector topology on a vector space $X$ over $F$ ( see [3] ) is a fuzzy topology $\gamma$ on $X$ such that the two functions

$\mu : (X, \gamma) \times (X, \gamma) \to (X, \gamma)$, such that $\mu(x, y) = x + y$, for all $x, y \in X$;

$\upsilon : (F, \Im_U) \times (X, \gamma) \to (X, \gamma)$, such that $\upsilon(\alpha, x) = \alpha . x$, for $\alpha \in F, x \in X$, are fuzzy continuous when $F$ is equipped with the usual fuzzy topology, ( $\Im_U$ is a fuzzy topology generated by the usual topology $U$ on $F$ ) and $F \times X, X \times X$ have the corresponding product fuzzy topologies. A vector space $X$ equipped with a fuzzy vector topology $\gamma$ is called a fuzzy topological vector space.

## 3. Main Results

**Theorem 3.1. [3]**

Let $\Phi$ be a family of balanced fuzzy sets in a vector space $X$ over $F$. Then $\Phi$ is a base at zero for a fuzzy vector topology if, and only if $\Phi$ satisfies the following conditions :

(1) $A(0) > 0$, for each $A \in \Phi$;

(2) for each non-zero constant fuzzy set $C_\beta$ and any $\alpha \in (0, \beta)$ there exists $A \in \Phi$ with $A \subseteq C_\beta$ and $A(0) > \alpha$;

(3) If $A, B \in \Phi$ and $\alpha \in (0, \min\{A(0), B(0)\})$, then there exists $D \in \Phi$ with $D \subseteq A \cap B$ and $D(0) > \alpha$;

(4) If $A \in \Phi$ and $t$ a non-zero scalar, then for each $\alpha \in (0, A(0))$ there exists $B \in \Phi$ with $B \subseteq tA$ and $B(0) > \alpha$;

(5) Let $A \in \Phi$ and $\alpha \in (0, A(0))$. Then, there exists $B \in \Phi$ such that $B(0) > \alpha$ and $B + B \subseteq A$;

(6) Let $A \in \Phi$ and $x_\circ \in X$. If $\alpha \in (0, A(0))$, then there exists a positive number $s$ such that $A(tx_\circ) > \alpha$, for all scalar $t \in R$ with $|t| \leq s$;

(7) For each $A \in \Phi$ there exists a fuzzy set $B$ in $X$ with $B \subseteq A$, $B(0) = A(0)$ such that for each $x \in X$ with $B(x) > 0$ and if $n \in (0, B(x))$, there exists $D \in \Phi$ with $D \subseteq -x + B$ and $D(0) > n$.

**Definition 3.2.**

Let $X$ be a vector space over $F$. A fuzzy set $A$ in $X$ is called an affine fuzzy set if $\lambda A + (1-\lambda)A \subset A$ for each $\lambda \in F$.

**Theorem 3.3.**

Let $A$ be a fuzzy set in a vector space $X$ over $F$ and $\lambda \in F$, then the following statement are equivalent.

(1) $A$ is an affine fuzzy set.

(2) for all $x, y$ in $X$, we have $A(\lambda x + (1-\lambda)y) \geq \min\{A(x), A(y)\}$.

**Proof :**

$(1) \Rightarrow (2)$

Suppose that $A$ is an affine fuzzy set, by (Definition 3.2) we have $\lambda A + (1-\lambda)A \subset A$ for each $\lambda \in F$. Now, for each $x, y \in X$

$$A(\lambda x + (1-\lambda)y) \geq (\lambda A + (1-\lambda)A)(\lambda x + (1-\lambda)y)$$

$$= \sup_{\lambda x + (1-\lambda)y = x_1 + y_1} \min\{(\lambda A)(x_1), ((1-\lambda)A)(y_1)\}$$

$$\geq \min\{(\lambda A)(\lambda x), ((1-\lambda)A)((1-\lambda)y)\} \geq \min\{A(x), A(y)\}.$$

$(2) \Rightarrow (1)$

Let $x \in X$, $(\lambda A + (1-\lambda)A)(x) = \sup_{x_1 + x_2 = x} \min\{(\lambda A)(x_1), ((1-\lambda)A)(x_2)\}$

(a) If $\lambda \neq 0$ and $1-\lambda \neq 0$, then

$$(\lambda A)(x_1) = A(\frac{1}{\lambda}x_1) \text{ and } ((1-\lambda)A)(x_2) = A(\frac{1}{(1-\lambda)}x_2)$$

Thus, $(\lambda A + (1-\lambda)A)(x) = \sup_{x_1 + x_2 = x} \min\{A(\frac{1}{\lambda}x_1), A(\frac{1}{(1-\lambda)}x_2)\}$

But, $\min\{A(\frac{1}{\lambda}x_1), A(\frac{1}{(1-\lambda)}x_2)\} \leq A(\lambda(\frac{1}{\lambda}x_1) + (1-\lambda)(\frac{1}{(1-\lambda)}x_2)) = A(x_1 + x_2) = A(x)$.

(b) If $\lambda \neq 0$ and $1-\lambda = 0$, then $(\lambda A)(x_1) = A(x_1)$ and

$$((1-\lambda)A)(x_2) = \begin{cases} 0 & , x_2 \neq 0 \\ \sup_{z \in X} A(z) & , x_2 = 0 \end{cases}$$

($i$) If $x_2 \neq 0$, then $((1-\lambda)A)(x_2) = 0$ and

$$(\lambda A + (1-\lambda)A)(x) = \sup_{x_1 + x_2 = x} \min\{A(x_1), 0\} = 0$$

Since $((1-\lambda)A)(x_2) \geq A(x_2)$, then $A(x_2) = 0$ and we get $\min\{A(x_1), A(x_2)\} = 0$.

($ii$) If $x_2 = 0$, then $((1-\lambda)A)(x_2) = \sup_{z \in X} A(z)$ and

$$(\lambda A + (1-\lambda)A)(x) = \sup_{x_1 = x} \min\{\lambda A(x_1), \sup_{z \in X} A(z)\} \leq \sup_{x_1 = x} A(x_1) = A(x).$$

(c) If $\lambda = 0$ and $1-\lambda \neq 0$, by the same way in (b).

(d) if $\lambda = 0$ and $1-\lambda = 0$, its impossible.

**Definition 3.4.**

Let $X$ be a vector space over $F$. A fuzzy set $A$ in $X$ is called $S-fuzzy\,set$ if $A$ is balanced and affine fuzzy set.

**Theorem 3.5.**

Let $X$ be a vector space over $F$ and $x_\circ \in X$, then the characteristic function of $\{x_\circ\}$ ( in symbol $\chi_{\{x_\circ\}}$) is $S-fuzzy\,set$.

**Proof :**

(1) To prove $\chi_{\{x_\circ\}}$ is an affine fuzzy set. Let $x, y \in X$, $\lambda \in F$ and suppose that

$\chi_{\{x_\circ\}}(\lambda x + (1-\lambda)y) < \min\{\chi_{\{x_\circ\}}(x), \chi_{\{x_\circ\}}(y)\}$

Then $\chi_{\{x_\circ\}}(\lambda x + (1-\lambda)y) = 0$ and $\min\{\chi_{\{x_\circ\}}(x), \chi_{\{x_\circ\}}(y)\} = 1$. Thus, $\lambda x + (1-\lambda)y \neq x_\circ$ and $x = y = x_\circ$ implies that $x_\circ \neq x_\circ$ which is impossible. Hence,

$\chi_{\{x_\circ\}}(\lambda x + (1-\lambda)y) \geq \min\{\chi_{\{x_\circ\}}(x), \chi_{\{x_\circ\}}(y)\}$. From ( Theorem 3.3), we get the result.

(2) To prove $\chi_{\{x_\circ\}}$ is a balanced fuzzy set. Let $\lambda \in F$ such that $|\lambda| \leq 1$, then from (1) we get for $x \in X$, $\chi_{\{x_\circ\}}(\lambda x) = \chi_{\{x_\circ\}}(\lambda x + (1-\lambda)0) \geq \min\{\chi_{\{x_\circ\}}(x), \chi_{\{x_\circ\}}(0)\}$. Now, the proof is completely if $\chi_{\{x_\circ\}}(x) < \chi_{\{x_\circ\}}(0)$. Suppose that $\chi_{\{x_\circ\}}(x) > \chi_{\{x_\circ\}}(0)$, then $\chi_{\{x_\circ\}}(x) = 1$ and $\chi_{\{x_\circ\}}(0) = 0$. That is mean $x = x_\circ$ for all $x \in X$ and $x_\circ \neq 0$. By other word $0 \notin X$ which contradiction .

**Theorem 3.5.**

Let $A$ be $S-fuzzy\,set$. Define $B: X \to I$, $B(x) = \sup\{A(tx) : t > 1\}$, then $B$ is $S-fuzzy\,set$. Moreover, $B \subseteq A$ and $B(0) = A(0)$.

**Proof :**

(1) To prove $B$ is a balanced fuzzy set. Let $\lambda \in F$ such that $|\lambda| \leq 1$, $x \in X$.

$B(\lambda x) = \sup\{A(\lambda tx) : t > 1\} \geq \sup\{A(tx) : t > 1) = B(x)$.

(2) To prove $B$ is an affine fuzzy set. Let $\lambda \in F$ and for any $x, y \in X$, we have

$$B(\lambda x + (1-\lambda)y) = \sup\{A(t(\lambda x + (1-\lambda)y)) : t > 1\} = \sup\{A(\lambda tx + (1-\lambda)ty) : t > 1\}$$

$$\geq \sup\{\min\{A(tx), A(ty)\} : t > 1\}$$

$\geq \min\{\sup A(tx), \sup A(ty)\}$ for $t > 1$.

$= \min\{B(x), B(y)\}$. Then $B$ is $S-$ *fuzzy set*.

(3) To prove $B \subseteq A$ and $B(0) = A(0)$. Let $x \in X$, since $A$ is a balanced fuzzy set, we get $B(x) = \sup_{t>1} A(tx) = \sup_{t>1} \frac{1}{t} A(x) \leq \sup_{t>1} A(x) = A(x)$. That is mean $B \subseteq A$.

$B(0) = \sup\{A(t0) : t > 1\} = A(0)$.

**Definition 3.6. [3]**

A fuzzy set $A$ in a vector space $X$, absorbs a fuzzy set $B$ if $A(0) > 0$ and for every $\theta < A(0)$, there exists $t > 0$ such that $C_\theta \cap tB \subseteq A$

**Definition 3.7. [3]**

A fuzzy set $A$ in a fuzzy topological vector space $X$ is called bounded if it is absorbs by every neighborhood of zero.

**Definition 3.8.**

A fuzzy topological vector space $X$ is called locally affine if it has a base at zero consisting of affine fuzzy sets.

**Theorem 3.9.[3]**

Let $A$ be a neighborhood of zero in a fuzzy topological vector space $X$. Then, there exist an open neighborhood $B_1$ of zero and a balanced neighborhood $B_2$ of zero such that $B_1(0) = B_2(0) = A(0)$, $B_1 \subseteq B_2 \subseteq A$ and $tB_1 \subseteq B_1$ for each scalar $t$ with $|t| \leq 1$.

**Remark.**

If a neighborhood $A$ of zero is an affine, then we called an affine neighborhood of zero. Likewise is called $S$-neighborhood if it is balanced and affine fuzzy set.

**Theorem 3.10.**

Let $A$ be an affine neighborhood of zero in a fuzzy topological vector space $X$. Then there exist $S$-neighborhood $B$ of zero with $B \subseteq A$ and $B(0) = A(0)$.

**Proof :**

By (Theorem 3.9.), there exists an open neighborhood $D_1$ of zero and a balanced neighborhood $D_2$ of zero with $D_1 \subseteq D_2 \subseteq A$ and $D_1(0) = D_2(0) = A(0)$. Let $B = span(D_2)$, then $B$ is $S - fuzzy\,set$ and $D_2 \subseteq B \subseteq A$. To get the result only prove that $B$ is a neighborhood of zero and this fact is true, since $D_1 \subseteq B$ and $D_1(0) = B(0) > 0$.

**Theorem 3.11.**

If $X$ is a fuzzy topological vector space, $x_\circ \in X$ then $\chi_{\{x_\circ\}}$ is bounded set.

**Proof:**

Let $x \in X$ and $A$ is a neighborhood of zero, then $A(0) > 0$. Let $\theta \in (0, A(0))$. By (6) of theorem 3.1., there exists a positive number $\delta$ such that $A(tx) > \theta$ for all scalars $t$ with $|t| \le \delta$. Take $t = \delta$, we see that $\min\{\theta, \chi_{\{x_\circ\}}(x)\} \le \theta < A(\delta x) = \frac{1}{\delta} A(x)$. Thus, $C_\theta \cap \delta\chi_{\{x_\circ\}} \subset A$. This complete the proof.

**Definition 3.12.**

A locally affine fuzzy topological vector space $X$ is called finological if every $S - fuzzy\ set$ in $X$ which absorbs bounded sets is a neighborhood of zero.

The next definition is related to a special kind of finological spaces that is the space which contains all $S - fuzzy\ sets$ which absorbs the characteristic function for $\{0\}$ ( in symbol $\chi_{\{0\}}$ or $\chi_\circ$ for short).

**Definition 3.13.**

A locally affine fuzzy topological vector space $X$ is called $S - space$ if every $S - fuzzy\ set$ in $X$ which absorbs $\chi_\circ$ is a neighborhood of zero.

**Theorem 3.14.**

Any $S-space$ has a base at zero.

**Proof :**

Let $(X,\gamma)$ be $S-space$ and let

$IB=\{\, A\in I^{X} : A \ is \ S-fuzzy \ set \ which \ absorbs \ \chi_{\circ} \,\}$

<u>To prove</u> $IB$ is a base at zero for $S-space \ \ X$. By another word it suffices to show that $IB$ satisfies the conditions (1)-(7) of (Theorem 3.1).

(1)

Let $A\in IB$ and let $B$ be a bounded set. If $A(0)=0$, then there is no $\theta<A(0)$ such that $C_{\theta}\cap tB\subseteq A$ for all $t>0$. That is mean $A$ is not absorbs $B$ but this impossible with assume. Thus, $A(0)>0$ for each $A\in IB$.

(2)

Let $0<\beta\le 1$ and suppose that $\alpha\in(0,\beta)$. Choose $\alpha_1\in(\alpha,\beta)$. Now, $C_{\alpha_1}$ is $S-fuzzy \ set$ and $C_{\alpha_1}\cap\chi_{\circ}\subseteq C_{\alpha_1}\subseteq C_{\beta}$. Thus, that $C_{\alpha_1}\in IB$ and $C_{\alpha_1}(0)=\alpha_1>\alpha$.

(3)

Let $A,B\in IB$ and $0<\alpha<\min\{A(0),B(0)\}$. Let $\alpha_1\in(\alpha,\min\{A(0),B(0)\})$, then $\alpha_1<A(0)$ and $\alpha_1<B(0)$ subsequently, there are $t_1,t_2>0$ such that $C_{\alpha_1}\cap t_1\chi_{\circ}\subseteq A$ and $C_{\alpha_1}\cap t_2\chi_{\circ}\subseteq B$. Choose $0<t<t_1,t_2$. Then $\left|t/t_1\right|<1$ and $\left|t/t_2\right|<1$. Since $\chi_{\{x_{\circ}\}}$ is a balanced fuzzy set then $t\chi_{\circ}\subseteq t_1\chi_{\circ}$ and $t\chi_{\circ}\subseteq t_2\chi_{\circ}$. Now, let $D=C_{\alpha_1}\cap t\chi_{\circ}$, we see that $D=C_{\alpha_1}\cap t\chi_{\circ}\subseteq(C_{\alpha_1}\cap t_1\chi_{\circ})\cap(C_{\alpha_1}\cap t_2\chi_{\circ})\subseteq A\cap B$. Likewise

$D(0)=(C_{\alpha_1}\cap t\chi_{\circ})(0)=\alpha_1>\alpha$.

(4)

Let $A\in IB$ and $0\ne t\in R$ . Suppose that $0<\alpha<A(0)$. Choose $\alpha_1\in(\alpha,A(0))$. Since $A\in IB$, then there exist $t_1>0$ such that $C_{\alpha_1}\cap t_1\chi_{\circ}\subseteq A$. That is implies $t(C_{\alpha_1}\cap t_1\chi_{\circ})\subseteq tA$. In other words $(C_{\alpha_1}\cap t_1\chi_{\circ})(x/t)\le(tA)(x)$ for all $x\in X$. Let $B=C_{\alpha_1}\cap t.t_1\chi_{\circ}$. Now, $B(x)=(C_{\alpha_1}\cap t.t_1\chi_{\circ})(x)=(C_{\alpha_1}\cap t_1\chi_{\circ})(x/t)\le tA(x)$. Then $B\subseteq tA$ and $B(0)=\alpha_1>\alpha$.

(5)

Suppose that $A \in IB$ and $0 < \theta < A(0)$. Choose $\theta_1 \in (\theta, A(0)$. Since $A \in IB$, then there exist $t > 0$ such that $C_{\theta_1} \cap t\chi_{\circ} \subseteq A$. Let $s = \frac{t}{2}$, $B = C_{\theta_1} \cap s\chi_{\circ}$. Now, to prove that $B$ which was to be demonstrated.

$$
\begin{aligned}
(B+B)(x) &= \sup_{x_1+x_2=x} \min\{B(x_1), B(x_2)\} \\
&= \sup_{x_1+x_2=x} \min\{(C_{\theta_1} \cap s\chi_{\circ})(x_1), (C_{\theta_1} \cap s\chi_{\circ})(x_2)\} \\
&= \sup_{y\in X} \min\{C_{\theta_1}, \min\{s\chi_{\circ}(y), s\chi_{\circ}(x-y)\}\} \\
&= \sup_{y\in X} \min\{C_{\theta_1}, \min\{\frac{t}{2}\chi_{\circ}(y), \frac{t}{2}\chi_{\circ}(x-y)\}\} \\
&= \sup_{y\in X} \min\{C_{\theta_1}, \min\{\chi_{\circ}(\frac{2y}{t}), \chi_{\circ}(\frac{2x-2y}{t})\}\} \\
&\le \sup_{y\in X} \min\{C_{\theta_1}, \chi_{\circ}((\frac{1}{2})(\frac{2y}{t}) + (\frac{1}{2})(\frac{2x-2y}{t}))\}, \ \chi_{\circ} \text{ is an affine fuzzy set.} \\
&= \sup_{y\in X} \min\{C_{\theta_1}, t\chi_{\circ}(y+x-y))\} = \min\{C_{\theta_1}, t\chi_{\circ}(x)\} \le A(x).
\end{aligned}
$$

Also, $B(0) = C_{\theta_1} \cap t\chi_{\circ}(0) = \min\{\theta_1, 1\} = \theta_1 > \theta$.

(6)

Let $x_{\circ} \in X$, $A \in IB$ and $0 < \theta < A(0)$. Choose $\theta_1 \in (\theta, A(0))$. Since the fuzzy set $\chi_{\{x_{\circ}\}}$ is bounded, then there is $t > 0$ such that $C_{\theta_1} \cap t\chi_{\{x_{\circ}\}} \subseteq A$. Now, if $|s| \le t$, then $A(sx_{\circ}) \ge A(tx_{\circ}) \ge \min\{\theta_1, \chi_{\{x_{\circ}\}}(x_{\circ}) = 1\} = \theta_1 > \theta$.

(7)

Let $A \in IB$. Define $B : X \to I$, $B(x) = \sup\{A(tx) : t > 1\}$. Form (Theorem 3.5) $B$ is $S-$*fuzzy set*. Moreover, $B \subseteq A$ and $B(0) = A(0)$. Also, $B$ absorbs $\chi_{\circ}$. In fact, let $0 < \theta < B(0)$. Since $A$ absorbs $\chi_{\circ}$, then there exists $t > 0$ such that $C_{\theta} \cap t\chi_{\circ} \subseteq A$. Now, $(C_{\theta} \cap t\chi_{\circ})(2x) \le A(2x) \le B(x)$. And so $C_{\theta} \cap (\frac{1}{2}t\chi_{\circ}) \subseteq B$ which proves that $B$ absorbs $\chi_{\circ}$. By other word $B \in IB$. If $0 < \theta < B(x)$, let $\theta_1 \in (\theta, B(x))$, there exists $t > 1$ such that $A(tx) > \theta_1$. Let $1 < s < t$. Then $B(sx) \ge A(tx) > \theta_1$ for $x \in X$.

Take $\eta = \frac{1}{s}$, $\mu = 1 - \eta$. Using the fact that $B$ is an affine fuzzy set and for $y \in X$, we get,

$(-x+B)(y)=B(x+y)=B(\eta(sx)+\mu(\mu^{-1}y))\geq \min\{B(sx),B(\mu^{-1}y)\}\geq \min\{\theta_1,(\mu B)(y)\}$ let $B_2=C_{\theta_1}\cap \mu B$. Now, $B_2\subseteq -x+B$ and $B_2(0)=\theta_1>\theta$. We just prove $B_2\in IB$. It is clear that $B_2$ is $S-$ *fuzzy set* and $C_\theta\cap \frac{1}{2}t\mu^{-1}\chi_\circ\subset C_{\theta_1}\cap \mu B=B_2$ which means $B_2$ absorbs $\chi_\circ$. Hereupon, Which complete the proof.

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