
KARBALA INTERNATIONAL
JOURNAL OF MODERN SCIENCE
Volume 2 Issue 2 June 2016
ISSN 2405-609X
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ScienceDirect
Karbala International Journal of Modern Science 2 (2016) 124–128
http://www.journals.elsevier.com/karbala-international-journal-of-modern-science/

Low-lying modes of trapped condensed atoms in anharmonic trap

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Received 28 January 2016; revised 7 March 2016; accepted 8 March 2016
Available online 14 April 2016

Abstract

We describe the physical properties of a Bose-Einstein condensate (BEc) confined by a three-dimensional harmonic plus cubic and quartic trap. To this end, we solve the time-dependent Gross–Pitaevskii (GP) equation within a variational approach in order to obtain a set of second ordinary differential equations for the condensate widths. We then discuss in detail the low-lying oscillation modes, in an anharmonic trap where the anharmonicity of the confining potential leads to significant effect on the collective excitations of the system.

Keywords: Bose–Einstein condensates; Gross–Pitaevskii equation; Anharmonic trap

1. Introduction

From a theoretical point of view [1–5], and for a wide range of experimentally relevant conditions [6–15], Bose-Einstein condensate is a typical topic of interest in the realm of ultra-low temperature. The dynamics of a BEC is well described by an effective mean-field theory. This approximation is simpler than dealing with the full many-body theory and describe quite accurately the static and dynamical properties of BECs. The relevant model is a classical nonlinear equation, the so-called GP equation [16–19]. Most of the BECs studies have been considered for a condensate trapped in a harmonic (parabolic) trap potential, for instance, Refs. [1,8,20–23]. Higher order for the trapping potential was neglected due to the large scale of generating magnetic field elements. The dynamics of a Bose gas in anharmonic trap are discussed explicitly within the exact solutions of the GP equation at zero temperature of Ref. [24]. The collective oscillations of one-dimensional BEc with repulsive two-body interaction in a harmonic trap with a quartic distortion were investigated in Refs. [25,26] by using variational approach. In particular, the dynamics of a BEc confined in anharmonic position jittering are considered in Ref. [27] to show how a small anharmonicity is effected on the periodic oscillation of the position of an anharmonic elongated trap potential. Ref. [28] was showed the effects of an anharmonic distortion on the collective oscillations of a dipolar Bose gas. The interacting two-boson system in one dimension, the ground state of the system as well as its quantum dynamics upon excitation under anharmonic trap was studied in Ref. [29]. Ref. [30] was studied numerically the Fermi-decay law for quantum fidelity in a anharmonic trapped BEC. In particular, the collective excitations of trapped anharmonically superfluid Fermi gas in the crossover from a Bardeen-Cooper-Schrieffer to a Bose-

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Peer review under responsibility of University of Kerbala.

http://dx.doi.org/10.1016/j.kijoms.2016.03.002

Einstein condensate (BCS-BEC) was considered in Ref. [31]. The stability and collective excitation of BECs with system of two- and three-body interactions in a two-dimensional anharmonic trap was investigated in Ref. [32]. According to the anharmonicities which occurs in a trap, the critical particle number of atoms in case of either repulsive or attractive two-body interaction strength was discussed in Refs. [33,34]. Motivated by this, the low-lying collective modes at zero temperature are described by the nonlinear GP equation for the condensate wavefunction. However, we discuss the significant affected how a deviation of the harmonic trap, outside the central region on the low-lying collective modes.

2. Method

We consider three-dimensional BEc in an anharmonic trap with two-body interaction strength. We analyze the dynamics of the condensate wave function by the Gross−Pitaevskii (GP) equation [16−19] which has the form

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t)=\left\{-\frac{\hbar^2}{2M}\nabla^2+V(\boldsymbol{r})\right\}\psi(\boldsymbol{r},t)+gN\,|\psi(\boldsymbol{r},t)|^2\psi(\boldsymbol{r},t) \tag{1}$$

where $\psi(\boldsymbol{r},t)$ is the macroscopic condensate wave function, M is the atomic mass, and g denotes the strength of the two-body interaction which is proportional to the s-wave scattering length a and can be written by $g=4\pi\hbar^2 a/M$. In particular, we consider the trapping potential has form as

$$V(\boldsymbol{r})=\frac{M\,\omega_\rho^2}{2}\left(\rho^2+\gamma^2 z^2\right)+\kappa M\,\omega_\rho^2\left(\rho^{2+n}+\gamma^2 z^{2+n}\right) \tag{2}$$

Here γ denotes the trap anisotropy of the confining potential and the anharmonic parameter κ shows how far the realized trap deviates from the center of harmonic trap. The anharmonic term labeled to the harmonic potential is cubic when $n=1$ and quartic when $n=2$. Equation (1) can be restated into a variational problem, which corresponds to the extremization of the action defined by the Lagrangian

$$L(t)=\int d\boldsymbol{r}\left[\frac{i\hbar}{2}\left(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi^*}{\partial t}\right)-\frac{\hbar^2}{2M}|\nabla|^2-V(\boldsymbol{r})-\frac{g}{2}|\psi(\boldsymbol{r},t)|^2+\right] \tag{3}$$

In order to analytically study the dynamics of BEC system, we use the Gaussian variational ansatz, which was introduced in Refs. [3,4]

$$\psi(\rho,z,t)=N(t)\,Exp\left(-\sum_{r=\rho,z}\left[\frac{1}{R_r^2}+i\beta_r\right]r^2\right) \tag{4}$$

where $N(t)=1/\sqrt{\pi^{3/2}R_\rho^2R_z}$ is a normalization factor, while R_i and β_i are variational parameters, representing the condensate widths and the corresponding phases, respectively. From Eqs. (3) and (4) and the corresponding Euler−Lagrange equations we obtain the equations of motion for all variational parameters. The phases β_ρ and β_z can be expressed explicitly in terms of first derivatives of the widths R_ρ and R_z according to

$$\beta_\rho=\frac{M\dot{R}_{\rho,z}}{2\hbar R_{\rho,z}}. \tag{5}$$

Inserting Eq. (6) into the Euler−Lagrange equations for the width of the condensates $R_{\rho,z}$, and after introducing dimensionless parameters according to $R_{\rho,z}\to l(R_{\rho,z})$, $t\to t\omega_\rho$, $\kappa=\kappa/(\hbar\omega_\rho)$ with the oscillating length $l=\sqrt{\hbar/(M\omega_\rho)}$ as well as the dimensionless two-body interaction strength has the form $\alpha=\sqrt{\pi/2}\,\frac{Na}{l}$, we obtain system of second order differential equations for R_ρ and R_z in the dimensionless form

$$\ddot{R}_\rho+R_\rho\left[1+\kappa(2+n)\Gamma\left(2+\frac{n}{2}\right)R_\rho^n\right]-\frac{1}{R_\rho^3}-\frac{\alpha}{R_\rho^3R_z}=0, \tag{6}$$

$$\ddot{R}_z+\gamma^2R_z-\frac{1}{R_z^3}-\frac{\alpha}{R_\rho^2R_z^2}+\frac{\kappa R_z\gamma^2(1+(-1)^n)(2+n)\Gamma\left(\frac{3+n}{2}\right)R_z^n}{2\sqrt{\pi}}=0, \tag{7}$$

3. Result and discussion

For ^{87}Rb BEC atoms [35], $M=1.44\times10^{-25}$ kg, $\omega_\rho=2\pi\times126$ Hz, $\omega_z=2\pi\times21$ Hz, $N=1\times105$ atoms, $a=100\,a_0$ where a_0 is Bohr radius. In principle, the value of interaction strength can be tuned to any value, large or small, positive or negative by applying external field through a Feshbach resonance. However, we consider κ a controllable parameter that the anharmonicity is in range of $\kappa\ll1$.

3.1. Equilibrium condensates

The stationary solution of Eqs. (6) and (7) is determined by setting the acceleration to zero we get

$$\ddot{R}_\rho = 0, \rho = R_{\rho 0}, \tag{8}$$

$$\ddot{R}_z = 0,\ R_z = R_{z0}. \tag{9}$$

Fig. 1 shows the equilibrium positions $R_{\rho 0}$, R_{z0}, Eqs. (8) and (9), respectively, as well as the aspect ratio $R_{\rho 0}/R_{z0}$ versus the anharmonicity of the trapping potential κ with cubic $n = 1$ and quartic $n = 2$ distortion term, respectively, with two-body interaction $\alpha = 1$ and trap anisotropy (a) $\gamma = 0.5$ and (b) $\gamma = 1.5$. We plot in Fig. 1(a) and (b) the equilibrium positions $R_{\rho 0}$, R_{z0} an anharmonicity function of the trapping potential κ for different trap anisotropy $\gamma = 0.5$ and $\gamma = 1.5$, We plot in Fig. 1(a) and (b) the equilibrium positions $R_{\rho 0}$, R_{z0} as a function of the anharmonicity of the trapping potential κ for different trap anisotropy $\gamma = 0.5$ and $\gamma = 1.5$, respectively. We read off that the equilibrium condensate widths depend strongly on the trap anisotropy γ and the quartic anharmonic term than the cubic term which has significant effect. A cigar-like condensate the axial equilibrium width deviates according to the quartic anharmonic term and significant effects in the radial equilibrium width, while the equilibrium condensate widths in a pancake-like condensate deviate in both cubic and quartic anharmonic trap. We show We show in Fig. 1(c) the aspect ratio of $R_{\rho 0}/R_{z0}$ as a function of the anharmonicity of the trapping potential κ for different trap anisotropy $\gamma = 0.5$ and $\gamma = 1.5$, respectively, and we see that the aspect ratio increase (decrease) for cigar- (pancake-) condensates due to the anharmonicity of the trapping potential.

3.2. Oscillation modes

In order to calculate frequencies of the low-lying collective modes analytically, we linearize equations (6) and (7) around the stationary solutions (8) and (9). If we expand the condensate widths as $R_\rho(t) = R_{\rho 0} + \delta R_\rho(t)$ and $R_z(t) = R_{z0} + \delta R_z(t)$ and by keeping only linear terms, we immediately get the frequencies for low-energy excitation modes,

$$\frac{\omega_{B,Q}}{\omega_\rho} = \sqrt{\frac{c_1 + c_3 \pm \sqrt{(c_1 - c_3)^2 + 8c_2^2}}{2}} \tag{10}$$

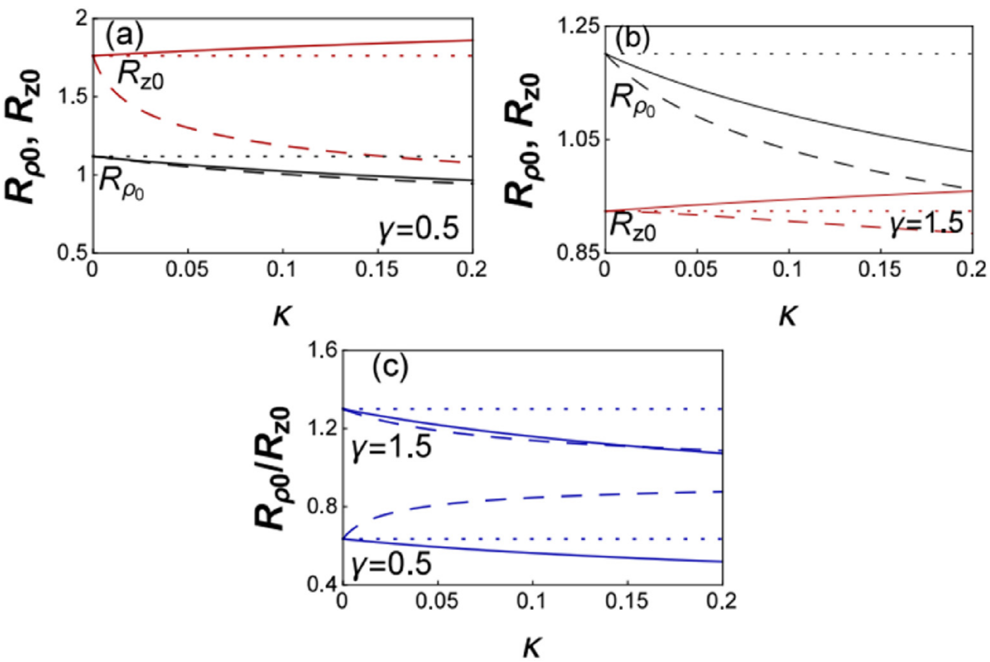

Fig. 1. Equilibrium condensate widths $R_{\rho 0}$ (black curves), R_{z0} (red curves) and $R_{\rho 0}/R_{z0}$ (blue curves) versus the anharmonicity of the trapping potential κ for $\alpha = 1$, cubic $n = 1$ and quartic $n = 2$ distortion with different trap anisotropy γ. Dotted curves correspond to the equilibrium condensate for harmonic trap (i.e. $\kappa = 0$), while dashed, and solid curves represent the anharmonic term labeled to the harmonic potential plus cubic term $n = 1$ and quartic term $n = 2$, respectively.

where the abbreviations c_1, c_2, and c_3 are obtained via Mathematica program.

The breathing mode in-phase oscillations, while quadrupole mode has a lower frequency and is characterized by out-of phase radial and axial oscillations. In Fig. 2(a) and (b) show the frequencies of collective modes as a function of the anharmonicity of the trapping potential κ for $\alpha = 0.5$, cubic $n = 1$ and quatric $n = 2$ distortion with different trap anisotropy (a) $\gamma = 0.5$ and (b) $\gamma = 1.5$. Fig. 2(c) as a function of the trap anisotropy γ for. $\alpha = 0.5$, $\kappa = 0.1$, cubic $n = 1$ and quartic $n = 2$ distortion. These frequencies in Eq. (10) depend strongly on both the anharmonicity of the trapping potential κ and the trap anisotropy γ. We can see in Fig. 2(a) and (b) that the frequencies deviated when the anharmonicity of the trapping potential κ emerge for cubic $n = 1$ and quartic $n = 2$ terms. For the cubic anharmonic trap $n = 1$ significant effect for quadrupole (breathing) mode frequency in cigar (pancake)-like condensate, while the breathing (quadrupole) mode frequency depend strongly on the anharmonicity of the trapping potential κ in cigar (pancake)-like condensate. For the quartic anharmonic trap $n = 2$, the frequencies change strongly with the anharmonicity of the trapping potential. In particular, Fig. 2(c) discuss the frequencies of collective mode as a function of γ and we read off that we have significant effect for cubic anharmonic trap $n = 1$, while the frequencies strong change in quartic anharmonic trap $n = 2$. Indeed the anharmonicity of the trapping potential effect strongly on the geometric resonance (see Ref. [22]) in case of quartic $n = 2$ than cubic $n = 1$ anharmonic trap. Therefore, in Fig. 2(c) we have a geometric resonance at $\gamma = 2.8$ due to the condition $\omega_B = 2\omega_Q$ in case of quartic anharmonic trap. $n = 2$.

4. Conclusions

We have studied low-lying collective modes of a BEc in a harmonic plus cubic and quartic trap by solving the time-dependent GP equation within a variational approaches and we have obtained the analytical expressions for the motion of the low-energy excitations. It has been shown that the lake of harmonics of trapping potential has a remarkable effect on the frequencies of collective modes.

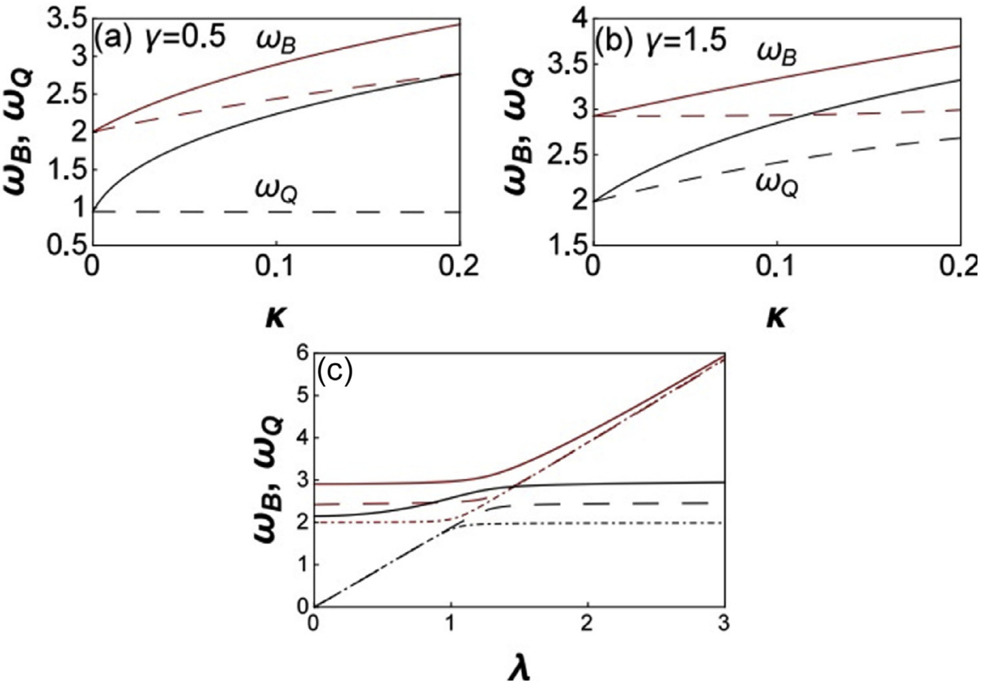

Fig. 2. (a) and (b) low-lying frequencies in unit of ω_ρ^{-1} for the quadrupole (black curves) and breathing (red curves) mode versus the anharmonicity of the trapping potential κ plus cubic term $n = 1$ and quartic term $n = 2$ distortion with two-body interaction $\alpha = 0.5$ and for different trap anisotropy (a) $\gamma = 0.5$ and (b) $\gamma = 1.5$. (c) The low-lying frequencies in unit of ω_ρ^{-1} for the quadrupole (black curves) and breathing (red curves) anharmonicity of the trapping potential κ for $\alpha = 0.5$, cubic $n = 1$ and quartic $n = 2$ distortion with different trap anisotropy (a) $\gamma = 0.5$ and (b) $\gamma = 1.5$. (c) discuss the frequencies of collective modes mode versus the trap anisotropy of the confining potential for different anharmonicity of the trapping potential $\kappa = 0$ (dotted curves) and 0.1 where the anharmonic term labeled to the harmonic potential is cubic when $n = 1$ (dashed curves) and quartic when $n = 2$ (solid curves).

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