# Study the impact of some variables on overtime hours for performance improving employees in Al-diwaniyah electricity organization using tobit regression model


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## Abstract

the main objective is to analyze the relationship between dependent variable and the covariates, traditionally there are many of regression models and use of these models depend on the nature of data ,for example the dependent variable for specific phenomenon is continuous quantity variable and the data spread is linearly then liner regression model considers the proper model, and if spread the phenomenon data spread nonlinearly then nonlinear regression model considers is a proper model ,and if dependent variable of the data is binary then the probit regression model is proper model ,but if the data of dependent variable for specific phenomenon censored from right side or left side then censored regression model considers the proper model ,and when regression model be censored from left at censored point equal to zero ,then tobit regression model represent the appropriate model(which represent special case from censored regression model). in this paper we will study the variables that effect on overtime hours for employees.

Al- diwaniyah electricity company where the data is censored at zero point so the model of tobit regression is mixed model as the cumulative function (cdf) for normal distribution at restricted part (at zero ) and probability density function (pdf) at the other part (at quantities).

## *1-Introduction:*

From known hours number actual working for majority employees in private and generic sector through one week arrival to approximately (35) hours ,where working hours weekly are distribute on week days equally ,where not less about (5) days and no increase about (7) working hours , any augmentation in requisitioned duration ,which it determined the working law .it considers overtime hours ,employee is deserved wage of overtime hours .also from known the employee is deserved week-end wage driven ,not less of 48 hour in case mandating the employee by working through week-end ,its considers for this employee as overtime hours ,he well get on this hours wage be determine by law. Overtime hours system is be applicable in aldiwnal company electricity .in this study response variable is represent overtime hours for each employee, there are employees is being their overtime hours is equal zero and some employee arrival their overtime hours more than( 5) hours in one week may arrival 20 hours ,therefor tobit regression model is be appropriate for this study because the response variable data is censored at zero point .

Since the seminal work of James Tobin (1958) ,Tobit regression has been the subject of great theoretical interest as well as numerous practical applications in a number of field such as econometrics, biological sciences, finance, medicine. Tobit regression (TR) model offers an active way of coping with left-censored data, and can be viewed as a linear regression model where only the data on the response variable is incompletely observed .In tobit model the response variable is censored at zero , mathematical form private in tobit regression model is contains two part ,first part representation cumulative distribution function (c.d.f) for normal distribution (discrete represents limited observations) and second part is representation probability density function ( p.d.f) for normal distribution continuous part (non limited observation), this means the tobit model is mixture between (c.d.f)and (p.d.f) for normal distribution ,the first part treatment massed-together data at censored point which equal zero and second part treatment quantitative data .in this paper we will studying the overtime for staff Al-dawina company electricity the response variable data in this study is be censored at zero point ,therefore the tobit model it is be best model for study this data .

The rest of the paper is ordered as follows in section 2 we offer some theoretical aspects for tobit regression model, in section 3 we present Mean and Marginal effect on the mean for tobit model in section 4 we present analysis and interpretation the results, in section 5 we conclude the paper with a brief discussion.

*2-1* ***tobit regression model***

Tobit regression model (T.R.M) is special case from censored regression model when the censored point equal zero, then general formulation of censored regression model as.

$$Y = \begin{cases} c & if\ y_i^* \leq c \\ y_i^* & if\ y_i^* > c \end{cases} \quad (1)$$

$$where\ \ y_i^* = x_i^T \beta + u_i, \qquad i = 1, \dots \dots, n \quad (2)$$

Where

$Y_i$ :is response variable it is censoring at $c$ ,

$c$: is censored point

$y_i^*$ is called the latent variable and $u_i$ *is the error term* distribute according to normal distribution. $u_i \sim N(0, \sigma^2)$ and $c$ is called the censored point. In the tobit model censored point $c = 0$, then the formula(1) becomes as follow:

$$Y = \begin{cases} 0 & if y_i^* \leq 0 \\ y_i^* & if y_i^* > 0 \end{cases} \quad (3)$$

$$y_i^* = x_i^T \beta + u_i, \qquad i = 1, \dots \dots, n$$

whereas latent variable distributed normal distribution for mean $(\alpha + x_i^T \beta)$ and variance$(\sigma^2)$, then the probability density function of $y_i^*$take the formula as follow,

If $pro(Y = y_i^*)\ if\ pro(y_i^* > 0)$

$$p(Y) = \frac{1}{\sqrt{2\pi\sigma^2}} Exp\left(\frac{\left(Y - x_i^T \beta\right)^2}{2\sigma^2}\right) \qquad (4)$$

The probability density function in (4) belong to continuous part(non limited observation) we can rewrite (4) as follows

$$p(Y) = \frac{1}{\sigma}\phi\left(\frac{Y - \left(x_i^T \beta\right)}{\sigma}\right) \qquad (5)$$

$where \phi(.)$is a Probability density function$(\text{p.d.f})$ see Long, J. Scott.( 1997)

and

If $pro(Y = 0) \quad if \quad pro(y_i^* \leq 0) \longrightarrow \Phi\left(\frac{Y-(x_i^T\beta)}{\sigma}\right) = \Phi\left(\frac{0-(x_i^T\beta)}{\sigma}\right)$

$$= \Phi\left(\frac{-(x_i^T\beta)}{\sigma}\right) = 1 - \Phi\left(\frac{(x_i^T\beta)}{\sigma}\right) \qquad (6)$$

where $\Phi(.)$ is a cumulative distribution function $(\text{c.d.f})$ .The equation (6) is cumulative distribution function (c.d.f) for normal distribution , this means the tobit model is mixed function of probability density function
and cumulative distribution function ,for normal distribution .

$$p(Y) = \left[\frac{1}{\sigma}\phi\left(\frac{Y - (x_i^T\beta)}{\sigma}\right)\right]\left[1 - \Phi\left(\frac{\left(x_i^T\beta\right)}{\sigma}\right)\right] \qquad (7)$$

Equation (7) representation Function of tobit model, which it is be mixed between continuous and discrete parts.Wherein $\left[\frac{1}{\sigma}\phi\left(\frac{Y-(x_i^T\beta)}{\sigma}\right)\right]$ :representation the traditional regression for the uncensored observations.
$\left[1 - \Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]$ :representation the cumulative distribution function for observation is censored.
Possible estimate the tobit model parameter via (M.L.E) as fallow

$$L = \prod_{i=1}^{N}\left[\frac{1}{\sigma}\phi\left(\frac{Y-(x_i^T\beta)}{\sigma}\right)\right]\left[1-\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]$$

$$\ln L = -N ln\sigma + \sum_{i=1}^{N} ln\left[\phi\left(\frac{Y-(x_i^T\beta)}{\sigma}\right)\right] + \sum_{i=1}^{N} ln\left[1-\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]$$

$$\frac{\partial lnL}{\partial\beta} = \sum_{I=1}^{N}\frac{\phi\left(\frac{(Y-x_i^T\beta)}{\sigma}\right)\frac{(Y-x_i^T\beta)}{\sigma}\left(\frac{-x_i^T}{\sigma}\right)}{\phi\left(\frac{(Y-x_i^T\beta)}{\sigma}\right)} + \frac{\phi\left(\frac{(Y-x_i^T\beta)}{\sigma}\right)\left(\frac{x_i^T}{\sigma}\right)}{1-\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)} = 0$$

$$\frac{\partial lnL}{\partial\beta} = \sum_{I=1}^{N}\frac{\phi\left(\frac{(Y-x_i^T\beta)}{\sigma}\right)\frac{(Y-x_i^T\beta)}{\sigma}\left(\frac{-x_i^T}{\sigma}\right)\left[1-\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right] + \left[\phi\left(\frac{(Y-x_i^T\beta)}{\sigma}\right)\right]^2\left(\frac{x_i^T}{\sigma}\right)}{\left[\phi\left(\frac{(Y-x_i^T\beta)}{\sigma}\right)\right]\left[1-\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]} = 0$$

$$\sum_{i=1}^{N}\phi\left(\frac{(Y-x_i^T\beta)}{\sigma}\right)\frac{(Y-x_i^T\beta)}{\sigma}\left(\frac{-x_i^T}{\sigma}\right)\left[1-\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right] + \left[\phi\left(\frac{(Y-x_i^T\beta)}{\sigma}\right)\right]^2\left(\frac{x_i^T}{\sigma}\right) = 0 \quad (8)$$

From equation (8)for parameter estimation of tobit model we use numerical method such that o Newton's method ,but estimation operation is become so routine and been included in many computer packages, packages("censReg") for analyzing data.

***2- :Mean of tobit model :***

In classical regression model ,predictive values ($\hat{y}_{i)}$ is defined as conditional mean for response variable, also in tobit regression model predictive value is conditional mean for response variable .According to (Sigelman and Zeng 1999) we need to work will treatment three parts to find mean of tobit model ,

***2-1: Mean of the latent variable$y_i^*$:***

From equation (2) the Expected value of latent variable $T_i$ is equal to

$$\mathrm{E}(y_i^*) = (\alpha + x_i^T\beta) \quad (9)$$

***2-2: Mean of the Y|Y*** $> 0$

$$E(Y|Y > o) = E(Y|\text{truncated}) = \beta X_i + \sigma\left(\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}\right) \quad (10)$$

See Appendix (A)

***2-3 :Mean of Y***

$$E(Y) = \Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\left[\alpha + x_i^T\beta + \sigma\left(\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(\alpha + x_i^T\beta)}{\sigma}\right)}\right)\right] \qquad (11)$$

$$where \quad \left(\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}\right) \text{is the inverse Mills ratio}$$

From The previous talk , we find there are three mean, the first is belong to latent variable for the continuous part, and second is belong to , censord point for discrete part ,and the third is belong to tobit model (Combines two parts, continuous and discrete ). As Greene (2007) he was believed the third mean is More important than the rest, Because it focuses on censored and uncensored data at one time.

***3:Marginal Effects expectation for Tobit Model***

Marginal effects explication summarizes the effects of $X_i$on $y_i$: (Greene 2007,) . Tobit model contains two parts ,first limited part censored observation at zero, and second non limit part represents non censored observation for latent variable $T_i$. The changing in $X_i$ possible to effect each part separately or together in the same time. From previous illustration there are three expected value for tobit model and three marginal Effects expectation for Tobit Model as follows:

***3-1 Marginal effects for latent variable*** $T_i$

when back the equation $T_i = x_i^T\beta + u_{i,}$ then $\frac{\partial T_i}{\partial x_i} = \beta$,This means if change one unit in explanatory variables $X_i$ this will effect the latent variable .

*3-2* ***Marginal effects for censored observations***

If we back to the truncated variable $(Y_i/Y_i > 0)$ and the mean of truncated variable $E(Y_i/Y_i > 0)$, we can find:

$$\frac{\partial E(Y_i/Y_i > 0)}{\partial X_i} = \beta\left\{1 - \lambda i\left[\left(\frac{(x_i^T\beta)}{\sigma}\right) + \lambda i\right]\right\} \qquad (12)$$

For prof equation (12) see Appendix (B)

***3-3: Marginal effect on the mean for y (censored and uncensored):***

We can find the marginal effect for $Y$ in tobit regression model as follows (Wooldridge, Jeffrey.( 2002)).

$$\frac{\partial(\mathrm{Y_i}|\mathrm{X_i})}{\partial X_i} = \beta\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)$$

The prove of that is

$$\mathrm{E(Y|X_i)} = \Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\left[\beta X + \sigma\left(\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}\right)\right]$$

$$\mathrm{E(Y|X_i)} = \Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)(\beta X) + \sigma\left(\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(\alpha + x_i^T\beta)}{\sigma}\right)}\right)\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)$$

$$\mathrm{E(Y|X_i)} = \Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)(\alpha + x_i^T\beta) + \sigma\left(\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right)$$

$$\frac{\partial(\mathrm{Y_i}|\mathrm{X_i})}{\partial X_i} = (x_i^T\beta)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{\beta}{\sigma} + \beta\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right) + \sigma\left(\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(x_i^T\beta)}{\sigma}\left(\frac{-\beta}{\sigma}\right)\right)$$

$$\frac{\partial(\mathrm{Y_i}|\mathrm{X_i})}{\partial X_i} = (x_i^T\beta)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{\beta}{\sigma} + \beta\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right) - \left(\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(x_i^T\beta)}{\sigma}(\beta)\right)$$

$$\frac{\partial(\mathrm{Y_i}|\mathrm{X_i})}{\partial X_i} = \frac{(x_i^T\beta)}{\sigma}\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\beta + \beta\Phi\left(\frac{x_i^T\beta}{\sigma}\right) - \left(\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(x_i^T\beta)}{\sigma}(\beta)\right)$$

$$\frac{\partial \mathrm{Y}}{\partial X_i} = \beta\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)$$

Mcdonal and Moffitt ( 1980) proposed analytical formula $\frac{\partial(\mathrm{Y_i}|\mathrm{X_i})}{\partial X_i}$ as follows

$$\frac{\partial(\mathrm{Y_i}|\mathrm{X_i})}{\partial X_i} = \beta\left[\Phi_\mathrm{i}\left(1 - \lambda i\left\{\frac{(x_i^T\beta)}{\sigma} + \lambda i\right\} + \phi_i\left(\frac{(x_i^T\beta)}{\sigma} + \lambda i\right)\right)\right] \qquad (13)$$

For prove equation (13) see Appendix(c)

In this case the change in $X_i$ has two effects ,the first effect on the conditional mean for latent variable $T_i$( part non limit in , distribution )

and the second effect on the limited part of the distribution ,his means the change in $X_i$ effect in all $\mathrm{Y_i}$ components .

## *4-the study Sample and Data Analysis*

Depend on the sample size (200) observation ,these data collected from Annals (Al- diwaniyah electricity company) for year (2013) each observation from this sample representation overtime hours as response variable

And rest the variables representation the independent variables are:

The variable of this sample are:

Y: number overtime hours in one week .

.$X_1$: age employee

.$X_2$: Monthly salary for employee.

.$X_3$: gender  employee.

.$X_4$: wage one overtime hour.

.$X_5$: education years.

$X_6$ :social state

To study and analysis the effect of  these six variables on : overtime hour  . through use tobit regression model, and for data analysis. we use R programming which is a free software. Through use the packages("censReg ") Arne. H (2013).

***4-1 results interpretation***

From results shown in table (1) we find the tobit model has high ability in the interpretation

**Table (1) display value of    The pseudo-R  square**

| | |
|---|---|
| **The pseudo-R  square** | 0.83120 |

Table(1)    The pseudo-R  square = 0.83120 means that the independent variables (age employee, Monthly salary for employee, gender  employee, wage one overtime hour, education years, social state) explain 81.55% from the

In variation (number of overtime hours weekly ) this indictor shows strength  tobit model in representation of studied phenomenon data.

**Table (2) table variance analysis**

| S.O.V | df | S.S | M.S | F |
|---|---|---|---|---|
| **Regression** | 6 | 156.451 | 26.075 | 85.941 |
| **Residual** | 193 | 58.97 | 0.305 | |
| **Total** | 199 | 215.421 | | |

Table (2) we find the value ($F_c$) grater than ($F_T$) at confidence level (0.01,0.05) this shows confidence the Tobit model .

**Table (3) shown parameter estimation and stander error and t value**

| observation total 200 | left-censored 85 | uncensored 115 | right censored 0 | |
|---|---|---|---|---|
| Variables | Estimate | Std. error | t value | Pr(> t) |
| Intercept | 8.17420 | 2.74145 | 2.982 | 0.00287 |
| $X_1$: employee age | -0.17933 | 0.07909 | -2.267 | 0.02337 |
| $X_2$: Monthly salary for employee | -0.55414 | 0.13452 | -4.119 | 0.0000038 |
| $X_3$: gender employee | -1.68622 | 0.40375 | -4.176 | 0.5421 |
| $X_4$: wage one overtime hour | 0.32605 | 0.25442 | 1.282 | 0.00001 |
| $X_5$: education years. | -2.28497 | 0.40783 | -5.603 | 0.00000000211 ($211 \times 10^{-11}$) |
| $X_6$ :social state | 2.10986 | 0.06710 | 31.444 | 0.24554 |
| Newton-Raphson maximisation, 7 iterations | | | | |

We know the value ($\beta$) our tell of the relationship between response variable and covariates (if ($+\beta$) this means positive relationship and if (-$\beta$) this means negative relationship ).

Through the result shown in table(3), we see the relationship between number of overtime hours and $X_1$ employee age it is negative .This means if increase employee age one unit the number overtime hours will decreases (-0.17933) and this variable (employee age) is significant effect on (number overtime hours).Also we see the relationship between number of overtime hours and $X_2$ Monthly salary for employee it is negative .This means if increase Monthly salary for employee one unit the number overtime hours will decreases(-0.55414) and this variable (Monthly salary for employee) is significant effect on (number overtime hours). Also we see the variable ($X_3$: gender employee ) is not significant effect on (number overtime hours) this means the variable ($X_3$: gender employee ) has weakness in impact on (number overtime hours).

we see the relationship between number of overtime hours and $X_4$: wage one overtime hour it is positive .where $X_4$: wage one overtime hour increase one unit so (number overtime hours) is increase (0.32605) .Also the variable $X_4$: wage one overtime hour is significant effect on (number overtime hours).

we see the relationship between number of overtime hours and $X_5$: education years.it is negative .This means if increase $X_5$: education years one unit the number overtime hours will decreases(-2.28497) and this variable ($X_5$: education years) is significant effect on (number overtime hours).but see the variable ($X_6$ social state) is not significant effect on (number overtime hours) this means the variable ($X_6$ :social state ) has weakness in impact on (number overtime hours).

## *5-**Conclusion:***

When study overtime hours as response variable ,we find this variable data is censored at (zero) this means the tobit model is appropriate model .from the results shown in table (3) we can limiting the variables according most effect on number overtime hours)as follow:

$X_5$: education years is come first ranked in effect on (number overtime hours). And $X_2$: Monthly salary for employee is come second ranked in effect on (number overtime hours). And $X_1$: employee age is come third ranked in effect on (number overtime hours).the variable $X_4$: wage one overtime hour is come forth ranked ranked in effect on (number overtime hours). And rest variables is not significant effect on (number overtime hours).

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## Appendix (A)

Prove

$$\mathrm{E[y/truncation]} = \mu + \sigma\lambda(\alpha)$$

To prove equation above we will depend on m.g.f.

$$\text{m.g.f.} = \mathrm{E}e^{ty} = \int e^{ty} f(y/y > a)$$

$$f\ (y/y > a) = \frac{f(y)}{prob[y>a]}$$

$$pro[y > a] = 1 - \Phi\left(\frac{a-\mu}{\sigma}\right) \rightarrow 1 - \Phi(\alpha)$$

f$(y/y > a) = \frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}}{1 - \emptyset(\alpha)}$

E$(e^{ty})$=$\frac{\int_a^\infty e^{ty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy}{1 - \Phi(\alpha)}$

We found integration for numerator

$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_a^\infty e^{ty}\, e^{-\frac{(y-\mu)^2}{2\sigma^2}}\, dy$

$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_a^\infty e^{ty}\, e^{-\frac{(y^2-2\mu y+\mu^2)}{2\sigma^2}}\, dy$

$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_a^\infty e^{ty}\, e^{\frac{(-y^2+2\mu y-\mu^2)}{2\sigma^2}}\, dy$

$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_a^\infty e^{\frac{2t\sigma^2 y-y^2+2\mu y-\mu^2}{2\sigma^2}} dy$

$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_a^\infty e^{\frac{-y^2+2\mu y+2t\sigma^2 y-\mu^2}{2\sigma^2}} dy$

$$=\frac{1}{\sqrt{2\pi\sigma^2}}\int_a^\infty e^{\frac{-y^2+2(M+t\sigma^2)y-M^2}{2\sigma^2}}dy$$

$$=\frac{1}{\sqrt{2\pi\sigma^2}}\int_a^\infty e^{\frac{-y^2+2(\mu+t\sigma^2)y-(\mu+t\sigma^2)^2+(\mu+t\sigma^2)^2-\mu^2}{2\sigma^2}}dy$$

$$=\frac{e^{\frac{(\mu+t\sigma^2)^2-\mu^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}\int_a^\infty e^{\frac{-(y^2-2(\mu+t\sigma^2))y-(\mu+t\sigma^2)^2}{2\sigma^2}}dy$$

$$=\frac{e^{\frac{(\mu+t\sigma^2)^2-\mu^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}\int_a^\infty e^{\frac{-(y-(\mu+t\sigma^2)^2}{2\sigma^2}}dy$$

Let $\acute{\mu}=(\mu+t\sigma^2)$

$$=e^{\frac{(\mu+t\sigma^2)^2-\mu^2}{2\sigma^2}}\int_a^\infty \frac{1}{\sigma}\emptyset\left(\frac{y-\acute{\mu}}{\sigma}\right)$$

$$=e^{\frac{(\mu+t\sigma^2)^2-\mu^2}{2\sigma^2}}[\emptyset(\frac{y-\acute{\mu}}{\sigma})]_a^\infty$$

$$=e^{\frac{(\mu+t\sigma^2)^2-\mu^2}{2\sigma^2}}[\emptyset(\alpha)^{-1}-\emptyset(\frac{a-\acute{\mu}}{\sigma})]$$

$$=e^{\frac{(\mu+t\sigma^2)^2-\mu^2}{2\sigma^2}}[1-\emptyset(\frac{a-\acute{\mu}}{\sigma})]$$

$$=e^{\frac{\mu^2+2\mu t\sigma^2+t^2\sigma^2-\mu^2}{2\sigma^2}}[1-\emptyset(\frac{a-\acute{\mu}}{\sigma})]$$

$$=e^{\mu t+\frac{t^2\sigma^2}{2}}[1-\emptyset(\frac{a-\acute{\mu}}{\sigma})]$$

$$E[e^{ty}]=e^{\mu t+\frac{t^2\sigma^2}{2}}[\frac{[1-\emptyset(\frac{a-\acute{\mu}}{\sigma}-t\sigma)]}{[1-\Phi(\frac{a-\mu}{\sigma})]}]$$

$$\mu=\acute{\mu}+t\sigma^2$$

$$E[e^{ty}]=e^{\mu t+\frac{t^2\sigma^2}{2}}[\frac{[1-\emptyset(\frac{a-\mu}{\sigma}-t\sigma)]}{[1-\Phi(\frac{a-\mu}{\sigma})]}]$$

$$\text{m.g.f.}=e^{\mu t+\frac{t^2\sigma^2}{2}}[\frac{\left[1-\emptyset\left(\frac{a-\mu}{\sigma}-t\sigma\right)\right]}{\left[1-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]}]$$

to find E[y/truncation] we derive the first derivation for m.g.f

$$\acute{M}(t)/_{t=0}=e^{\mu t+\frac{t^2\sigma^2}{2}}\left[\frac{\left[-\emptyset\left(\frac{a-\mu}{\sigma}-t\sigma\right)-\sigma\right]}{\left[1-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]}\right]+\left[\frac{\left[1-\emptyset\left(\frac{a-\mu}{\sigma}-t\sigma\right)\right]}{\left[1-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]}\right]\quad e^{\mu t+\frac{t^2\sigma^2}{2}}\ \mu+t\sigma^2$$

$$\acute{M}(t)/_{t=0}=e^{\mu(0)+\frac{(0)^2\sigma^2}{2}}\left[\frac{\left[\sigma\emptyset\left(\frac{a-\mu}{\sigma}-(0)\sigma\right)\right]}{\left[1-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]}\right]+\left[\frac{\left[1-\emptyset\left(\frac{a-\mu}{\sigma}-(0)\sigma\right)\right]}{\left[1-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]}\right]\quad e^{M(0)+\frac{(0)^2\sigma^2}{2}}\ \mu+(0)\sigma^2$$

$$= 1^*\ \sigma\ \frac{\emptyset\left(\frac{a-\mu}{\sigma}\right)}{1-\Phi\left(\frac{a-\mu}{\sigma}\right)}+1*1*\mu+0$$

$$= \mu+\sigma\ \frac{\emptyset\left(\frac{a-\mu}{\sigma}\right)}{1-\Phi\left(\frac{a-\mu}{\sigma}\right)}\ \lambda(\alpha)=\frac{\emptyset\left(\frac{a-\mu}{\sigma}\right)}{1-\Phi\left(\frac{a-\mu}{\sigma}\right)}$$

in tobit model $\lambda(\alpha)$ is be $\lambda(\alpha)=\frac{\emptyset\left(\frac{\beta X_i}{\sigma}\right)}{1-\Phi\left(\frac{\beta X_i}{\sigma}\right)}$ therefor

E[y/truncation]= $\beta X_i$+ $\sigma\ \lambda(\alpha)$.

## Appendix (B)

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta\left\{1-\lambda i\left[\left(\frac{(x_i^T\beta)}{\sigma}\right)+\ \lambda i\quad\right]\right\}$$

Which we can prove it as follows:

Whereat $\lambda i=\frac{\phi\left(\frac{(0-x_i^T\beta)}{\sigma}\right)}{1-\Phi\left(\frac{(0-x_i^T\beta)}{\sigma}\right)}=\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}$

$$E(Y_i/Y_i>0)=x_i^T\beta+\sigma\left(\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}\right)$$

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta+\sigma\left[\frac{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(x_i^T\beta)}{\sigma}\left(\frac{-\beta}{\sigma}\right)-\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\left(\frac{\beta}{\sigma}\right)}{\left[\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]^2}\right]$$

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta+\sigma\left(\frac{\beta}{\sigma}\right)\left[\frac{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(x_i^T\beta)}{\sigma}(-)-\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\left[\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]^2}\right]$$

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta+\beta\left[\frac{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(x_i^T\beta)}{\sigma}(-)-\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\left[\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]^2}\right]$$

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta\left\{1+\left[\frac{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(\beta X_i)}{\sigma}(-)-\phi\left(\frac{(\beta X_i)}{\sigma}\right)\phi\left(\frac{(\beta X_i)}{\sigma}\right)}{\left[\Phi\left(\frac{(\beta X_i)}{\sigma}\right)\right]^2}\right]\right\}$$

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta\left\{1+\left[\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(x_i^T\beta)}{\sigma}(-)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}-\frac{\left[\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]^2}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}\right]\right\}$$

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta\left\{1+\left[\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\frac{(x_i^T\beta)}{\sigma}(-)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}-\frac{\left[\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]^2}{\left[\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)\right]^2}\right]\right\}$$

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta\left\{1+\left[-\lambda i\left(\frac{(x_i^T\beta)}{\sigma}\right)-\ \lambda i^2\right]\right\}$$

$$\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}=\beta\left\{1-\lambda i\left[\left(\frac{(x_i^T\beta)}{\sigma}\right)+\ \lambda i\right]\right\}.$$

## Appendix (c)

$$\frac{\partial(\mathrm{Y_i|X_i})}{\partial X_i}=\beta\left[\Phi_i\left(1-\lambda i\left\{\frac{(x_i^T\beta)}{\sigma}+\lambda i\right\}+\phi_i(\frac{(x_i^T\beta)}{\sigma}+\lambda i\right)\right]$$

whereat $\lambda i=\frac{\phi\left(\frac{(0-x_i^T\beta)}{\sigma}\right)}{1-\Phi\left(\frac{(0-(x_i^T\beta)}{\sigma}\right)}=\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}$

$$\frac{\partial(\mathrm{Y_i|X_i})}{\partial X_i}=\mathrm{pro}(\mathrm{Y}>0)*\frac{\partial E(Y_i/Y_i>0)}{\partial X_i}+\frac{\partial\mathrm{pro}(\mathrm{Y}>0)}{\partial X_i}*E(Y_i/Y_i>0)$$

$$\text{pro}(\mathrm{Y} > 0) = 1 - \Phi\left(\frac{(0 - (x_i^T\beta))}{\sigma}\right) = 1 - \Phi\left(\frac{(-(x_i^T\beta))}{\sigma}\right) = 1 - (1 - \Phi\left(\frac{((x_i^T\beta))}{\sigma}\right) = \Phi\left(\frac{((x_i^T\beta))}{\sigma}\right) = \Phi_i$$

$$\frac{\partial E(Y_i/Y_i > 0)}{\partial X_i} = \beta\left\{1 - \lambda i\left[\left(\frac{(x_i^T\beta)}{\sigma}\right) + \ \lambda i \quad \right]\right\}$$

$$\frac{\partial \text{pro}(\mathrm{Y} > 0)}{\partial X_i} = \phi\left(\frac{(x_i^T\beta)}{\sigma}\right) * \frac{\beta}{\sigma} = \phi_i * \frac{\beta}{\sigma}$$

$$\mathrm{E}(Y|Y > o) = \alpha + \beta X_i + \sigma\left(\frac{\phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}{\Phi\left(\frac{(x_i^T\beta)}{\sigma}\right)}\right)$$

$$\mathrm{E}(Y|Y > o) = \alpha + x_i^T\beta + \sigma\lambda i$$

After collocation above border we gets on

$$\frac{\partial(\mathrm{Y_i|X_{i)}}}{\partial X_i} = \Phi_i\left(\beta\left\{1 - \lambda i\left[\left(\frac{(x_i^T\beta)}{\sigma}\right) + \ \lambda i \quad \right]\right\}\right) + \left(\phi_i * \left(\frac{\beta}{\sigma}\right)\right)(x_i^T\beta + \sigma\lambda i)$$

$$\frac{\partial(\mathrm{Y_i|X_{i)}}}{\partial X_i} = \Phi_i\left(\beta\left\{1 - \lambda i\left[\left(\frac{(x_i^T\beta)}{\sigma}\right) + \ \lambda i \quad \right]\right\}\right) + \left(\phi_i(\beta)\right)(\frac{x_i^T\beta}{\sigma} + \lambda i)$$

$$\frac{\partial(\mathrm{Y_i|X_{i)}}}{\partial X_i} = \mathrm{\beta}\left\{\Phi_i\left(\left\{1 - \lambda i\left[\left(\frac{(x_i^T\beta)}{\sigma}\right) + \ \lambda i \quad \right]\right\}\right) + (\phi_i)(\frac{x_i^T\beta}{\sigma} + \lambda i)\right\}$$

$$\frac{\partial(\mathrm{Y_i|X_{i)}}}{\partial X_i} = \mathrm{\beta}\{\Phi_i(\{1 - \lambda i[\alpha_i + \ \lambda i \quad ]\}) + (\phi_i)(\alpha_i + \lambda i)\}$$

where $\alpha_i = \frac{x_i^T\beta}{\sigma}$