ON CENTRALIZERS ON SOME GAMMA RING

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الخلاصة العربية: قدمنا في هذا البحث دراسة حول تطبيق جوردان المركزي على بعض الحلقات

ABSTRACT

Let M be a 2-torsion free Γ-ring satisfies the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$ and $\alpha,\beta\in\Gamma$. In section one ,we prove if M be a completely prime Γ-ring and T:M→M an additive mapping such that $T(a\alpha a)=T(a)\alpha a$ (resp., $T(a\alpha a)=a\alpha T(a)$)holds for all $a\in M,\alpha\in\Gamma$.Then T is a left centralizer or M is commutative (res.,a right centralizer or M is commutative) and so every Jordan centralizer on completely prime Γ-ring M is a centralizer .In section two ,we prove this problem but by another way. In section three we prove that every Jordan left centralizer(resp., every Jordan right centralizer) on Γ-ring has a commutator right non-zero divisor(resp., on Γ-ring has a commutator left non-zero divisor)is a left centralizer(resp., is a right centralizer) and so we prove that every Jordan centralizer on Γ-ring has a commutator non –zero divisor is a centralizer .

Key wards : Γ-ring, prime Γ-ring,semi-prime Γ-ring , left centralizer, Right centralizer, centralizer, Jordan centralizer.

1-INTRODUCTION

Throughout this paper,M will represent Γ-ring with center Z .In [7] B.Zalar proved that any left (resp.,right)Jordan centralizer on a 2-torsion free semi-prime ring is a left (resp.,right)Centralizer.In [3] authors prove the same question on the condition that R has a commutator right (resp., left) non- zero divisor .And J.Vukman in [6] proved that if R is2-torsion free semi-prime ring and T:R→R be an additive mapping such that $2T(x^2)=T(x)x+xT(x)$ holds for all $x,y\in R$ .Then T is left and right centralizer.In this paper we define Jordan centralizer on Γ-ring and we show that the existence of a non-zero Jordan centralizer Ton a 2-torsion free completely prime Γ-ring M which satisfies the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$ and $\alpha,\beta\in\Gamma$ implies either T is centralizer or M is commutative Γ-ring.

Let M and Γ be additive abelian groups, M is called a Γ-ring if for any x,y,z $\in M$ and $\alpha,\beta\in\Gamma$,the following conditions are satisfied

(1)$x\alpha y\in M$

(2)$(x+y)\alpha z=x\alpha z+y\alpha z$

$x(\alpha+\beta)z=x\alpha z+x\beta z$

$x\alpha(y+z)=x\alpha y+x\alpha z$

(3)$(x\alpha y)\beta z=x\alpha(y\beta z)$

The notion of Γ-ring was introduced by Nobusawa[5] and generalized by Barnes[1],many properties of Γ-ring were obtained by many research such as [2]

Let A,B be subsets of a Γ-ringM and Λ a subset of Γ we denote $A\Lambda B$ the subset of M consisting of all finite sum of the form $\sum a_i\lambda_i b_i$ where $a_i\in A, b_i\in B$ and $\lambda_i\in\Lambda$ .Aright ideal(resp.,left ideal) of a Γ-ring M is an additive subgroup I of

M such that $I\Gamma M\subset I$(resp.,$M\Gamma I\subset I$).If I is a right and left ideal inM,then we say that I is an ideal .M is called a 2-torsion free if 2x=0 implies x=0 for all $x\in M$.A Γ-ringM is called prime if $a\Gamma M\Gamma b=0$ implies a=0 or b=0 and M is called completely prime if $a\Gamma b=0$ implies a=0 or b=0(a,b $\in M$),Since $a\Gamma b\Gamma a\Gamma b\subset a\Gamma M\Gamma b$,then every completely prime Γ-ring is prime.A Γ-ring M is called semi-prime if $a\Gamma M\Gamma a=0$ implies a=0and M is called completely semi-prime if $a\Gamma a=0$ implies a=0($a\in M$)

Let R be a ring,an additive mapping D:R→R is called derivation if D(xy)=D(x)y+xD(y) holds for all x,y$\in$R.A left(right) centralizer of R is an additive mapping T:R →R which satisfies T(xy)=T(x)y(T(xy)=xT(y)) ***for all x,y$\in$R.A Jordan centralizer be an additive mapping T which satisfies $T(x\circ y)=T(x)\circ y=x\circ T(y)$.***
A Centralizer of R is an additive which is both left and right centralizer.An easy computation shows that every centralizer is also a Jordan centralizer.Many Papers work about the problem every Jordan centralizer be centralizer such as in[7] .In this paper ,we work this problem on some kind of Γ-ring.
Now ,we shall give the following definition which are basic in this paper.
<u>Definition1.1</u>:-Let M be a Γ-ring and let D:M → M be an additive map,D is called a Derivation if for any a,b $\in$M and $\alpha\in\Gamma$,if the following condition satisfy
$D(a\alpha b)=D(a)\alpha b+a\alpha D(b)$
<u>Definition1.2</u>:- Let M be a Γ-ring and let T:M → M be an additive map ,T is called <u>Left centralizer</u> of M,if for any a,b $\in$M and $\alpha\in\Gamma$, the following condition satisfy
$T(a\alpha b)=T(a)\alpha b$,
<u>Right centralizer</u> of M,if for any a,b $\in$M and $\alpha\in\Gamma$, the following condition satisfy
$T(a\alpha b)=a\alpha T(b)$,
<u>Jordan left centralizer</u> if for all a$\in$M and $\alpha\in\Gamma$, the following condition satisfy
$T(a\alpha a)=T(a)\alpha a$
<u>Jordan Right centralizer</u> if for all a$\in$M and $\alpha\in\Gamma$, the following condition satisfy
$T(a\alpha a)=a\alpha T(a)$
<u>Jordan centralizer</u> of M,if for any a,b $\in$M and $\alpha\in\Gamma$, the following condition satisfy $T(a\alpha b+b\alpha a)=T(a)\alpha b+b\alpha T(a)=a\alpha T(b)+T(b)\alpha a$

A centralizer of M is an additive mapping which is both left and right centralizer.An easy computation shows that every centralizer is also a Jordan centralizer but the converse is not true .In this paper we prove this problem when M is 2-torsion free completely prime Γ-ring.Now we shall prove the following Lemmas which are necessarily to prove our main result in this paper.
<u>Lemma 1.3</u>:-Let M be a 2-torsion free Γ-ring and let T:M → M be an additive mapping which satisfies $T(a\alpha a)=T(a)\alpha a$,(resp., $T(a\alpha a)=a\alpha T(a)$) for all a$\in$M and $\alpha\in\Gamma$,then the following statement holds for all a,b,c$\in$M and $\alpha,\beta\in\Gamma$,

(i) $T(a\alpha b+b\alpha a)=T(a)\alpha b+T(b)\alpha a$
(resp., $T(a\alpha b+b\alpha a)=a\alpha T(b)+b\alpha T(a)$)
(ii) Especially if M is 2-torsion free and $a\alpha b\beta c=a\beta b\alpha c$ for all a,b,c$\in$M and $\alpha,\beta\in\Gamma$ then
$T(a\alpha b\beta a)=T(a)\alpha b\beta a$ (resp., $T(a\alpha b\beta a)=a\alpha b\beta T(a)$)
(iii) $T(a\alpha b\beta c+c\alpha b\beta a)=T(a)\alpha b\beta c+T(c)\alpha b\beta a$.
(resp., $T(a\alpha b\beta c+c\alpha b\beta a)=a\alpha b\beta T(c)+c\alpha b\beta T(a)$

<u>Proof</u>:-(i) Since $T(a\alpha a)=T(a)\alpha a$ for all a$\in$M and $\alpha\in\Gamma$,…….(1)
Replace a by a+b in (1),we get

$T(a\alpha b+b\ \alpha a)=T(a)\ \alpha b+T(b)\ \alpha a$……………………………….(2)

(ii) by replacing b by $a\ \beta b+b\ \beta a$, $\beta \in \Gamma$

$W=T(a\alpha(a\ \beta b+b\ \beta a)+(a\ \beta b+b\ \beta a)\ \alpha a)$

$=T(a)\ \alpha(a\ \beta b+b\ \beta a)+T(a\ \beta b+b\ \beta a)\ \alpha a$

$= T(a)\ \alpha(a\ \beta b)+T(a)\ \alpha\ (b\ \beta a)+(T(a)\ \beta\ b+T(b)\ \beta\ a)\ \alpha a$

$= T(a)\ \alpha(a\ \beta b)+T(a)\ \alpha\ (b\ \beta a)+T(a)\ \beta\ b\alpha a+T(b)\ \beta\ a\ \alpha a$

Since $a\alpha b\beta c=a\beta b\alpha c$,then

$W= T(a)\ \alpha(a\ \beta b)+2T(a)\ \alpha\ (b\ \beta a)+T(b)\ \beta\ a\ \alpha a$

On the other hand

$W = T(a\alpha(a\ \beta b+b\ \beta a)+(a\ \beta b+b\ \beta a)\ \alpha a)$

$=T(a\alpha(a\ \beta b)+a\alpha(b\ \beta a)+(a\ \beta b)\ \alpha a+(b\ \beta a)\ \alpha a$

$=(a\alpha a\beta b+b\beta a\alpha a)+2T(a\alpha b\beta a)$

By comparing these two expression of W, we get

$2T(a\alpha b\beta a)= 2T(a)\ \alpha b\ \beta a$

Since M is 2-torsion free ,then

$T(a\alpha b\beta a)= T(a)\ \alpha b\ \beta a$………………………………(3)

(iii)In (3) replace a by a+c,to get

$T(a\alpha b\beta c+c\alpha b\beta a)=T(a)\ \alpha b\beta c+T(c)\ \alpha b\beta a$………………(4)

<u>Theorem 1.4:</u>- Let M be a 2-torsion free completely prime Γ-ring which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$, α , $\beta\in\Gamma$, and let $T:M\rightarrow M$ be an additive mapping which satisfies $T(a\alpha a)=T(a)\ \alpha a$,for all $a\in M$ and $\alpha\in\Gamma$,then $T(a\alpha b)=T(a)\ \alpha b$, for all $a ,b\in M$ and $\alpha\in\Gamma$ or M is commutative Γ-ring.

<u>Proof</u>:-By [Lemma 1.3,iii] ,we have

$T(a\alpha b\beta c+c\alpha b\beta a)=T(a)\ \alpha b\beta c+T(c)\ \alpha b\beta a$

Replace c by $a\alpha b$

$W=T(a\alpha b\beta(a\alpha b)+(a\alpha b)\alpha b\beta a)$

$=T(a)\ \alpha b\beta\ a\alpha b\ +T(a\alpha b)\ \alpha b\beta a$

On the other hand

$W= T((a\alpha b)\beta(a\alpha b)+a\alpha(b\alpha b)\beta a)$

$= T(a\alpha b)\beta a\alpha b+T(a)\alpha b\alpha b\beta a$

By comparing these two expression of W, we get

$T(a\alpha b)\beta(a\alpha b-b\ \alpha\ a) + T(a)\alpha b\ \beta\ (b\alpha a-a\ \alpha\ b)=0$

$T(a\alpha b)\beta(a\alpha b-b\ \alpha\ a) - T(a)\alpha b\ \beta\ (a\alpha b-b\ \alpha\ a)=0$

$(T(a\alpha b)- T(a)\alpha b)\beta(a\alpha b-b\ \alpha\ a)=0$………………….(5)

Since M is completely prime Γ-ring ,then

either $T(a\alpha b)- T(a)\alpha b=0$ or $a\alpha b-b\ \alpha\ a=0$

if $T(a\alpha b)- T(a)\alpha b=0$then $T(a\alpha b)= T(a)\alpha b$

and if $a\alpha b-b\ \alpha\ a=0$ for all $a, b\in M$ and $\alpha\in\Gamma$,then M is commutative Γ-ring◙

<u>Theorem 1.5:</u>- Let M be a 2-torsion free completely prime Γ-ring which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$, α , $\beta\in\Gamma$, and and let $T:M\rightarrow M$ be an additive mapping which satisfies $T(a\alpha a)= a\ \alpha\ T(a)$for all $a\in M$ and $\alpha\in\Gamma$,then $T(a\alpha b)= a\ \alpha\ T(b)$ for all $a,b\in M$ and $\alpha\in\Gamma$ or M is commutative Γ-ring.

<u>Proof</u>:- From[Lemma 1.3,iii],we have for all $a,b,c\in M$ and α , $\beta\in\Gamma$,

$T(a\ \alpha b\ \beta c+c\ \alpha b\ \beta a)=a\ \alpha b\ \beta T(c)+c\ \alpha b\ \beta T(a)$……………(6)

In (6) replace c by b α a, then
W= T(a α b β (b α a) +(b α a) α b β a)
=a α b β T(b α a)+ b α a β b α T(a)
on the other hand
W = T(a α (b β b) α a +(b α a) α (b β a))
=a α b β b α T(a)+b α a β T(b α a)
by comparing these two expression of W ,we get
a α b β (T(b α a)-b α T(a)) -b α a β (T(b α a)-b α T(a))=0
(a α b-b α a) β (T(b α a)-b α T(a))=0…………………………….(7)
since M is completely prime Γ -ring,then
either (T(b α a)-b α T(a))=0 $\Rightarrow$ T(b α a)=b α T(a)
or a α b-b α a=0 $\Rightarrow$ a α b=b α a $\Rightarrow$ M is commutative Γ -ring◙

<u>Corrolary 1.6</u>:- Every Jordan centralizer of 2-torsion free completely prime Γ -ring M which satisfy the condition x α y β z=x β y α z for all x,y,z $\in$ M , α , $\beta \in \Gamma$, is a centralizer on M.

<u>2-The second result</u>

In this section we again divided the proof in few lemmas.

<u>Lemma2.1</u>:- Let M be a semi-prime Γ -ring and D a derivation of M and a $\in$ M some fixed element.
(i)D(x) α D(y)=0 for all x,y $\in$ M, $\alpha \in \Gamma$ implies that D=0 on M
(ii) a α x-x α a $\in$ Z, for all x $\in$ M, $\alpha \in \Gamma$ implies that a $\in$ Z.

<u>Proof</u>:-
(i) since D(x) α D(y)=0 for all x,y $\in$ M, $\alpha \in \Gamma$.
and D(y α x)=D(y) α x+ y α D(x)
and so D(x) α D(y α x)=0,then
D(x) α D(y) α x+ D(x) α y α D(x)=0
since D(x) α D(y)=0,then
D(x) α y α D(x)=0 for all x,y $\in$ M, $\alpha \in \Gamma$
And since M be a semi-prime Γ -ring ,then
D(x)=0 for all x $\in$ M.
(ii)define D(x)= a α x-x α a
it is easy to see that D is derivation on M
since D(x) $\in$ Z for all x $\in$ M,we have
D(y) α x=x α D(y)…………………………………..(8)
Replace y by y α z in (8)
D(y α z) α x=x α D(y α z)
D(y) α z α x+y α D(z) α x=x α D(y) α z+x α y α D(z)
D(y) α (z α x-x α z)=D(z) α (x α y-y α x)
Now, take z=a, then it is easy to see that D(a)=0,so
D(y) α (a α x-x α a)=0
D(y) α D(x)=0,then from (i),we get D=0 and hence a $\in$ Z◙

<u>Lemma 2.2</u>:- Let M be a semi-prime Γ -ring and a $\in$ M some fixed element. If T(x)= a α x +x α a, for all x $\in$ M, $\alpha \in \Gamma$ is a Jordan centralizer ,then a $\in$ Z

<u>Proof</u>:-from [definition 1.2]
T(x α y+y α x)=T(x) α y+y α T(x)
Gives us
T(x α y)+T(y α x)=T(x) α y+y α T(x)
a α x α y +a α y α x+x α y α a+y α x α a=

$=(a\alpha x+x\alpha a)\alpha y+y\alpha(a\alpha x+x\alpha a)$

$=a\alpha x\alpha y+x\alpha a\alpha y+y\alpha a\alpha x+y\alpha x\alpha a$

Then

$a\alpha y\alpha x-x\alpha a\alpha y+x\alpha y\alpha a-y\alpha a\alpha x=0$

*$(a\alpha y-y\alpha a)\alpha x-x\alpha(a\alpha y-y\alpha a)=0$ **for all** $x,y\in M,\alpha\in\Gamma$*

***Then** $a\alpha y-y\alpha a\in Z$ **and so by [Lemma 2.1,ii],we get** $a\in Z$.*

Lemma 2.3:- Let M be a semi-prime Γ-ring ,then every Jordan centralizers of M maps from Z into Z.

Proof:-take any $c\in Z$ and denote a=t(c)

$2T(c\alpha x)=T(c\alpha x+x\alpha c)$

$=T(c)\alpha x+x\alpha T(c)=a\alpha x+x\alpha a$

Then $S(x)=2T(c\alpha x)$ is also a Jordan centralizer ,by[lemma 2.2],we get $a\in Z$.

Then $T(c)\in Z$◙

Lemma 2.4:- Let M be a semi-prime Γ-ring and a ,b $\in M$ two fixed elements.

If $a\alpha x=x\alpha b$ for all $x\in M$, $\alpha\in\Gamma$ then $a=b\in Z$.

Proof:-Since $x\alpha b=a\alpha x$

Replace x by $x\alpha y$

$x\alpha y\alpha b=a\alpha x\alpha y$

$x\alpha y\alpha b=x\alpha b\alpha y$

*$x\alpha(y\alpha b-b\alpha y)=0$**,and so***

$(y\alpha b-b\alpha y)x\alpha(y\alpha b-b\alpha y)=0$

Since M is semi-prime Γ-ring, then

$(y\alpha b-b\alpha y)=0$

$y\alpha b=b\alpha y$ for all $y\in M$, then $b\in Z$

since $a\alpha x=x\alpha b=b\alpha x$

it is easy to see that

$(a-b)\alpha x=0$ for all $x\in M$

and$(a-b)\alpha x\alpha(a-b)=0$ for all $x\in M$

again since M is semi-prime Γ-ring then $a-b=0\Rightarrow a=b\in Z$◙

Proposition 2.5:-everyJordan centeralizerof 2-torsion free completely prime Γ-ringM is a centralizer.

Proof:-Let T be a Jordan centeralizer,i.e

$T(x\alpha y+y\alpha x)=T(x)\alpha y+y\alpha T(x)=x\alpha T(y)+T(y)\alpha x$

If we replace y by $x\alpha y+y\alpha x$, then the left side

$W=T(x\alpha(x\alpha y+y\alpha x)+(x\alpha y+y\alpha x)\alpha x)$

$=T(x)\alpha(x\alpha y+y\alpha x)+(x\alpha y+y\alpha x)\alpha T(x)$

$=T(x)\alpha(x\alpha y)+T(x)\alpha y\alpha x+x\alpha y\alpha T(x)+y\alpha x\alpha T(x)$

and the right side

$W=x\alpha T(x\alpha y+y\alpha x)+T(x\alpha y+y\alpha x)\alpha x$

$=x\alpha T(x)\alpha y+x\alpha y\alpha T(x)+T(x)\alpha y\alpha x+y\alpha T(x)\alpha x$

Then

$T(x)\alpha x\alpha y+y\alpha x\alpha T(x)-x\alpha T(x)\alpha y-y\alpha T(x)\alpha x=0$

$(T(x)\alpha x-x\alpha T(x))\alpha y+y\alpha(x\alpha T(x)-T(x)\alpha x)=0$

Then

$(T(x)\alpha x-x\alpha T(x))\alpha y=y\alpha(T(x)\alpha x-x\alpha T(x))$ for all $x,y\in M$, $\alpha\in\Gamma$.

And so $(T(x)\alpha x-x\alpha T(x))\in Z$

then we must prove that

$T(x)\alpha x-x\alpha T(x)=0$

Take any $c\in Z$

$2T(c\alpha x)=T(c\alpha x+x\alpha c)$

$= T(c)\alpha x+x\alpha T(c)$

$=2T(x)\ \alpha c$

Using[Lemma 2.3]and since M is 2-torsion free Γ *- ring*

$T(c\alpha x)= T(x)\ \alpha c= T(c)\ \alpha x$

$(T(x)\ \alpha x\text{-}x\alpha\ T(x))\ \alpha c= T(x)\ \alpha x\ \alpha c\text{-}x\alpha T(x)\alpha c$

$=T(c)\ \alpha x\alpha x\text{-}x\alpha T(c)\ \alpha x=0$

then $(T(x)\ \alpha x\text{-}x\alpha\ T(x))\ \alpha c\alpha (T(x)\ \alpha x\text{-}x\alpha\ T(x)) =0$

since M is semi-prime Γ *- ring ,then* $T(x)\ \alpha x\text{-}x\alpha\ T(x) =0$

$2T(x\alpha x)=T(x\alpha x+x\alpha x) =T(x)\ \alpha x+x\alpha\ T(x)$

$=2T(x)\ \alpha x\ =2x\alpha T(x)$

Since M is 2-torsion free ,then

$T(x\alpha x)= T(x)\ \alpha x=x\alpha T(x)$

And so by [Theorem 1.4,Theorem1.5],we get the result.

3-JORDAN CENTRALIZERS ON SOME GAMMA RING

***Theorem 3.1:**- Let M be a 2-torsion free* Γ *-ring which satisfy the condition* $x\alpha y\beta z=x\beta y\alpha z$ *for all* $x,y,z\in M$ *,* $\alpha,\ \beta\in\Gamma$ *and has a commutator right non-zero divisor and let T:M→M be an additive mapping which satisfies* $T(a\alpha a)= T(a)\ \alpha\ a$ *for all* $a\in M$ *and* $\alpha\in\Gamma$*,then* $T(a\alpha b)= T(a)\ \alpha\ b$ *for all* $a,b\in M$ *and* $\alpha\in\Gamma$*.*

***Proof:**- from (5),we have*

$(T(a\alpha b)\text{-}\ T(a)\alpha b\ )\beta (a\alpha b\text{-}b\ \alpha\ a)=0$

if we suppose that

$\delta(a,b)= T(a\alpha b)\text{-}\ T(a)\alpha b$ *and* $[a,b]= a\alpha b\text{-}b\ \alpha\ a$

then $\delta(a,b)\ \beta\ [a,b]=0$ *for all* $a,b\in M$ *and* $\alpha,\beta\in\Gamma$ *……………(9)*

Since M has a commutator right non-zero divisor ,then $\exists x,y\in M,\ \alpha\in\Gamma$ *such that if for every* $c\in M,\ \beta\in\Gamma$

$c\beta[x,y]=0\Rightarrow c=0$

by (9),we have $\delta(x,y)\ \beta\ [x,y]=0$ *and so* $\delta(x,y)=0$*………………(10)*

replace a by a+x

$\delta(a+x,b)\ \beta\ [a+x,b]=0$ *and so by (9) and (10)*

$\delta(x,b)\ \beta\ [a,b]+ \delta(a,b)\ \beta\ [x,b]=0$*……………..(11)*

Now replace b by b+y

$\delta(x,b+y)\ \beta\ [a,b+y]+ \delta(a,b+y)\ \beta\ [x,b+y]=0$

and so by (10) and (11),we get

$\delta(x,b)\ \beta\ [a,y]+ \delta(a,y)\ \beta\ [x,b]+\delta(a,b)\ \beta\ [x,y]+\delta(a,y)\ \beta\ [x,y]=0$

$\delta(a,b)\ \beta\ [x,y]+ \delta(a,y)\ \beta\ [x,y]=0$

by (11),we get

$\delta(a,b)\ \beta\ [x,y]\text{-}\ \delta(x,y)\ \beta\ [a,y]=0$

then

$\delta(a,b)\ \beta\ [x,y]=0$*,and so* $\delta(a,b) =0$ *for all* $a,b\in M$ *and* $\alpha\in\Gamma$

$T(a\alpha b)= T(a)\alpha b\Rightarrow T$ *is left centralizer of M.*

***Theorem 3.2:**- Let M be a 2-torsion free* Γ *-ring which satisfy the condition* $x\alpha y\beta z=x\beta y\alpha z$ *for all* $x,y,z\in M$ *,* $\alpha,\ \beta\in\Gamma$ *and has a commutator left non-zero divisor and let T:M→M be an additive mapping which satisfies*

$T(a\alpha a)= a \alpha T(a)$ for all $a\in M$ and $\alpha \in \Gamma$,then $T(a\alpha b)= a \alpha T(b)$ for all $a,b\in M$ and $\alpha \in \Gamma$.

<u>Proof</u>:- From[Lemma 1.3,iii],we have

$T(a \alpha b \beta c+c \alpha b \beta a)=a \alpha b \beta T(c)+c \alpha b \beta T(a)$……………(12)

In (12) replace c by $b \alpha a$,then

$W= T(a \alpha b \beta (b \alpha a) +(b \alpha a) \alpha b \beta a)$

$=a\alpha b\beta T(b \alpha a)+ b \alpha a \beta b \alpha T(a)$

on the other hand

$W= T(a \alpha (b \beta b) \alpha a +(b \alpha a) \alpha (b \beta a))$

$=a\alpha b\beta b\alpha T(a)+b\alpha a\beta T(b\alpha a)$

by comparing these two expression of W ,we get

$a\alpha b\beta (T(b\alpha a)-b\alpha T(a)) -b\alpha a\beta (T(b\alpha a)-b\alpha T(a))=0$

then if we suppose $B(b,a)= (T(b\alpha a)-b\alpha T(a))$

$[a,b] \beta B(b,a)= [a,b]\beta B(a,b)=0$ for all $a,b\in M , \alpha , \beta \in \Gamma$ …………………….(13)

Since M has a commutator left non-zero divisor then $\exists x,y\in M, \alpha \in \Gamma$ such that if for every $c\in M, \beta \in \Gamma$, $[x,y] \beta c=0 \Rightarrow c=0$

then by (13),we have

$[x,y] \beta B(x,y)=0 \Rightarrow B(x,y)=0$…………………(14)

in (13) replace a by a+x

$[a+x,b] \beta B(a+x,b)=0$

then by (13)

$[x,y] \beta B(a,b)+[a,b] \beta B(x,b)=0$…………………(15)

Now replace b by b+y

$[x,b+y] \beta B(a,b+y)+[a,b+y] \beta B(x,b+y)=0$

then by using (14) and (15),we get

$[x,y] \beta B(a,b)=0$

and since [x,y] is a commutator left non-zero divisor then

$B(a,b)=0 \Rightarrow T(a\alpha b)=a\alpha T(b)$ which is mean that T is right centralizer

<u>Corrolary3.7</u>:- Let M be a 2-torsion free Γ-ring which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M , \alpha , \beta \in \Gamma$, has a commutator non-zero divisor and let $T:M\rightarrow M$ be a Jordan centralizer then T is centralizer◙

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<u>References</u>

[1]W.E.Barnes"On the Γ-ring of Nabusawa",PacificJ.Math.,18(1966),411-422.

[2]Y.Ceven,"Jordan left derivations on completely prime Gamma rings",Fenbilimleri Dergisi(2002)cilt 23say12.

[3]W.Cortes and C.Haetinger"On Lie ideal and Jordan left centralizers of 2-torsion free rings",URL:http://ensino.univates.br/~Chaet.

[4]I.N.Herstein"Topics in ring theory "University of Chicago press,1969.

[5]N. Nabusawa"On a generalization of the ring theory",OsakaJ.Math.,1(1964).

[6] J.Vukam"An identity related to centralizers in semi-prime rings", Comment .Math.Univ.Carolinae,40,3(1999)447-456.

[7]B.Zalar"On centralizers of semi-prime rings" Comment.Math.Univ.Carolinae 32(1991)609-614.