On Symmetric Left Bi-(σ,τ)-Derivation on Gamma Rings

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Abstract

In this paper ,we introduce the concept of symmetric left bi- (σ,τ)- derivation on gamma rings and study this concept on completely prime Gamma ring also we study it if acts as homomorphism and we show that if D nonzero Jordan left bi-(σ,σ)derivation on 2-torsion free completely prime gamma ring M then M is commutative or D be left bi-(σ,σ) derivation .
Key words: Gamma ring, Prime ring, Derivation ,Left derivation .

الخلاصة

في هذا البحث ,قدمنا مفهوم الاشتقاق اليساري الثنائي المتناظر-(σ,τ)على الحلقات من نوع كاما ودرسنا هذا المفهوم على الحلقات الاولية الكاملة من نوع كاما وكذلك درسناه اذا كان يعمل كتشاكل واثبتنا اذا كان Dاشتقاق جوردان اليساري الثنائي-(σ,σ)غير صفري على الحلقة الاولية الكاملة وطليقة الالتواء من النمط الثاني من نوع كاما Mفانه اما أن تكونM حلقة ابدالية او يكون Dاشتقاق يساري ثنائي-(σ,σ).
الكلمات المفتاحية: حلقة كاما، الحلقة الاولية، الاشتقاق، اشتقاق يساري.

1-Introduction

Throughout this paper ,M will represent an associative Γ- ring and Z will be its center. Let R be a ring and let x,y$\in$R the commutator xy-yx will be denoted by [x,y],we will also use the identities[xy,z]=x[y,z]+[x,z]y and [x,yz]=y[x,z]+[x,y]z .Let A be anon empty subset of R then a map f:R$\to$R is said to be commuting (resp.,Centralizing) on A if [f(x),x]=0(resp .,[f(x),x] $\in$Z) for all x$\in$A. We recall that R is semi-prime if a Ra=0 implies that a=0 and R is prime if a R b=0 implies that a=0 or b=0.it is easy to see that every prime ring be semi-prime but the converse is not true.

An additive map d: R$\to$R is called a derivation(resp.left derivation) if d(xy)=d(x)y+xd(y)(resp d(xy)=xd(y)+yd(x)) for all x,y$\in$R. A bi-derivation mean that a bi-additive map D:R$\times$R $\to$R(D is additive in both arguments) which satisfy the relations

D(xy,z)=D(x,z)y+xD(y,z)

D(x,yz)=D(x,y)z+yD(x,z) $\forall$ x,y $\in$R.

Let D be symmetric that is D(x,y)=D(y,x) $\forall$ x,y $\in$R. the map T:R$\to$R defined by T(x)=D(x,x) $\forall$ x $\in$R is called the trace of D. Many paper study the concept of symmetric bi-derivation such as the paper of [Mohammed Ashraf 1999],[Vukman. J 1989],[Vukman. J 1990].In [Breaser 2004,Theorem3.5] prove that ,if R is a non-commutative 2-torsion free prime ring and D:R$\times$ R $\to$R is a symmetric bi-derivation then D=0.

Let R be a ring then the bi-additive map D:R$\times$R $\to$R is called a left bi-derivation if D(xy,z)= x D(y,z)+y D(x,z) and D(x,yz)= y D(x,z) +z D(x,y) $\forall$ x,y,z $\in$R.

We should mentioned the reader that the notion of left bi-derivation was introduced by authors in [Nagy and others 2010,Definition 3.1]and see that a prime ring of characteristic $\neq$2,3that admit anon-zero Jordan left bi-derivation is commutative also if R is semi prime ring every bi-left derivation be an ordinary bi-derivationthat maps from R into its center.

Let M and Γ be additive abelian groups, M is called a Γ -ring if for any x,y,z $\in$M and α , β $\in$ Γ,the following conditions are satisfied

(1)x α y $\in$M

(2)(x+y) α z=x α z+ y α z

x(α + β)z=x α z+x β z

x α (y+z)=x α y+x α z

(3)(x α y) β z=x α (y β z)

The notion of Γ -ring was introduced by [Nabusawa, 1964] and generalized by [Barnes, 1966], many properties of Γ -ring were obtained by many research such as Ceven, 2002. Let A,B be subsets of a Γ-ring M and Λ be a subset of Γ we denote AΛB the subset of M consisting of all finite sum of the form $\sum a_i\lambda_i b_i$ where $a_i \in A, b_i \in B$ and $\lambda_i \in \Gamma$.A right ideal(resp.,left ideal) of aΓ-ring M is an additive subgroup Iof M such that IΓM$\subset$I(resp.,MΓI$\subset$I) . If I is a right and left ideal inM,then we say that I is an ideal .M is called a 2-torsion free if 2x=0 implies x=0, for all x$\in$M.AΓ-ringM is called prime if aΓMΓb=0 implies a=0 or b=0 and M is called completely prime if a Γb=0 implies a=0 or b=0(a,b $\in$M),Since a Γb Γa Γb $\subset$aΓM Γb, then every completely prime Γ-ring is prime. AΓ-ring M is called semi-prime if aΓMΓa=0 implies a=0and M is called completely semi-prime if a Γa=0 implies a=0(a$\in$M)

In this paper ,we give a new definition which is the definition of Symmetric bi-left (σ,τ) derivation on Γ-ring and study this concept on completely prime Γ-ring in section Two and in section three we study this concept if it is acts as left or right homomorphism also we introduce the concept of Jordan left bi- (σ ,σ) derivation on Γ-ring and study it on completely prime Γ-ring .

2-Symmetric left bi -(σ,τ)derivation on completely prime Γ-rings.

In this section, we introduce the definition of symmetric left bi -(σ,τ) derivation on gamma rings as follows

Definition 2.1:-

An additive mapping D: $M \times M \rightarrow M$ is called a symmetric left bi (σ,τ)-derivation if there exist $\sigma , \tau : M \rightarrow M$ such that

D(xα y,z)= τ (x) α D(y,z)+ σ (y) α D(x,z)

And

D(x,yα z)= τ (y) α D(x,z)+ σ (z) α D(x,y) $\forall$ x,y,z$\in$M

Lemma 2.2:-

Let M be a2-torsion free completely primeΓ- ring and I a non-zero left (or right ideal of M).Let D: $M \times M \rightarrow M$ be symmetric left bi -(σ,τ)-derivation $\sigma , \tau : M \rightarrow M$, σ (I)=I, and d the trace of D .Suppose that d(x)=0 $\forall$ x $\in$I. Then d=0 and so D=0.

Proof:-Since we have d(x)=0 $\forall$ x $\in$I…………..(2.1)

By replacing x by x+y and use (1)

We get 2D(x,y)=0 $\forall$ x,y $\in$I

Since M is 2-torsion free,

We get D(x,y)=0 $\forall$ x,y $\in$I…………………….(2.2)

Replace y by ry, r$\in$M

$D(x,r\alpha y)=0$

$\tau(r)\ \alpha\ D(x,y)+\ \sigma(y)\ \alpha\ D(x,r)=0$

By (2.2),we get

$\sigma(y)\ \alpha\ D(x,r)=0$

$\sigma(I)\ \alpha\ D(x,r)=0\ \forall\ x,y\in I,\ \forall\ r\in M$

Since $I\neq 0$ and σ is automorphism then $\sigma(I)\neq 0$ and since M is completely prime Γ-ring then

$D(x,r)=0$…………..(2.3)

Replace x by $x\alpha r$

$D(x\alpha r,r)=0$

$D(x\alpha r,r)=\ \tau(x)\ \alpha\ D(r,r)+\ \sigma(r)\ \alpha\ D(x,r)=0$

By (2.3),we get

$\tau(x)\ \alpha\ D(r,r)=0$

Since M is completely prime Γ-ring

Then either $\tau(x)=0$ or $d(r)=0$

Since $I\neq 0$ and τ is automorphism then $\tau(I)\neq 0$ is an ideal of M

Then d (r) =0

$d(r)=0\ \forall\ r\in M$

Now replace r by r+s ,$s\in M$

$d(r+s)=d(r)+d(s)+2D(r,s)$

And since $d(r)=0\ \forall\ r\in M$

Then2D(r,s)=0

Since M is 2-torsion free ,we get

$D(r,s)=0\ \forall\ r,s\in M$ .which is D=0.

Theorem 2.3 :-Let M be a 2-torsion free completely prime Γ- ring and I a non-zero (non-commutitive)ideal of M and Let D: $M\times M\to M$ be symmetric left bi -(σ,τ)-derivation ,$\sigma,\tau:M\to M$,$\sigma(I)=\ \tau(I)=I$, and d the trace of D .

If $[x,d(x)]_\alpha=0\ \forall\ x\in I$ Then D=0.

Proof:-

Since $[x,d(x)]_\alpha=0\ \forall\ x\in I$……………… (2.4)

Linearizing x by x+ y ,to get

$x\alpha d(x)+x\alpha d(y)+2x\alpha D(x,y)+y\alpha d(x)+y\alpha d(y)+2y\alpha D(x,y)-d(x)\ \alpha\ x-d(x)\ \alpha\ y-d(y)\ \alpha\ x-d(y)\ \alpha\ y-2D(x,y)\ \alpha\ x-2D(x,y)\ \alpha\ y=0$

Then

$[x,d(y)]_\alpha+[y,d(x)]_\alpha+2[x,D(x,y)]_\alpha+2[y,D(x,y)]_\alpha=0$…………(2.5)

Replace x by –x in (2.5) to get

$-[x,d(y)]_\alpha-[y,d(x)]_\alpha+2[x,D(x,y)]_\alpha-2[y,D(x,y)]_\alpha=0$ …………(2.6)

By comparing (2.5) and (2.6),to get

$4[x,D(x,y)]_\alpha=0$

Since M is 2-torsion free

$[x,D(x,y)]_\alpha=0$……………………………..(2.7)

Replace y by $y\beta z$,

$[x,D(x,y\beta z)]_\alpha=0$

$[x,\ \tau(y)\ \beta D(x,z)+\ \sigma(z)\ \beta D(x,y)]_\alpha=0$

$[x,\ \tau(y)\ \beta D(x,z)]_\alpha+[x,\sigma(z)\ \beta D(x,y)]_\alpha=0$

$[x, \tau(y)]_\alpha \beta D(x,z) + \tau(y) \beta [x,D(x,z)]_\alpha + [x,\sigma(z)]_\alpha \beta D(x,y) + \sigma(z) \beta [x,D(x,y)]_\alpha = 0$

Then by (2.7),we get

$[x, \tau(y)]_\alpha \beta D(x,z) + [x,\sigma(z)]_\alpha \beta D(x,y) = 0$

By replacing z by y and use the fact σ (I)=T(I)=I,we get

$2[x, \sigma(y)]_\alpha \beta D(x,y) = 0$

Since R is 2-torsion free ,

$[x, \sigma(y)]_\alpha \beta D(x,y) = 0$

Since M is completely prime Γ-ring, then we get

Either $[x, \sigma(y)]_\alpha = 0$ or $D(x,y)=0$

If $[x, \sigma(y)]_\alpha = 0$ then $[x,y]_\alpha = 0 \; \forall x,y \in I$ then I is commutative which is contradiction with I is non-commutative then $D(x,y)=0 \; \forall x,y \in I$ and so D=0 on M. by proof of [lemma 2.2]

Theorem2.4 :- Let M be a 2-torsion free and 3-torsion free prime Γ- ring and Let D: $M \times M \to M$ be symmetric left bi -(σ,τ)-derivation $\sigma, \tau : M \to M$, σ (x)= τ (x) and d the trace of D .if $[x, d(x)]_\alpha \in Z(R) \; \forall x \in M$.Then D=0.

Proof:-

Since we have $[x, d(x)]_\alpha \in Z(M) \; \forall x \in M$…………………..(2.8)

Replace x by x+y

$[d(x),y]_\alpha + [d(y),x]_\alpha + 2[D(x,y),x]_\alpha + 2[D(x,y),y]_\alpha \in Z(M)$………..(2.9)

Replace x by –x ,to get

$[d(x),y]_\alpha - [d(y),x]_\alpha + 2[D(x,y),x]_\alpha - 2[D(x,y),y]_\alpha \in Z(M)$…………..(2.10)

By comparing (2.9) and (2.10)

$2[d(x),y] + 4[D(x,y),x] \in Z(M)$

Since R is 2-torsion free,

$[d(x),y]_\alpha + 2[D(x,y),x]_\alpha \in Z(M)$

Replace y by $x \beta x$,to get

$[d(x), x\beta x]_\alpha + 2[D(x, x\beta x),x]_\alpha \in Z(M)$

Since D is a symmetric left bi -(σ,τ)- derivation

Then

$[d(x), x\beta x]_\alpha + 2[\sigma(x) \beta D(x,x) + \tau(x) \beta D(x,x),x]_\alpha \in Z(M)$

$[d(x), x\beta x]_\alpha + 2[\sigma(x) \beta d(x) + \tau(x) \beta d(x),x]_\alpha \in Z(M)$

$[d(x), x\beta x]_\alpha + 2[\sigma(x) \beta d(x),x]_\alpha + 2[\tau(x) \beta d(x),x]_\alpha \in Z(M)$

$[d(x),x]_\alpha \beta x + x\beta [d(x),x]_\alpha + 2[\sigma(x),x]_\alpha \beta d(x) + 2\sigma(x) \beta [d(x),x]_\alpha + 2[\tau(x),x]_\alpha \beta d(x) + 2\tau(x) \beta [d(x),x]_\alpha \in Z(M)$

Since $[d(x),x]_\alpha \in Z(M)$and σ (x)= τ (x)=x

$2x\beta [d(x),x]_\alpha + 4[\sigma(x),x]_\alpha \beta d(x) + 4\sigma(x) \beta [d(x),x]_\alpha \in Z(M)$

Then $2x\beta [d(x),x]_\alpha + 4[\sigma(x) \beta d(x),x]_\alpha \in Z(M)$

If σ (x)=x then

$6x\beta [d(x),x]_\alpha \in Z(M)$……………………….(2.11)

Since M is 2-torsion and 3-torsion free ,then by (2. 11) and (2.8),we get

$[x,y]_\lambda\ \beta\ [d(x),x]_\alpha=0$
If $x \notin Z(M)$ then $[x,y]_\lambda \neq 0$
Since M is completely prime Γ-ring, then
$[d(x),x]_\alpha=0$ and so by [Theorem 2.3]
We get d=0 which leads to D=0.

3-Symmetric Left BI- (σ,τ)-Derivation acts as homomorphism.

Definition3.1 :-Let M be a Γ- ring and I a non-zero left (resp.right)ideal of M. we shall say that a mapping D: $M \times M \to M$ acts as a left (resp.right)a (σ,τ)-homomorphism on I if (D(r α x,y)= σ(x) α D(x,y) and D(x,rα y)= σ(r) α D(x,y))(resp.D(xα r,y)=D(x,y) α α τ(r) and D(x,yα r)=D(x,y) α τ(r) $\forall$ x,y $\in$I and r $\in$M.

Let S be a set ,L (S) (resp.r (S)) will denote the left (resp.right) annihilator of S.

Theorem3.2 :-Let M be a ring and I a non –zero left (resp.right)ideal of R. such that r (I)=0 (resp. L (I)=o).Let D: $M \times M \to M$ be a symmetric left bi -(σ,τ) derivation if D acts as a left (resp. right)-homomorphism on I then D=0.

Proof:- Suppose that I is a left ideal such that L (I)=o and D acts as a left homomorphism on I then

σ(r) α D(x,y)=D(rα x,y)
= τ(x) α D(r,y)+ σ(r) α D(x,y)
Then T(x) α D(r,y)=0 $\forall$ x,y $\in$I and r $\in$M, $\alpha \in \Gamma$.
Then D(r,y) $\in$ r (I)=0
Then D(r,y)=0 $\forall$ y $\in$I and r $\in$M.
0=D(sα x,r)= τ(x) α D(s,r)+ σ(s) α D(x,r)=0
Then τ(x)D(s,r)=0 $\forall$ x $\in$I and r,s $\in$M.
And since τ is automorphism then τ(x) is an ideal
Then D(s,r) $\in$ r (I)=0
Then D(s,r)=0 $\forall$ s,r$\in$M,and so D=0.
And by the same way for the right ideal and right homomorphism.

We should mentioned the reader that the above theorem be true for any Γ- ring M then it is easy to see that it is true for prime Γ- ring and semi-prime Γ- ring .

4- Jordan Left Bi-(σ,σ)Derivation on Gamma Ring

Definition4.1_:- Let M be a Γ-ring then the bi-additive mapping D:$M \times M \to M$ is called a Jordan left bi-(σ,τ) derivation if there exist σ, τ : M→M

D(xα x,y)= τ(x) α D(x,y)+ σ(x) α D(x,y)

and

D(x,yα y)= τ(y) α D(x,y)+ σ(y) α D(x,y) $\forall$ x,y$\in$M.

It is easy to see that every left bi-(σ,τ) derivation be Jordan left bi-(σ,τ) derivation but the converse is not true. In this section we study this problem.

Lemma 4.2:- Let M be a Γ-ring, D: $M \times M \to M$ be a Jordan left bi-(σ,σ) derivation then the following statements hold:

(i)D(xα z+zα x,y)=2 σ (x)α D(z ,y)+ 2σ (z)α D(x ,y)

Especially if M is 2-torsion free and $x\alpha y\beta z=x\beta y\alpha z$, for all $x,y,z\in M$ and $\alpha,\beta\in\Gamma$, then

(ii) $D(x\beta z\alpha x\ ,\ y)=\sigma(x)\alpha\sigma(x)\beta D(z,y)+3\sigma(x)\alpha\sigma(z)\beta D(x,y)-\sigma(z)\beta\sigma(x)\alpha D(x,y)$

(iii) $D(x\alpha z\beta w+w\alpha z\beta x,y)=\sigma(x)\alpha\sigma(w)\beta D(z,y)+\sigma(w)\alpha\sigma(x)\beta D(z,y)+3\sigma(x)\alpha\sigma(z)\beta D(w,y)+3\sigma(w)\alpha\sigma(z)\beta D(x,y)-\sigma(z)\beta\sigma(w)\alpha D(x,y)-\sigma(z)\beta\sigma(x)\alpha D(w,y)$

(iv) $[\sigma(x),\sigma(z)]_\alpha\beta D(x\alpha z,y)=\sigma(x)\alpha[\sigma(x),\sigma(z)]_\alpha\beta D(z,y)+\sigma(z)\beta[\sigma(x),\sigma(z)]_\alpha\alpha D(x,y)$

(v) $[\sigma(x),\sigma(z)]_\alpha\beta\sigma(x)\alpha D(x,y)=\sigma(x)\alpha[\sigma(x),\sigma(z)]_\alpha\beta D(x,y)$

Proof :-(i)Since D is a Jordan left bi-(σ,σ) derivation then

$D(x\alpha x,y)=2\sigma(x)\alpha D(x,y)$,for all $x,y\in M$ and $\alpha\in\Gamma$,……………(4.1)
by linearizing (4.1),we get

$D(x\alpha z+z\alpha x,y)=2\sigma(x)\alpha D(z,y)+2\sigma(z)\alpha D(x,y)$….(4.2)

(ii)In (4.2) replace z by $x\beta z+z\beta x$, $\beta\in\Gamma$.

$W=D(x\alpha(x\beta z+z\beta x)+(x\beta z+z\beta x)\alpha x,y)$

$=2\sigma(x)\alpha D(x\beta z+z\beta x,y)+2\sigma(x\beta z+z\beta x)\alpha D(x,y)$

$=2\sigma(x)\alpha(2\sigma(x)\beta D(z,y)+2\sigma(z)\beta D(x,y))+2\sigma(x\beta z+z\beta x)\alpha D(x,y)$

$=4\sigma(x)\alpha\sigma(x)\beta D(z,y)+6\sigma(x)\alpha\sigma(z)\beta D(x,y))+$

$+2\sigma(z)\beta\sigma(x)\alpha D(x,y)$

On the other hand,

$W=D(x\alpha(x\beta z+z\beta x)+(x\beta z+z\beta x)\alpha x,y)$

$=D(x\alpha x\beta z+2x\beta z\alpha x+z\beta x\alpha x,y)$

$=D((x\alpha x)\beta z+z\beta(x\alpha x),y)+2D(x\beta z\alpha x,y)$

$=2\sigma(x\alpha x)\beta D(z,y)+2\sigma(z)\beta D(x\alpha x,y)+2D(x\beta z\alpha x,y)$

$=2\sigma(x)\alpha\sigma(x)\beta D(z,y)+4\sigma(z)\beta\sigma(x)\alpha D(x,y)+2D(x\beta z\alpha x,y)$

By comparing these two expression of W ,we get

$2D(x\beta z\alpha x,y)=2\sigma(x)\alpha\sigma(x)\beta D(z,y)+6\sigma(x)\alpha\sigma(z)\beta D(x,y)-$

$2\sigma(z)\beta\sigma(x)\alpha D(x,y)$

And since M is 2-torsion free ,then

$D(x\beta z\alpha x\ ,\ y)=\ \sigma\ (x)\alpha\ \sigma(\ x)\ \beta D(z\ ,y)+3\sigma\ (x)\alpha\ \sigma\ (z)\beta D(x\ ,y)-\ \sigma\ (\ z)\beta\ \sigma(x)\alpha\ D(x\ ,y)$ ………(4.3)

(iii) by linearizing (4.3) on x ,we get

$Y=D((x+w)\ \alpha z\beta(x+w)),y)$

$=\sigma\ (x+w)\alpha\ \sigma(\ x+w)\ \beta D(z\ ,y)+3\sigma\ (x+w)\alpha\ \sigma\ (z)\beta D(x+w\ ,y)$

$-\ \sigma\ (\ z)\beta\ \ \sigma(\ x+w)\alpha\ D(x+w\ ,y)$

$=\sigma\ (x)\ \alpha\ \sigma\ (x)\ \beta\ D(z\ ,y)+\ \sigma\ (w)\ \alpha\ \sigma\ (w)\ \beta D(z\ ,y)+$

$\sigma\ (x)\ \alpha\ \sigma\ (w)\ \beta D(z\ ,y)+\ \sigma\ (w)\ \alpha\ \sigma\ (x)\ \beta D(z\ ,y)+$

$3\sigma\ (x)\ \alpha\ \sigma\ (z)\ \beta D(x\ ,y)+3\sigma\ (x)\ \alpha\ \sigma\ (z)\ \beta D(w\ ,y)+3\sigma\ (w)\ \alpha\ \sigma\ (z)\ \beta D(x\ ,y)+3\sigma\ (w)\ \alpha\ \sigma\ (z)\ \beta D(w\ ,y)-\ \sigma\ (z)\ \beta\ \sigma\ (x)\ \alpha D(x\ ,y)-\ \sigma\ (z)\ \beta\ \sigma\ (w)\ \alpha D(x\ ,y)-\ \sigma\ (z)\ \beta\ \sigma\ (x)\ \alpha D(w\ ,y)-\ \sigma\ (z)\ \beta\ \sigma\ (w)\ \alpha D(w\ ,y)$

On the other hand

$Y=\ D((x+w)\ \alpha z\beta(x+w)),y)$

$=D(x\ \alpha z\ \beta x,y)+D(x\ \alpha z\ \beta w+w\ \alpha z\ \beta x,y)+D(w\ \alpha z\ \beta w,y)$

By comparing these two expression of Y ,we get

$D(x\ \alpha z\ \beta w+w\ \alpha z\ \beta x,y)=\ \sigma\ (x)\ \alpha\ \sigma\ (w)\ \beta D(z\ ,y)+\ \sigma\ (w)\ \alpha\ \sigma\ (x)\ \beta D(z\ ,y)+3\sigma\ (x)\ \alpha\ \sigma\ (z)\ \beta D(w\ ,y)+3\sigma\ (w)\ \alpha\ \sigma\ (z)\ \beta D(x\ ,y)-\ \sigma\ (z)\ \beta\ \sigma\ (w)\ \alpha D(x\ ,y)-\ \sigma\ (z)\ \beta\ \sigma\ (x)\ \alpha D(w\ ,y)$………………..(4.4)

(iv)Now replace w by $x\alpha z$ in (4.4)

$Y=\ D(x\ \alpha z\ \beta(\ x\alpha z\ )+(\ x\alpha z)\ \alpha z\ \beta x,y)=\ \sigma\ (x)\ \alpha\ \sigma\ (x\alpha z)\ \beta D(z\ ,y)+\ \sigma\ (x\alpha z)\ \alpha\ \sigma\ (x)\ \beta D(z\ ,y)+3\sigma\ (x)\ \alpha\ \sigma\ (z)\ \beta D(x\alpha z\ ,y)+3\sigma\ (x\alpha z)\ \alpha\ \sigma\ (z)\ \beta D(x\ ,y)-\ \sigma\ (z)\ \beta\ \sigma\ (x\alpha z)\ \alpha D(x\ ,y)-\ \sigma\ (z)\ \beta\ \sigma\ (x)\ \alpha D(x\alpha z\ ,y)$

On the other hand

$Y=\ D((x\ \alpha z)\ \beta(\ x\alpha z\ )+\ x\alpha(z\ \alpha z\ )\beta x\ ,y)$

$=2\ \sigma\ (x\alpha z)\ \beta\ D(x\alpha z\ ,y)+\ \sigma\ (x)\ \alpha\ \ \sigma\ (x)\ \beta\ D(z\ \alpha z\ ,y)+3\ \sigma\ (x)\ \alpha\ \ \sigma\ (z\ \alpha z)\ \beta\ D(x\ ,y)-\ \sigma\ (z\ \beta z)\ \alpha\ \ \sigma\ (x)\ \alpha\ \ D(x\ ,y)$

$=2\ \sigma\ (x)\ \alpha\ \ \sigma\ (z)\ \beta\ D(x\alpha z\ ,y)+2\ \sigma\ (x)\ \alpha\ \ \sigma\ (x)\ \beta\ \ \sigma\ (z)\ \alpha\ \ D(z\ ,y)+3\ \sigma\ (x)\ \alpha\ \sigma\ (z)\ \alpha\ \ \sigma\ (z)\ \beta\ D(x\ ,y)-\ \sigma\ (z)\ \beta\ \ \sigma\ (z)\ \alpha\ \ \sigma\ (x)\ \alpha\ \ D(x\ ,y)$

By comparing these two expression of Y ,we get

$[\sigma(x), \sigma(z)]_\alpha \beta D(x\alpha z,y) - \sigma(z)\alpha\sigma(x)\alpha\sigma(z)\alpha D(x,y) + \sigma(x)\alpha\sigma(z)\alpha\sigma(x)\beta D(z,y) - \sigma(x)\alpha\sigma(x)\beta\sigma(z)\alpha D(z,y) + \sigma(z)\beta\sigma(z)\alpha\sigma(x)\alpha D(x,y) = 0$

Then

$[\sigma(x), \sigma(z)]_\alpha \beta D(x\alpha z,y) = \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(z,y) + \sigma(z)\beta[\sigma(x), \sigma(z)]_\alpha \alpha D(x,y)$…………..(4.5)

(v)Now replace z by x+z in (4.5)

$[\sigma(x), \sigma(x+z)]_\alpha \beta D(x\alpha(x+z),y) = \sigma(x)\alpha[\sigma(x), \sigma(x+z)]_\alpha \beta D(x+z,y) + \sigma(x+z)\beta[\sigma(x), \sigma(x+z)]_\alpha \alpha D(x,y)$

$= \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(x,y) + \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \alpha D(z,y) + \sigma(x)\beta[\sigma(x), \sigma(z)]_\alpha \alpha D(x,y) + \sigma(z)\beta[\sigma(x), \sigma(z)]_\alpha \alpha D(x,y)$

On the other hand

$Y = [\sigma(x), \sigma(z)]_\alpha \beta D(x\alpha x,y) + [\sigma(x), \sigma(z)]_\alpha \beta D(x\alpha z,y)$

$= 2[\sigma(x), \sigma(z)]_\alpha \beta\sigma(x)\alpha D(x,y) + [\sigma(x), \sigma(z)]_\alpha \beta D(x\alpha z,y)$

$= \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(x,y) + \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(z,y) + \sigma(x)\beta[\sigma(x), \sigma(z)]_\alpha \alpha D(x,y) + \sigma(z)\beta[\sigma(x), \sigma(z)]_\alpha \alpha D(x,y)$

By (4.5)and by comparing these two expression of Y,we get

$2[\sigma(x), \sigma(z)]_\alpha \beta\sigma(x)\alpha D(x,y) + \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(z,y) + \sigma(z)\beta[\sigma(x), \sigma(z)]_\alpha \alpha D(x,y) = \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(x,y) + \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(z,y) + \sigma(x)\beta[\sigma(x), \sigma(z)]_\alpha \alpha D(x,y) + \sigma(z)\beta[\sigma(x), \sigma(z)]_\alpha \alpha D(x,y)$.

$2[\sigma(x), \sigma(z)]_\alpha \beta\sigma(x)\alpha D(x,y) = 2\sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(x,y)$

Since M is 2-torsion free

$[\sigma(x), \sigma(z)]_\alpha \beta\sigma(x)\alpha D(x,y) = \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(x,y)$…….(4.6)

Theorem4.3:-Let M be a 2-torsin free completely prime Γ-ring and let D: $M \times M \to M$ be a Jordan left bi-(σ,σ) derivation then either M is commutative or D is a left bi-(σ,σ) derivation

Proof:-from (4.6),we have

$[\sigma(x), \sigma(z)]_\alpha \beta\sigma(x)\alpha D(x,y) = \sigma(x)\alpha[\sigma(x), \sigma(z)]_\alpha \beta D(x,y)$

By replacing x by x+z ,we get

$[\sigma(x+z),\sigma(z)]_\alpha \beta \sigma(x+z)\alpha D(x+z,y)=\sigma(x+z)\alpha[\sigma(x+z),\sigma(z)]_\alpha \beta D(x+z,y)$

Then

$W=[\sigma(x),\sigma(z)]_\alpha \beta \sigma(x)\alpha D(x,y)+[\sigma(x),\sigma(z)]_\alpha \beta \sigma(z)\alpha D(x,y)+[\sigma(x),\sigma(z)]_\alpha \beta \sigma(x)\alpha D(z,y)+[\sigma(x),\sigma(z)]_\alpha \beta \sigma(z)\alpha D(z,y)$

On the other hand

$W=\sigma(x)\alpha[\sigma(x),\sigma(z)]_\alpha \beta D(x,y)+\sigma(x)\alpha[\sigma(x),\sigma(z)]_\alpha \beta D(z,y)+\sigma(z)\alpha[\sigma(x),\sigma(z)]_\alpha \beta D(x,y)+\sigma(z)\alpha[\sigma(x),\sigma(z)]_\alpha \beta D(z,y)$

By comparing these two expression of W ,we get

$[\sigma(x),\sigma(z)]_\alpha \beta \sigma(z)\alpha D(x,y)+[\sigma(x),\sigma(z)]_\alpha \beta \sigma(x)\alpha D(z,y)$

$=\sigma(x)\alpha[\sigma(x),\sigma(z)]_\alpha \beta D(z,y)+\sigma(z)\alpha[\sigma(x),\sigma(z)]_\alpha \beta D(x,y)$

from (4.6),we get

$[\sigma(x),\sigma(z)]_\alpha \beta(D(x\alpha z,y)-\sigma(x)\alpha D(z,y)-\sigma(z)\alpha D(x,y))=0$

Since M is completely prime gamma ring, then

Either $[\sigma(x),\sigma(z)]_\alpha=0$ or $D(x\alpha z,y)-\sigma(x)\alpha D(z,y)-\sigma(z)\alpha D(x,y)=0$

If $[\sigma(x),\sigma(z)]_\alpha=0$ then M is commutative(since σ is automorphism) and if $D(x\alpha z,y)-\sigma(x)\alpha D(z,y)-\sigma(z)\alpha D(x,y)=0$ then D is a left bi-(σ,σ) derivation.

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