# *Pointwise Ker-injective Modules*

**By**

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**Abstract**

G.F. Birkenmeier (1978) introduced the concept of ker-injective modules as a generalization of injective modules. Also injectivity was generalized to pointwise injectivity by S.A. Gataa (1999). In this paper, pointwise ker-injective modules is defined as a proper generalization of both ker-injective modules and pointwise injective modules. Many characterizations and properties of pointwise ker-injective modules are obtained. New characterizations of semi-simple Artinian rings in terms of pointwise ker-injectivity are introduced. Further Osofsky's theorem and Birkenmeier's corollary are generalized.

**§0: Introduction**

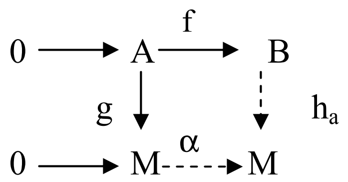
In [4] a generalization of injective modules, noted ker-injective modules is introduced. G.F. Birkenmeier proved that an R-module M is ker-injective if and only if for each R-monomorphism f:A→B (where A and B are R-modules) and for each R-homomorphism g:A→M, there exist an R-monomorphism α:M→M and an R-homomorphism h:B→M such that (hof)(a)=(αog)(a), for all a in A [4]. Also in [7], the concept of a pointwise injective modules is introduced as a generalization of injective modules. An R-module M is called pointwise injective if for each R-monomorphism f:A→B (where A and B are two R-modules), each R-homomorphism g:A→M and for each a∈A, there exists an R-homomorphism $h_a$:B→M ($h_a$ may depend on a) such that ($h_a$ of)(a)=g(a).

Through out this paper, R will denote an associative ring with unit and every module over a ring R will be understood to be unitary (left) R-module. For an R-module M, E(M) denote the injective envelope of M.

## §1: Pointwise Ker-injective Modules

As a generalization of both ker-injective modules and pointwise injective modules we introduce the following:

**Definition (1.1)**: An R-module M is said to be pointwise ker-injective if for each R-monomorphism f:A→B (where A and B are R-modules), each R-homomorphism g:A→M and for each a∈A, there exist an R-monomorphism α:M→M and an R-homomorphism $h_a$:B→M ($h_a$ may depend on a) such that ($h_a$of)(a)=(αog)(a). In other word the following diagram is commutative pointwisely.




A ring R is called self pointwise ker-injective, if it is pointwise ker-injective R-module.

## Remarks (1.2):

**(1)**It is clear that every ker-injective module (also, pointwise injective module) is a pointwise ker-injective module.

**(2)**The concept of pointwise ker-injective modules is a proper generalization of both ker-injective modules and pointwise injective modules; for examples:

**(i)**Let $R=\prod_{p\in \mathbf{P}} Z/pZ$ where Z is the ring of integers and **P** is the set of all positive prime integers. Let $M=\oplus_{p\in \mathbf{P}} Z/pZ$ by [7] , then M is a pointwise injective R-module and hence M is a pointwise ker-injective R-module. Assume that M is ker-injective R-module. Since M is a semi-simple R-module [10, Ex. 1.12], thus M is a semi-simple R-module and ker-injective . Hence M is injective R-module [4] and this contradiction since M is not injective R-module [9],thus M is not ker-injective R-module. Therefore M is a pointwise ker-injective R-module, but it is not ker-injective R-module.

**(ii)**Let M=Z⊕∏Q (where ∏Q is an infinite direct product of copies of Q as Z-module). M is ker-injective Z-module [4], hence M is a pointwise ker-injective Z-module. If M is a pointwise injective Z-module, then by [7] we have that Z is a pointwise injective Z-module and this contradiction with [7, Ex. (1.2.2)]. Therefore M is not pointwise injective Z-module but it is pointwise ker-injective Z-module.

**(3)**Let M be a nonpointwise injective R-module and ∏E(M) be an infinite direct product of copies of E(M). Then every R-module of the form M⊕∏E(M) is a pointwise ker-injective R-module but not pointwise injective.

**Proof**: Since M⊕∏E(M) is a ker-injective R-module [4], thus M⊕∏E(M) is a pointwise ker-injective R-module. If M⊕∏E(M) is pointwise injective R-module, thus M is pointwise injective R-module [7] and this contradiction, therefore M⊕∏E(M) is not pointwise injective R-module. □

**(4)** The reader can easily check that, pointwise ker-injectivity is an algebraic property.

Recall that an R-homomorphism f:A→B is pointwise split if for each a∈A, there exists an R-homomorphism $g_a$:B→A ($g_a$ may depend on a) such that ($g_a$of)(a)=a [7]. As a generalization we introduce:

**Definition(1.3):**Let A and B are two R-modules. An R-homomorphism f:A→B is said to be pointwise ker-split if for each a∈A, there exist an R-monomorphism α:A →A and an R-homomorphism $g_a$:B→A ($g_a$ may depend on a) such that ($g_a$of)(a)=α(a).

Thus we have the following theorem (compare with [7, P.15])

**Theorem (1.4)**: The following statements are equivalent for an R-module M:

(1)M is a pointwise ker-injective R-module.

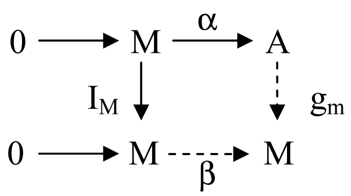
(2)For each R-module A, each R-monomorphism α:M→A is a pointwise ker-split.

(3)For each injective R-module A, each R-monomorphism $\alpha:M\rightarrow A$ is a pointwise ker-split.

(4)Each R-monomorphism $\alpha:M\rightarrow E(M)$ is a pointwise ker-split.

**Proof:**

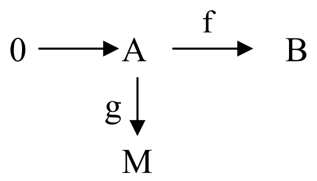
(1)⇒(2). Assume that M is a pointwise ker-injective R-module. Let A be any R-module and $\alpha: M\rightarrow A$ by any R-monomorphism. Consider the following diagram:




Since M is a pointwise ker-injective, thus for each $m\in M$ there exist an R-monomorphism $\beta:M\rightarrow M$ and an R-homomorphism $g_m:A\rightarrow M$ such that $(g_m o\alpha)(m)=(\beta oI_M)(m)$. Since $\beta oI_M$: $M\rightarrow M$ is an R-monomorphism, therefore $\alpha$ is a pointwise ker-split.

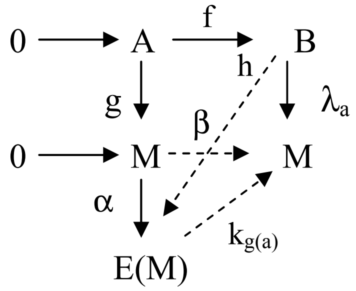
(2)⇒(3) and (3) ⇒(4) are obvious.

(4)⇒(1). Assume that each R-monomorphism $\alpha:M\rightarrow E(M)$ is a pointwise ker-split. Consider the diagram (1) with exact row, where A and B are R-modules and $g:A\rightarrow M$ be any R-homomorphism.




(diagram (1))

Since E(M) is an extension of M [12], thus there is an R-monomorphism $\alpha$: $M\rightarrow E(M)$. Hence we have the following diagram:



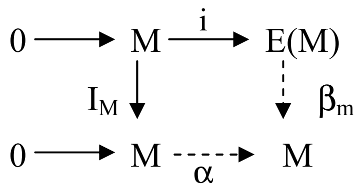

Let $a\in A$. By injectivity of E(M), there exists an R-homomorphism $h:B\rightarrow E(M)$ such that $hof=\alpha og$ . By (4) $\alpha$ is a pointwise ker-split , thus there exist an R-monomorphism

$\beta$: M→M and an R-homomorphism $k_{g(a)}$: E(M)→M such that $(k_{g(a)} o\alpha)(g(a)) = \beta(g(a))$. Put $\lambda_a = k_{g(a)}oh$ :B→M, thus we have that $(\lambda_a of)(a) = ((k_{g(a)}oh)of)(a) = (k_{g(a)}o(hof))(a) = (k_{g(a)}o(\alpha og))(a) = ((k_{g(a)}o\alpha)(g(a))) = \beta(g(a)) = (\beta og)(a)$. Therefore for diagram (1) and for each $a\in A$, we get an R-monomorphism $\beta$: M→M and an R-homomorphism $\lambda_a$: B→M such that $(\lambda_a of)(a)=(\beta og)(a)$. Hence M is a pointwise ker-injective R-module. □

The following proposition gives another characterization of pointwise ker-injective modules.

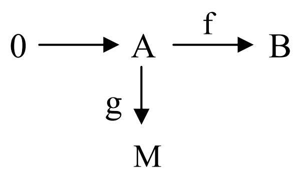
**Proposition(1.5):** An R-module M is pointwise ker-injective if and only if for each $m\in M$, there exist an R-monomorphism $\alpha$:M→M and an R-homomorphism $\beta_m$:E(M)→M such that $\beta_m(m)=\alpha(m)$.

**Proof**: Assume that M is a pointwise ker-injective R-module. Consider the following diagram where i is the inclusion mapping.



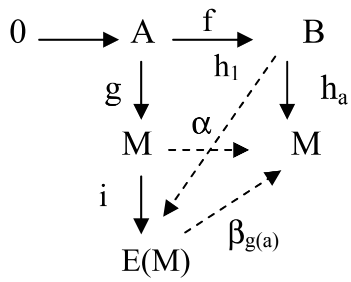

For each $m\in M$, since M is a pointwise ker-injective R-module, thus there exist an R-monomorphism $\alpha$:M→M and an R-homomorphism $\beta_m$: E(M)→M such that $(\beta_m oi)(m)=(\alpha o I_M)(m)$ and hence $\beta_m(m)=\alpha(m)$.

Conversely, consider the diagram (1) with exact row where A and B are R-modules.




(diagram (1))

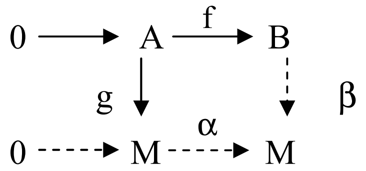
By injectivity of E(M), there exists an R-homomorphism $h_1$:B→E(M) such that $h_1 of=iog$. Thus we have the diagram (2) where i is the inclusion mapping.




(diagram (2))

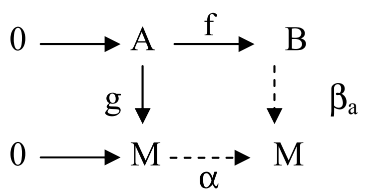
For each $a \in A$, since $g(a) \in M$ thus by our assumption there exist an R-monomorphism $\alpha: M \to M$ and an R-homomorphism $\beta_{g(a)}: E(M) \to M$ such that $\beta_{g(a)}(g(a)) = \alpha(g(a))$.Put $h_a =: \beta_{g(a)} o h_1 : B \to M$. Thus we have that $(h_a o f)(a) = ((\beta_{g(a)} o h_1) o f)(a) = \beta_{g(a)}((h_1 o f)(a)) = \beta_{g(a)}((i o g)(a)) = \beta_{g(a)}(g(a)) = \alpha(g(a)) = (\alpha o g)(a)$. Therefore for diagram(1) we get that, for each $a \in A$ there exist an R-monomorphism $\alpha: M \to M$ and an R-homomorphism $h_a: B \to M$ such that $(h_a o f)(a) = (\alpha o g)(a)$. Hence M is a pointwise ker-injective R-module. □

**Theorem (1.6)**: An R-module M is pointwise ker-injective module if and only if for each diagram with exact row (where A be a cyclic R-module and B any R-module),



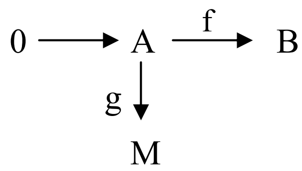

there exist an R-monomorphism $\alpha: M \to M$ and an R-homomorphism $\beta: B \to M$ such that $\beta o f = \alpha o g$.

**Proof**: (⇒) Let M be a pointwise ker-injective R-module. Consider the following diagram with exact row:

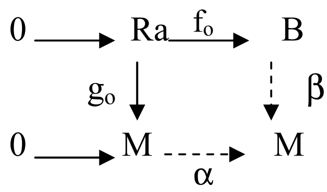



(where A and B are R-modules with A=Ra is a cyclic R-module). Since M is pointwise ker-injective, thus there exist an R-homomorphism $\beta_a: B \to M$ and an R-monomorphism $\alpha: M \to M$ such that $(\beta_a o f)(a) = (\alpha o g)(a)$.For each $x \in A$,implies $x = ra$ for some $r \in R$.Hence for each $x \in A$ we have that $(\alpha o g)(x) = (\alpha o g)(ra) = r((\alpha o g)(a)) = r((\beta_a o f)(a)) = (\beta_a o f)(ra) = (\beta_a o f)(x)$, then $\beta_a o f = \alpha o g$.

(⇐) To prove that M is pointwise ker-injective R-module. Consider the following diagram with exact row:



(where A and B are R-modules). For each $a \in A$, $Ra$ is a cyclic R-submodule of A. Consider $g_o = g/_{Ra} : Ra \to M$ and $f_o = f/_{Ra} : Ra \to B$. Thus we have the following diagram with exact row:



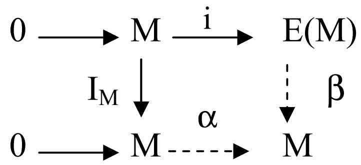
By our assumption, there exist an R-homomorphism $\beta: B \to M$ and an R-monomorphism $\alpha: M \to M$ such that $(\beta o f_o)(x) = (\alpha o g_o)(x)$, $\forall x \in Ra$. Then $(\beta o f)(a) = (\alpha o g)(a)$. Therefore M is pointwise ker-injective R-module. □

**Proposition (1.7):** For a cyclic R-module M, the following statements are equivalent:

1) M is an injective R-module.
2) M is a pointwise injective R-module.
3) M is a pointwise ker-injective R-module.
4) M is ker-injective R-module.

**Proof:** (1)⇒(2) and (2)⇒(3) are obvious.

(3)⇒(4). Let M be a cyclic pointwise ker-injective R-module. Consider the following diagram:



Since M is a pointwise ker-injective module and cyclic, thus by (theorem (1.6)) there exist an R-homomorphism $\beta: E(M) \to M$ and an R-momomorphism $\alpha: M \to M$ such that $(\beta o i)(x) = (\alpha o I_M)(x)$, $\forall x \in M$. Since $\alpha o I_M: M \to M$ is an R-monomorphism, thus $\ker(\alpha o I_M) = \{0\} \Rightarrow \ker(\beta o i) = \{0\}$, thus $\ker(\beta) \cap M = \{0\}$. Since $M \subset^{e} E(M)$, then $\ker(\beta) = \{0\}$ and hence $\beta$ is an R-monomorphism . Hence M and E(M) are subisomorphisms. Therefore M is ker-injective R-module, by theorem (1.1) in [4].

(4)⇒(1). Since R is a commutative ring by hypothesis, thus by prop. (1.2) in [4] M is an injective R-module. □

Immediately from proposition (1.7), we have the following corollary:

**Corollary (1.8):** For a ring R, the following statements are an equivalent:
1) R is a self-injective ring.
2) R is a self pointwise injective ring.
3) R is a self pointwise ker-injective ring.
4) R is a self ker-injective ring. □

**Corollary (1.9):** Every cyclic Z-module is not pointwise ker-injective (In particular Z and $Z_n$ as Z-module are not pointwise ker-injective, $n \in Z^+$).

**Proof:** Assume that a cyclic Z-module M is a pointwise ker-injective, thus by proposition (1.7) M is injective Z-module and this contradiction since every finitely generated Z-module is not injective [12]. Therefore every cyclic Z-module is not pointwise ker-injective module. □

An R-module M is called fully stable if for each R-submodule N of M and each R-homomorphism f:N→M, then f(N)⊆N [1]. Also, an R-module M is called bounded if there exists an element x∈M such that $\underset{R}{ann}(M) = \underset{R}{ann}(x)$ where $\underset{R}{ann}(M) = \{r \in R \mid rm = 0, \forall m \in M\}$ [2].

**Corollary (1.10):** If M is a pointwise ker-injective R-module, then M is a cyclic if and only if M is a fully stable bounded R-module.

**Proof:** Let M be a cyclic pointwise ker-injective R-module. Since every cyclic R-module is multiplication [3] and by proposition (1.7), thus we have that M is a multiplication pointwise injective R-module. By Proposition (2.17) in [7], then M is fully stable R-module. Hence M is a cyclic fully stable R-module and this implies that M is a bounded R-module [2]. Therefore M is a fully stable bounded R-module. The converse which is appear in [2]. □

It is well-known that a direct product of ker-injective R-modules is ker-injective [4]. In the following proposition we show that this result is true in case of pointwise ker-injective modules.

**Proposition (1.11):** Let $\{M_\lambda\}_{\lambda\in\wedge}$ be a family of pointwise ker-injective R-modules then:

1) $\prod\limits_{\lambda\in\wedge} M_\lambda$ is a pointwise ker-injective R-module.

2)If $\wedge$ is a finite set or R is a Neotherian ring, then $\underset{\lambda\in\wedge}{\oplus} M_\lambda$ is a pointwise ker-injective R-module.

**Proof:**

**1)**For each $m=(m_\lambda)_{\lambda\in\wedge}\in\prod\limits_{\lambda\in\wedge} M_\lambda$. Since $m_\lambda\in M_\lambda$ and $M_\lambda$ is a pointwise ker-injective R-module, for each $\lambda\in\wedge$, thus by proposition (1.5) there exist an R-monomorphism $g_\lambda : M_\lambda \to M_\lambda$ and an R-homomorphism $f_\lambda : E(M_\lambda)\to M_\lambda$ such that $f_\lambda(m_\lambda)=g_\lambda(m_\lambda)$, for each $\lambda\in\wedge$. Define $f_m=\prod\limits_{\lambda\in\wedge} f_\lambda : \prod\limits_{\lambda\in\wedge} E(M_\lambda)\to\prod\limits_{\lambda\in\wedge} M_\lambda$ by $f_m((m'_\lambda)_{\lambda\in\wedge})=(f_\lambda(m'_\lambda))_{\lambda\in\wedge}$ for all $(m'_\lambda)_{\lambda\in\wedge}\in\prod\limits_{\lambda\in\wedge} E(M_\lambda)$ and $g((m_\lambda)_{\lambda\in\wedge})=(g_\lambda(m_\lambda))_{\lambda\in\wedge}$ for all $(m_\lambda)_{\lambda\in\wedge}\in\prod\limits_{\lambda\in\wedge} M_\lambda$. It is clear that $f_m$ is an R-homomorphism and g is an R-monomorphism. Therefore $f_m(m)=f_m((m_\lambda)_{\lambda\in\wedge})=(f_\lambda(m_\lambda))_{\lambda\in\wedge}=(g_\lambda(m_\lambda))_{\lambda\in\wedge}=g((m_\lambda)_{\lambda\in\wedge})=g(m)$. Since $E(\prod\limits_{\lambda\in\wedge} M_\lambda)=\prod\limits_{\lambda\in\wedge} E(M_\lambda)$ [12], thus for each $m\in\prod\limits_{\lambda\in\wedge} M_\lambda$ there exist an R-monomorphism $g:\prod\limits_{\lambda\in\wedge} M_\lambda\to\prod\limits_{\lambda\in\wedge} M_\lambda$ and an R-homomorphism $f_m : E(\prod\limits_{\lambda\in\wedge} M_\lambda)\to\prod\limits_{\lambda\in\wedge} M_\lambda$ such that $f_m(m)=g(m)$. Therefore $\prod\limits_{\lambda\in\wedge} M_\lambda$ is a pointwise ker-injective R-module by prop. (1.5). □

**2)**

i) If $\wedge$ is a finite set, then $\underset{\lambda\in\wedge}{\oplus} M_\lambda=\prod\limits_{\lambda\in\wedge} M_\lambda$ [8] and hence by (1) we have that $\underset{\lambda\in\wedge}{\oplus} M_\lambda$ is a pointwise ker-injective R-module.

ii)If R is a Neotherian ring, then $E(\underset{\lambda\in\wedge}{\oplus} M_\lambda)=\underset{\lambda\in\wedge}{\oplus} E(M_\lambda)$ [12] and by same way of the proof of (1) we have that $\underset{\lambda\in\wedge}{\oplus} M_\lambda$ is a pointwise ker-injective R-module. □

**Remark (1.12):** The converse of proposition (1.11) is not true in general, for example let M=Z⊕∏E(Z) where ∏E(Z) is an infinite direct product of copies of E(Z) as Z-module and let N=Z⊕(0). It is clear that N is a direct summand of a pointwise ker-injective Z-module M. Since N≅Z and Z is not pointwise ker-injective Z-module by corollary (1.9), thus by remark ((1.2), 4) N is not pointwise ker-injective Z-module.

## §2: Pointwise Ker-injective Modules and Semi-Simple Artinian Rings

In this section we give new characterizations of semi-simple Artinian rings.

It is know that a ring R is semi-simple Artinian iff every R-module is injective [12] iff every R-module is ker-injective [4] iff every R-module is pointwise injective [7]. The following theorem is a generalization of above statements.

**Theorem (2.1):** The following statements are equivalent for a ring R:

1) R is a semi-simple Artinian ring.
2) Every R-module is a pointwise ker-injective.
3) Every cyclic R-module is a pointwise ker-injective.

**Proof:** (1) ⇒ (2) and (2) ⇒ (3) are obvious.

(3) ⇒ (1). Let M be a cyclic R-module. By (4) we have that M is a pointwise ker-injective R-module. Hence by proposition (1.7), then M is an injective R-module. Therefore every cyclic R-module is an injective R-module and this implies that R is a semi-simple Artinian ring [11]. □

**Corollary (2.2):** The following statements are equivalent for a ring R:

1) R is a semi-simple Artinian ring.
2) R is a pointwise ker-injective ring and each quotient of a pointwise ker-injective R-module is pointwise ker-injective. □

Recall that a ring R is hereditary if and only if each quotient of an injective R-module is injective [5].

**Corollary (2.3):** Let R be a self injective ring. Then R is hereditary if and only if each quotient of pointwise ker-injective R-module is pointwise ker-injective. □

B.L. Osofsky in [6] has noted that a ring R is semi-simple Artinian iff for each R-module M, if $N_1$ and $N_2$ are injective R-submodules of M, then $N_1 \cap N_2$ is also injective R-module.

In the following theorem we give a new characterization of semi-simple Artinian rings which a generalization of Osofsky's theorem in [6].

**Theorem (2.4):** The following statements are equivalent for a ring R:

1) R is a semi-simple Artinian ring.

2) For each R-module M, if $N_1$ and $N_2$ are pointwise ker-injective R-submodules of M, then $N_1 \cap N_2$ is pointwise ker-injective R-module.

3) For each R-module M, if $N_1$ and $N_2$ are injective R-submodules of M, then $N_1 \cap N_2$ is pointwise ker-injective R-module.

**Proof:** (1) $\Rightarrow$ (2). It is follows from theorem (2.1).

(2) $\Rightarrow$ (3). It is obvious.

(3) $\Rightarrow$ (1). Let M be any R-module and E=E(M) is the injective envelope of M. Let $Q=E\oplus E$, $K=\{(x,x)\in Q | x\in M\}$ and let $\overline{Q} = Q/K$. Put $M_1=\{y+K\in \overline{Q} | y\in E\oplus 0\}$ and let $M_2=\{y+K\in \overline{Q} | y\in 0\oplus E\}$. It is clear that $\overline{Q}=M_1+M_2$. Define $\alpha_1$: E $\rightarrow$ $M_1$ and $\alpha_2$: E$\rightarrow$$M_2$ by $\alpha_1(y)=(y,0)+K$ and $\alpha_2(y)=(0,y)+K$ for all y in E. Since $(E\oplus 0)\cap K=0$ and $(0\oplus E)\cap K=0$, thus we have $\alpha_1$ and $\alpha_2$ are R-isomorphisms. Since E is an injective R-module, therefore $M_i$ is injective R-submodule of $\overline{Q}$, for i=1,2 [6]. Thus by (3), we have $M_1 \cap M_2$ is a pointwise ker-injective R-module. Define f: $M\rightarrow M_1\cap M_2$ by f(m)= (m,0)+K for all m in M. Since $M_1\cap M_2=\{y+K\in \overline{Q} | y\in M\oplus 0\}$, therefore it is easy to prove that f is an R-isomorphism. Thus M is a pointwise ker-injective R-module, by remark ((1.2), 4). Hence every R-module is pointwise ker-injective and this implies that R is a semi-simple Artinian ring, by theorem (2.1). □

**Corollary (2.5)** (Osofsky's theorem [6]): A ring R is a semi-simple Artinian if and only if for each R-module M, if $N_1$ and $N_2$ are injective R-submodules of M, then $N_1\cap N_2$ is also injective R-module. □

The following theorem is a new characterization of semi-simple Artinian rings which is a generalization of G.F Birkenmeier statement in [4].

**Theorem (2.6):** The following statements are equivalent for a ring R:

1) R is a semi-simple Artinian ring.
2) Every pointwise ker-injective R-module is injective.
3) Every pointwise ker-injective R-module is pointwise injective.
4) Every ker-injective R-module is pointwise injective

**Proof:** (1) ⇒ (2), (2) ⇒ (3) and (3) ⇒ (4) are obvious.

(4) ⇒ (1). Let M be any R-module. Thus by [4], M⊕∏E(M) is ker-injective R-module where ∏E(M) is an infinite direct product of copies of E(M). Therefore by (4), M⊕∏E(M) is a pointwise injective R-module. Consequently, M is a pointwise injective R-module [7]. Hence every R-module is pointwise injective and this implies that R is a semi-simple Artinian ring [7]. □

**Corollary (2.7)** (Birkenmeier Corollary [4]): A ring R is a semi-simple Artinian ring if and only if every ker-injective R-module is injective. □

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# الموديولات إغمارية النواة نقطياً

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**الخلاصة:**

بيركنمير في سنة 1978 قدم مفهوم الموديولات اغمارية النواة كتعميم للموديولات الإغمارية. كذلك الموديولات الإغمارية قد تم تعميمها إلى الموديولات نقطية الإغمار من قبل سعد عبد الكاظم كاطع في سنة 1999. في هذا البحث تم تقديم مفهوم الموديولات إغمارية النواة نقطياً كتعميم فعلي لكل من الموديولات إغمارية النواة والموديولات نقطية الإغمار. أعطينا جملةمن التشخصيات والخواص للموديولات إغمارية النواة نقطياً. جملة من التميزات للحلقات الارتينية شبه البسيطة بدلالة هذا النوع من الموديولات قد أعطيت. أكثر من ذلك تم تعميم مبرهنة اوسوفسكي ونتيجة بيركنمير.

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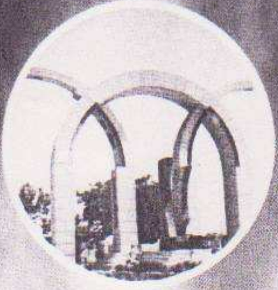

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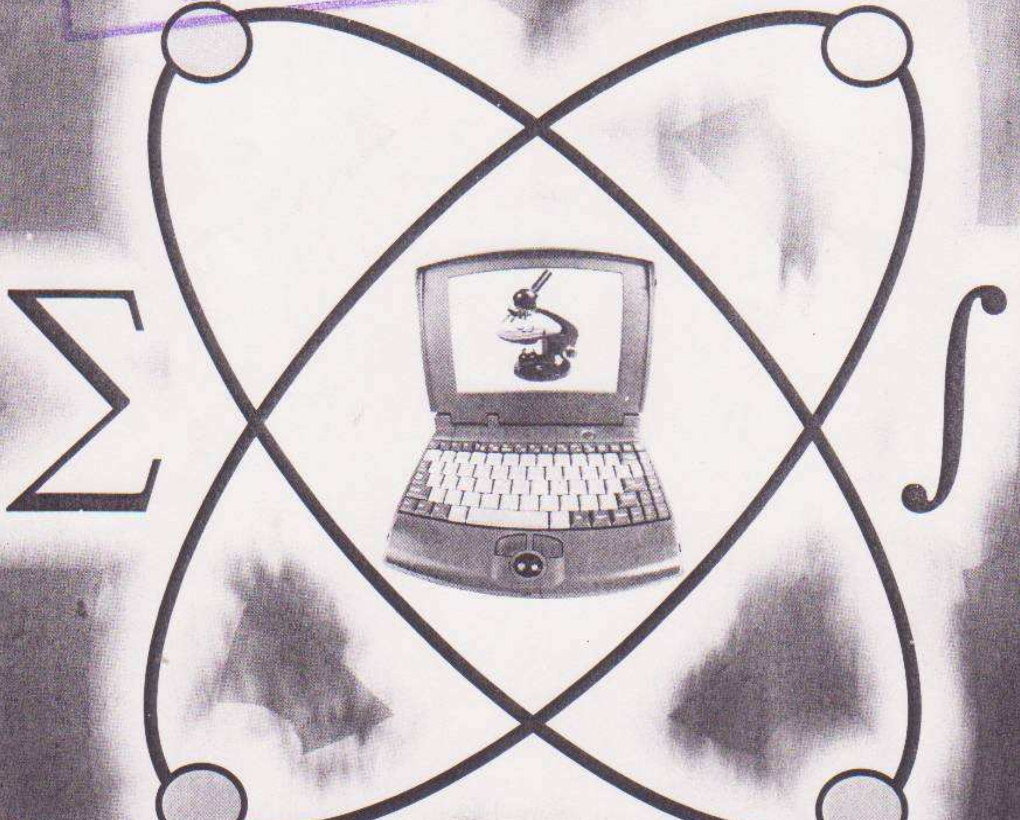


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