On D-Compact Smarandache Groupoids

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Abstract

In the present paper , we have introduced some new definitions On D-compact M Smarandache groupoid , D-compact G Smarandache groupoid and D-compact cyclic G Smarandache groupoid , we obtain some results related to D-compact M Smarandache groupoid , D-compact G Smarandache groupoid and D-compact cyclic G Smarandache groupoid .

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1. Introduction

The notion of groupoid was introduced by H. Brandt [Math. Ann., 96(1926), 360-366 ; MR 1512323] . A groupoid $(G,*)$ is a set on which is defined a non associative binary operation which is closed on G, the groupoid $(G,*)$ is a semigroup if the binary operation $*$ is associative , the semigroup $(G,*)$ with identity is called a monoid , the monoid which is every element is invertible is a group [3] .

Vasantha Kandasamy [4] , introduced details on a Smarandache structure on a set **G** means a weak structure **W** on **G** , where there exists a proper subset **H** of **G** embedded with a strong structure **S** . we investigated on D-compact M Smarandache groupoid , D-compact G Smarandache groupoid and D-compact cyclic G Smarandache groupoid , we obtain some good results related to these concepts above .

2. Definitions

[1]Definition 1

1- A *Smarandache G. groupoid* $(G,*)$ is a groupoid which has a proper subset $S \subset G$ such that $(S,*)$ is a group (with respect to the same induced operation) .

2- Let $(G,*)$ be a Smarandache G. groupoid , if every proper subset $S \subset G$ which is a group is cyclic then we say $(G,*)$ is a *Smarandache cyclic groupoid* .

3- Let $(G,*)$ be a Smarandache G. groupoid , if there exists at least a proper subset $S \subset G$ which is a cyclic group (untrivial) then we say $(G,*)$ is a *Smarandache weakly cyclic groupoid* .

[1] similar to definition Smarandache (weakly) cyclic semigroup [5] .

Definition 2

Let $(G,*)$ be a groupoid and I be an indexed (I is a finite or an infinite set) , we say that ;

1- $\{(M_i,*); M_i \subset G, (M_i,*)$ is a monoid, $\forall i \in I\}$, is a *D-cover monoids* of $(G,*)$ if $G = \bigcup_{i\in I} M_i$.

2- $\{ (G_i,*) ;\ G_i \subset G , (G_i,*)$ is a group , $\forall i \in I \}$, is a *D-cover groups* of $(G,*)$ if $G = \bigcup_{i\in I} G_i$.

3- $\{ (G_i,*) ;\ G_i \subset G , (G_i,*)$ is a cyclic group , $\forall i \in I \}$, is a *D-cover cyclic groups* of $(G,*)$ if $G = \bigcup_{i\in I} G_i$.

Definition 3

Let $(G,*)$ be a groupoid and $\{(M_i,*) ;\ M_i \subset G , (M_i,*)$ is a monoid , $\forall i \in I\}$, is a *D-cover monoids* of $(G,*)$, we say that ;

1- $(G,*)$ is a *D-compact weakly M Smarandache* groupoid (*D-CWMS groupoid*) if there is a finite *D-cover monoids* of $(G,*)$.

2- $(G,*)$ is a *D-compact M Smarandache* groupoid (*D-CMS groupoid*) if for every *D-cover monoids* of $(G,*)$ there is a finite *sub-D-cover monoids* of $(G,*)$.

3- $(G,*)$ is a *D-compact weakly Lindelöff M Smarandache* groupoid (*D-CWLMS groupoid*) if there is a countable *D-cover monoids* of $(G,*)$.

4- $(G,*)$ is *D-compact Lindelöff M Smarandache* groupoid (*D-CLMS groupoid*) if for every *D-cover monoids* of $(G,*)$ there is a countable *sub-D-cover monoids* of $(G,*)$.

5- $(G,*)$ is a *D-locally compact M Smarandache* groupoid (*D-LCMS groupoid*) if for every element x of G there is a subset G_x of G include x , such that $(G_x,*)$ is a monoid with respect to the same operation $*$ on G .

6- $(G,*)$ is a *D- strong locally compact M Smarandache* groupoid (*D-SLCMS groupoid*) if for every element x of G (except the unite element) there is a unique subset G_x of G include x , such that $(G_x,*)$ is a monoid with respect to the same operation $*$ on G .

Definition 4

Let $(G,*)$ be a groupoid and $\{(G_i,*) ;\ G_i \subset G , (G_i,*)$ is a group , $\forall i \in I\}$, is a *D-cover groups* of $(G,*)$, we say that ;

1- $(G,*)$ is a *D-compact weakly G Smarandache* groupoid (*D-CWGS groupoid*) if there is a finite *D-cover groups* of $(G,*)$.

2- $(G,*)$ is a *D-compact G Smarandache* groupoid (*D-CGS groupoid*) if for every *D-cover groups* of $(G,*)$ there is a finite *sub-D-cover groups* of $(G,*)$.

3- $(G,*)$ is a *D-compact weakly Lindelöff G Smarandache* groupoid (*D-CWLGS groupoid*) if there is a countable *D-cover groups* of $(G,*)$.

4- $(G,*)$ is *D-compact Lindelöff G Smarandache* groupoid (*D-CLGS groupoid*) if for every *D-cover groups* of $(G,*)$ there is a countable *sub-D-cover groups* of $(G,*)$.

5- $(G,*)$ is a *D-locally compact G Smarandache* groupoid (*D-LCGS groupoid*) if for every element x of G there is a subset G_x of G include x , such that $(G_x,*)$ is a group with respect to the same operation $*$ on G .

6- $(G,*)$ is a *D- strong locally compact G Smarandache* groupoid (*D-SLCGS groupoid*) if for every element x of G (except the unite element) there is a unique subset G_x of G include x , such that $(G_x,*)$ is a group with respect to the same operation $*$ on G .

Definition 5

Let $(G,*)$ be a groupoid and $\{(G_i,*) ;\ G_i \subset G , (G_i,*)$ is a cyclic group , $\forall i \in I\}$, is a *D-cover cyclic groups* of $(G,*)$, we say that ;

1- $(G,*)$ is a *D-compact weakly cyclic G Smarandache* groupoid (*D-CWCGS groupoid*) if there is a finite *D-cover cyclic groups* of $(G,*)$.

2- $(G,*)$ is a *D-compact cyclic G Smarandache* groupoid (*D-CCGS groupoid*) if for every *D-cover cyclic groups* of $(G,*)$ there is a finite *sub-D-cover cyclic groups* of $(G,*)$.

3- $(G,*)$ is a *D-compact weakly Lindelöff cyclic G Smarandache* groupoid (*D-CWLCGS groupoid*) if there is a countable *D-cover cyclic groups* of $(G,*)$.

4- $(G,*)$ is *D-compact Lindelöff cyclic G Smarandache* groupoid (*D-CLCGS groupoid*) if for every *D-cover cyclic groups* of $(G,*)$ there is a countable *sub-D-cover cyclic groups* of $(G,*)$.

5- $(G,*)$ is a *D-locally compact cyclic G Smarandache* groupoid (*D-LCCGS groupoid*) if for every element x of G there is a subset G_x of G include x , such that $(G_x,*)$ is a cyclic group with respect to the same operation $*$ on G .

6- $(G,*)$ is a *D- strong locally compact cyclic G Smarandache* groupoid (*D-SLCCGS groupoid*) if for every element x of G (except the unite element) there is a unique subset G_x of G include x , such that $(G_x,*)$ is a cyclic group with respect to the same operation $*$ on G .

Definition 6

Let $(G,*)$ be a groupoid and $(H,*)$ be a subgroupoid of $(G,*)$. Then we say that ;

1- $(H,*)$ is a *Smarandache G. subgroupoid (Smarandache cyclic subgroupoid , Smarandache weakly cyclic subgroupoid) ,if* $(H,*)$ is a *Smarandache G. groupoid (Smarandache cyclic groupoid , Smarandache weakly cyclic groupoid),* respectively .

2- $(H,*)$ is a *D-CMS subgroupoid (D-CWMS subgroupoid , D-CWLMS subgroupoid , D-CLMS subgroupoid , D-LCMS subgroupoid , D-SLCMS subgroupoid),if* $(H,*)$ *is a D-CMS groupoid* (*D-CWMS groupoid , D-CWLMS groupoid , D-CLMS groupoid , D-LCMS groupoid , D-SLCMS groupoid*) , respectively .

3- $(H,*)$ is a *D-CGS subgroupoid* (*D-CWGS subgroupoid , D-CWLGS subgroupoid , D-CLGS subgroupoid , D-LCGS subgroupoid , D-SLCGS subgroupoid*) , *if* $(H,*)$ *is a D-CGS groupoid* (*D-CWGS groupoid , D-CWLGS groupoid , D-CLGS groupoid , D-LCGS groupoid , D-SLCGS groupoid*) , respectively .

4- $(H,*)$ is a *D-CCGS subgroupoid* (*D-CWCGS subgroupoid , D-CWLCGS subgroupoid , D-CLCGS subgroupoid , D-LCCGS subgroupoid , D-SLCCGS subgroupoid*) , *if* $(H,*)$ *is a D-CCGS groupoid* (*D-CWCGS groupoid , D-CWLCGS groupoid , D-CLCGS groupoid , D-LCCGS groupoid , D-SLCCGS groupoid*) , respectively .

[2]**Definition 7**

1- Suppose Λ is non-empty set and $(G_\lambda,*_\lambda)$ is a groupoid for each $\lambda \in \Lambda$. Their *product* is $\prod_{\lambda\in\Lambda} G_\lambda$ with multiplication given by $(\boldsymbol{x} \otimes \boldsymbol{y}) = x_\lambda *_\lambda y_\lambda$ for each $x_\lambda , y_\lambda \in G_\lambda$ and $\lambda \in \Lambda$.

2- If $G_\lambda = G$, $\forall \lambda \in \Lambda$, then we denoted that $G^\Lambda = \prod_{\lambda\in\Lambda} G_\lambda$.

Definition 8

Let $(G,*)$ and $(\bar{G} ,\bar{*})$ are two groupoids , we say that

1- $f : (G,*) \to (\bar{G},\bar{*})$ is a *groupoid homomorphism* if $f(x * y) = f(x) \bar{*} f(y)$ $\forall x,y \in G$.

2- $f : (G,*) \to (\bar{G},\bar{*})$ is a *groupoid isomorphism* if f is a *bijective* and *groupoid homomorphism* .

3- $(G,*)$ is an isomorphic to $(\bar{G} ,\bar{*})$, denoted that $(G,*) \cong (\bar{G},\bar{*})$, if there is a groupoid isomorphism $f : (G,*) \to (\bar{G},\bar{*})$.

[2] See [5] , p54 .

3. Main Results

The prove of all the following lemmas are direct from definitions .

Lemma 1

1- If $(G,*)$ is a *Smarandache cyclic groupoid* , then $(G,*)$ is a *Smarandache weakly cyclic groupoid* .

2- If $(G,*)$ is a *Smarandache weakly cyclic groupoid* , then $(G,*)$ is a *Smarandache G groupoid* .

Lemma 2

1- If $(G,*)$ is a *D-CMS groupoid* (*D-CGS groupoid* , *D-CCGS groupoid*) , then $(G,*)$ is a *D-CWMS groupoid* (*D-CWGS groupoid* , *D-CWCGS groupoid*) .

2- If $(G,*)$ is a *D-CMS groupoid* (*D-CGS groupoid* , *D-CCGS groupoid*) , then $(G,*)$ is a *D-CLMS groupoid* (*D-CLGS groupoid* , *D-CLCGS groupoid*) .

3- If $(G,*)$ is a *D-CWMS groupoid* (*D-CWGS groupoid* , *D-CWCGS groupoid*) , then $(G,*)$ is a *D-CWLMS groupoid* (*D-CWLGS groupoid* , *D-CWLCGS groupoid*) .

Lemma 3

1- If $(G,*)$ is a *D-SLCMS groupoid* (*D-SLCGS groupoid* , *D-SLCCGS groupoid*) , then $(G,*)$ is a *D-LCMS groupoid* (*D-LCGS groupoid* , *D-LCCGS groupoid*) .

2- If $(G,*)$ is a *D-CLMS groupoid* (*D-CLGS groupoid* , *D-CLCGS groupoid*) , then $(G,*)$ is a *D-CWLMS groupoid* (*D-CWLGS groupoid* , *D-CWLCGS groupoid*) .

3- If $(G,*)$ is a *D-CWLMS groupoid* (*D-CWLGS groupoid* , *D-CWLCGS groupoid*) , then $(G,*)$ is a *D-LCMS groupoid* (*D-LCGS groupoid* , *D-LCCGS groupoid*) .

Lemma 4

1- If $(G,*)$ is a *D-CCGS groupoid* (*D-CWCGS groupoid* , *D-CWLCGS groupoid* , *D-CLCGS groupoid* , *D-LCCGS groupoid* , *D-SLCCGS groupoid*) , then $(G,*)$ is a *D-CGS groupoid* (*D-CWGS groupoid* , *D-CWLGS groupoid* , *D-CLGS groupoid* , *D-LCGS groupoid* , *D-SLCGS groupoid*) .

2- If $(G,*)$ is a *D-CGS groupoid* (*D-CWGS groupoid* , *D-CWLGS groupoid* , *D-CLGS groupoid* , *D-LCGS groupoid* , *D-SLCGS groupoid*) , then $(G,*)$ is a *D-CMS groupoid* (*D-CWMS groupoid* , *D-CWLMS groupoid* , *D-CLMS groupoid* , *D-LCMS groupoid* , *D-SLCMS groupoid*) .

The following theorems are direct from definitions and order of groupoid ,

Theorem 1

If $(G,*)$ is a finite groupoid , then the following are equivalent ;

1) $(G,*)$ is a *D-CMS groupoid* ,
2) $(G,*)$ is a *D-CWMS groupoid* ,
3) $(G,*)$ is a *D-CWLMS groupoid* ,
4) $(G,*)$ is a *D-CLMS groupoid* ,
5) $(G,*)$ is a *D-LCMS groupoid* .

Theorem 2

If $(G,*)$ is a finite groupoid , then the following are equivalent ;

1) $(G,*)$ is a *D-CGS groupoid* ,
2) $(G,*)$ is a *D-CWGS groupoid* ,
3) $(G,*)$ is a *D-CWLGS groupoid* ,

4) $(G,*)$ is a *D-CLGS groupoid* ,

5) $(G,*)$ is a *D-LCGS groupoid* .

Theorem 3

If $(G,*)$ is a finite groupoid , then the following are equivalent ;

1) $(G,*)$ is a *D-CCGS groupoid* ,

2) $(G,*)$ is a *D-CWCGS groupoid* ,

3) $(G,*)$ is a *D-CWLCGS groupoid* ,

4) $(G,*)$ is a *D-CLCGS groupoid* ,

5) $(G,*)$ is a *D-LCCGS groupoid* .

[3]**Theorem 4**

Any D-compact group is a *D-CGS groupoid* .

Proof .

Let $(G,*)$ is a D-compact group so any D-cover group of the group $(G,*)$ there is a finite sub-D-cover group of $(G,*)$, but any group is a groupiod so $(G,*)$ is a groupoid and any D-cover group of $(G,*)$ there exists a finite sub-D-cover group of $(G,*)$, therefore $(G,*)$ is a *D-CGS groupoid* . □

[4]Let $N_o(\mathbb{Z}_p) = \{(a,b)\ ;\ a,b \in \mathbb{Z}_p\}$ the collection of all open natural intervals ,
$N_c(\mathbb{Z}_p) = \{[a,b]\ ;\ a,b \in \mathbb{Z}_p\}$ the collection of all closed natural intervals ,
$N_{oc}(\mathbb{Z}_p) = \{(a,b]\ ;\ a,b \in \mathbb{Z}_p\}$ the collection of all open-closed natural intervals , and
$N_{co}(\mathbb{Z}_p) = \{[a,b)\ ;\ a,b \in \mathbb{Z}_p\}$ the collection of all closed-open natural intervals .

Theorem 5

If p is a prime number , then $(N_o(\mathbb{Z}_p),\times_p)$ is a D-CCGS groupoid .

Proof .

It is clear that $(N_o(\mathbb{Z}_p),\times_p)$ is a groupoid and
$N_o(\mathbb{Z}_p) = \{(a,b);\ a,b \in \mathbb{Z}_p\backslash\{0\}\} \cup \{(0,b);\ b \in \mathbb{Z}_p\backslash\{0\}\} \cup \{(a,0);\ a \in \mathbb{Z}_p\backslash\{0\}\} \cup \{(0,0)\}$,
and we know that $(\{(0,0)\},\ \times_p)$, $(\{(a,0);\ a \in \mathbb{Z}_p\backslash\{0\}\},\ \times_p)$ and $(\{(0,b);\ b \in \mathbb{Z}_p\backslash\{0\}\}\ ,\ \times_p)$ are cyclic groups . From theorem 2.7 in [6] we have $(\{(a,b);\ a,b \in \mathbb{Z}_p\backslash\{0\}\}\ ,\ \times_p)$ is a cyclic group . Therefore $(N_o(\mathbb{Z}_p),\times_p)$ is a D-CCGS groupoid . □

Remark 1

We can in the theorem 5 replace $N_o(\mathbb{Z}_p)$ by $N_c(\mathbb{Z}_p)$ or $N_{oc}(\mathbb{Z}_p)$ or $N_{co}(\mathbb{Z}_p)$ and still the theorem true .

Theorem 6

If $(G,*) \cong (\bar{G},\bar{*})$, then $(G,*)$ is a *D-CGS groupoid* $\Leftrightarrow$ $(\bar{G},\bar{*})$ is a *D-CGS groupoid* .

Proof .

$(\Rightarrow)$ Let $(\bar{G}_\iota,\bar{*})$ be any *D-cover group* of the groupoid $(\bar{G},\bar{*}) \Rightarrow \bar{G} = \bigcup_{i\in I} \bar{G}_\iota$, but f is an isomorphism $\Rightarrow f(G) = \bar{G} = \bigcup_{i\in I} \bar{G}_\iota \Rightarrow G = f^{-1}(\bigcup_{i\in I} \bar{G}_\iota) = \bigcup_{i\in I} f^{-1}(\bar{G}_\iota)$, and $f^{-1}(\bar{G}_\iota)$ is a group $\forall i \in I$, but $(G,*)$ is a *D-CGS groupoid* so there is a finite set J such that

[3] Definition of the D-compact group in [1]

[4] See [6]

[5] $G = \bigcup_{j\in J} f^{-1}(\bar{G}_j) = f^{-1}(\bigcup_{j\in J} \bar{G}_j) \Rightarrow \bar{G} = f(G) = f\left(f^{-1}(\bigcup_{j\in J} \bar{G}_j)\right) = \bigcup_{j\in J} \bar{G}_j$ and $(\bar{G}_j,\bar{*})$ is a group $\forall j \in J \Rightarrow (\bar{G},\bar{*})$ is a *D-CGS groupoid* .

($\Leftarrow$) Let $(G_i,*)$ be any *D-cover group* of the groupoid $(G,*) \Rightarrow G = \bigcup_{i\in I} G_i$, but f is an isomorphism $\Rightarrow \bar{G} = f(G) = f(\bigcup_{i\in I} G_i) = \bigcup_{i\in I} f(G_i)$, and $f(G_i)$ is a group $\forall i \in I$, but $(\bar{G},\bar{*})$ is a *D-CGS groupoid* so there is a finite set J such that

[6] $\bar{G} = \bigcup_{j\in J} f(G_j) = f(\bigcup_{j\in J} G_j) \Rightarrow G = f^{-1}(\bar{G}) = f^{-1}\left(f(\bigcup_{j\in J} G_j)\right) = \bigcup_{j\in J} G_j$ and $(G_j,*)$ is a group $\forall j \in J \Rightarrow (G,*)$ is a *D-CGS groupoid* . □

Corollary 1

1- If $f : (G,*) \longrightarrow (\bar{G},\bar{*})$ is an isomorphism and $(H,*)$ is a *D-CGS subgroupoid* of $(G,*)$, then $f(H)$ is a *D-CGS subgroupoid* of $(\bar{G},\bar{*})$.

2- If $f : (G,*) \longrightarrow (\bar{G},\bar{*})$ is an isomorphism and $(S,*)$ is a *D-CGS subgroupoid* of $(\bar{G},\bar{*})$. Then $f^{-1}(S)$ is a *D-CGS subgroupoid* of $(G,*)$.

Theorem 7

If $(A,\circledast)$ is a group and $(G,*)$ is a *D-CGS groupoid* , then $(A \times G, \otimes)$ is a *D-CGS groupoid* .

Proof .

Let $\{ (A \times G_i, \otimes) ; G_i \subset G, (A \times G_i, \otimes)$ is a group $, \forall i \in I \}$ be any *D-cover group* of $(A \times G, \otimes)$, where $(G_i,*)$ is a groups and $A \times G = \bigcup_{i\in I}(A \times G_i) = A \times (\bigcup_{i\in I} G_i) \Rightarrow G = \bigcup_{i\in I} G_i$, but $(G,*)$ is a *D-CGS groupoid* , so there is a finite set J such that $G = \bigcup_{j\in J} G_j$, and hence $A \times G = A \times (\bigcup_{j\in J} G_j) = \bigcup_{j\in J} A \times G_j \Rightarrow (A \times G, \otimes)$ is a *D-CGS groupoid* . □

Theorem 8

If $(G,*)$ and $(\bar{G},\bar{*})$ are two *D-LCCGS groupoids* , then $(G \times \bar{G}, \otimes)$ is a *D-LCCGS groupoid* .

Proof .

Let $(x,y) \in G \times \bar{G} \Rightarrow x \in G$ and $y \in \bar{G}$, but $(G,*)$ is a *D-LCCGS groupoid* $\Rightarrow \exists G_x \subseteq G$ such that $x \in G_x$ and $(G_x,*)$ is a cyclic group , also $(\bar{G},\bar{*})$ is a *D-LCCGS groupoid* $\Rightarrow \exists \bar{G}_y \subseteq \bar{G}$ such that $y \in \bar{G}_y$ and $(\bar{G}_y,\bar{*})$ is a cyclic group . $\Rightarrow (x,y) \in G_x \times \bar{G}_y \subseteq G \times \bar{G}$ and $G_x \times \bar{G}_y$ is a cyclic group $\Rightarrow (G \times \bar{G}, \otimes)$ is a *D-LCCGS groupoid* . □

Theorem 9

If $(G,*)$ and $(\bar{G},\bar{*})$ are two *D-CGS groupoids* , then $(G \times \bar{G}, \otimes)$ is a *D-CGS groupoid* .

Proof .

Let $(G,*)$ and $(\bar{G},\bar{*})$ are *D-CGS groupoids* $\Rightarrow$ there exists a *D-cover group* of $(G,*)$ say $\{G_a\}_{a\in A}$ and a *D-cover group* of $(\bar{G},\bar{*})$ say $\{\bar{G}_b\}_{b\in B}$

[7] $\Rightarrow G \times \bar{G} = (\bigcup_{a\in A} G_a) \times (\bigcup_{b\in B} \bar{G}_b) = \bigcup_{a\in A, b\in B}(G_a \times \bar{G}_b)$

$\Rightarrow \{G_a \times \bar{G}_b\}_{a\in A, b\in B}$ is a *D-cover group* of $(G \times \bar{G}, \otimes)$.

Let $\{\mathcal{W}_i\}_{i\in I}$ be any *D-cover group* of $(G \times \bar{G}, \otimes) \Rightarrow G \times \bar{G} = \bigcup_{i\in I} \mathcal{W}_i$, such that $\mathcal{W}_i = \mathcal{U}_i \times \mathcal{V}_i$, where $\{\mathcal{U}_i\}_{i\in I}$ and $\{\mathcal{V}_i\}_{i\in I}$ are groups with respect to the same induced operations $*$ and $\bar{*}$, respectively . But $(G,*)$ is a *D-CGS groupoid* , so there is a *D-cover group* of $(G,*)$ contains $\{\mathcal{U}_i\}_{i\in I}$ which have a finite *sub-D-cover group* (*i.e.* there is a finite set J) such that $G = \bigcup_{j\in J} \mathcal{U}_j$, let $\mathcal{U}_{j_1} \in \{\mathcal{U}_j\}_{j\in J} \Rightarrow \{\mathcal{U}_{j_1} \times \mathcal{V}_i\}_{i\in I}$ is a *D-cover group* of the *D-CGS groupoid* $(\mathcal{U}_{j_1} \times \bar{G}, \otimes)$

[5] See [2]
[6] See [2]
[7] See [2]

(from Theorem 7 since $(\mathcal{U}_{j_l},*)$ is a group and $(\bar{G},\bar{*})$ is a *D-CGS groupoid*) , so there is a finite set S such that $\mathcal{U}_{j_l} \times \bar{G} = \cup_{s\in S}(\mathcal{U}_{j_l} \times \mathcal{V}_s) = \mathcal{U}_{j_l} \times (\cup_{s\in S} \mathcal{V}_s)$

$\Rightarrow \cup_{j\in J}(\mathcal{U}_j \times (\cup_{s\in S} \mathcal{V}_s)) = (\cup_{j\in J} \mathcal{U}_j) \times (\cup_{s\in S} \mathcal{V}_s) = G \times \bar{G}$

$\Rightarrow G \times \bar{G} = (\cup_{j\in J} \mathcal{U}_j) \times (\cup_{s\in S} \mathcal{V}_s) = \cup_{j\in J, s\in S} (\mathcal{U}_j \times \mathcal{V}_s)$.

Therefore $G \times \bar{G}$ is a *D-CGS groupoid* . □

It is easy to prove the following Corollary ;

Corollary 2

If $(G,*)$ is a *D-CGS groupoid* ,then $(G^2, \otimes)$ is a *D-CGS groupoid* .

By induction we can prove the following theorem ,

Theorem 10

The product of any finite collection of *D-CGS groupoids* is *D-CGS groupoid* .

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