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Wave Equation Applications in Peridynamic Model

Hisham M. Ali Hasan
University of Al-Qadisiya
College of Computer Science and Mathematics, Department of Mathematics
E-mail: houshammah@gmail.com

Husham Rahman
University of Al-Qadisiya
College of computer Science and Mathematics, Department of Mathematics
E-mail: Hishamrehman@rambler.ru

Rasha Hamza Abed
University of Al-Qadisiya
College of computer Science and Mathematics, Department of Mathematics
E-mail: Lordfive89@yahoo.com

Abstract

In this paper, We develop a functional analytical framework for a linear peridynamic model of a spring system in one dimension. Using wave equation in peridynamic model, and discuss the convergence of the solution of wave equation by convergence the horizon.

Keyword: Peridynamic , Mathematical analysis , Convergence Solution.

1. Intiroduction

The peridynamics, continuum theory that employs a nonlocal model of force interaction. Specifically, the stress/strain relationship of classical elasticity is replaced by an integral operator that sums internal forces separated by a finite distance [1] . The peridynamic theory is alternative based on integral[3].The relation between general linear peridynamic model and the classical Navier equation[5].Rather differential equation the purpose of peridynamic theory is provide more generalizes or other framework than the classical theory for problems involving discontinuities or other singularities in the deformation the integral equation express nonlocal force model that describes long-rang material interaction the convergence peridynamic model to classical elasticity theory by the limit small the horizon, i.e $\delta \to 0$ [8]. Such properties make peridynamic theory a powerful tool for modeling problems involving cracks, interfaces or defects. It explained in [11] how the general state-based peridynamic material model converges to the continuum elasticity model as the ratio of the peridynamic horizon to effective length scale decreases, assuming that the underlying deformation is sufficiently smooth.

In this paper, by mathematical analysis we discuss convergence of the solution of wave equation, because the mathematical analysis gives a good indication that the mouthed used to solve the model. We take the wave equation because its application to Newton's two Law.

2. The Predynamic Model [4][6][10]

The peridynamic is the second-order in time partial integro-differential equation

$$\rho \ddot{u} = \int_{R^\circ} f(x, x`, u(x,t), u(x`,t))dx` + b(x,t) \tag{1}$$

where ρ denotes the mass density , *u* the displacement field of the body , *f* the pairwise force function that describes the internal forces, and b an in homogeneity that collects all external forces per unit volume. By t >0, the time under consideration is denoted,and R^{o} denote the open ball of radius δ where δ>0 is the so-called peridynamic horizon of interaction such that :

$$R^\circ = \{x \in \Omega : |x`-x| \le \delta\}$$

Where Ω is sub region of R(the set of real number)

The assumption of no explicit time dependence, and Newton's third law lead to

$$f(x, x`, u, u`) = f(x`-x, u`-u) \tag{2}$$

with

$$f(x`, x, -\eta) = -f(x, x`, \eta), \forall x, x`, \eta = u`-u.. \tag{3}$$

It is typical for the peridynamic model to require

$$f(x, x`, \eta) = 0, if\,|x`-x| \ge \delta... \tag{4}$$

A first-order approximation justifies for small relative displacements

$$f(x, x`, \eta) = f_0(x, x`) + C(x, x`).\eta \tag{5}$$

with the stiffness tensor (or micromodulus function)[12] C =C(x, x`) and denoting forces in the reference configuration $f_0$ without loss of generality, we may assume $f \equiv 0$ since otherwise $f_0$ can be incorporated into the right-hand side b In general, the stiffness tensor C is neither definite nor depending on only $|x`-x|$ the length However, C has to be symmetric with respect to its arguments as well as with respect to its tensor structure such that

$$C(x`, x) = C(x, x`)..$$

$$C(x`, x) = C(x, x`).. \tag{6}$$

$$C(x`, x)^T = C(x`, x)... \tag{7}$$

$$C(x`, x) = 0 if\,|x`-x| \ge \delta$$ view of (4), we shall require

The stiffness tensor can be shown to read as:

$$C(x, x`) = \lambda_\delta(|x`-x)|(x`-x) \otimes (x`-x) \tag{8}$$

For the special case of proportional materials the equation (8)take the form[2] :

$$C(x, x`) = \frac{c_\delta}{|x`-x|^3} \tag{9}$$

The linear peridynamic equation of motion (1) now reads as

$$\rho \ddot{u} = c_\delta \int_{R^\circ} \frac{(x`-x) \otimes (x`-x)}{|x`-x|^3} u(x`,t) - u(x,t)dx` \tag{10}$$

In this paper we discuss case steady –state ,one dimensional and homogenous and linear model ,along with " boundary" condition, the eq (10) and eq(1) reduces to:

$$\rho \ddot{u} = \frac{1}{\delta^2} \int_{R^\circ} \frac{u(x`,t) - u(x,t)}{|x`-x|^3} dx`. + b \tag{11}$$

We denote $L_\delta = \frac{1}{\delta^2} \int_{R^\circ} \frac{u(x`,t) - u(x,t)}{|x`-x|^3} dx`$ (12)

We called $\sigma|x`-x| = \frac{1}{|x`-x|^3}$ is kernel function of the peridynamic integral operator which also determines the micromodulus function.

3. Mathematical Analysis for Peridynamic Model[5]

In section one we defined the displacement $\eta = u(x`) - u(x) = (\nabla u)^T (x`-x)$

Since the deformation is elastic homogenous material $(\nabla u)^T = (\nabla u)$

The Taylor expansion of

$$u(x`) - u(x) = \frac{((x`-x)\nabla)u\ (x)}{1!} + \frac{((x`-x)\nabla)^2 u\ (x)}{2!} + \frac{((x`-x)\nabla)^3 u(x)}{3!} + ..$$

$$= \sum_{k=1}^{m} \frac{((x`-x)\nabla)^k u(x)}{k!} \tag{13}$$

Inserting Taylor expansion into the definition of L_δ in eq(12) yields

$$L_\delta u(x) = \sum_{k=1}^{m} L_\delta^k u(x). \tag{14}$$

$$L_\delta^k u(x) = \frac{1}{k!\delta^2} \int_{x-\delta}^{x+\delta} \frac{1}{|x`-x|} ((x`-x)\nabla)^k u(x) dx` \tag{15}$$

We can write (15) form:

$$L_\delta^k u(x) = \frac{1}{k!\delta^2} \int_{x-\delta}^{x+\delta} \frac{1}{|x`-x|} (x`-x)^k u^k (x) dx` \tag{16}$$

$$L^1 u(x) = \frac{1}{\delta^2} \int_{x-\delta}^{x+\delta} \frac{(x`-x)}{|x`-x|} dx` u`(x) = \frac{1}{\delta^2} |x`-x|_{x-\delta}^{x+\delta} u``(x) = 0 \tag{17}$$

$$L^2 u(x) = \frac{1}{\delta^2} \int_{x-\delta}^{x+\delta} \frac{(x`-x)^2}{|x`-x|} dx` u``.$$

$$u(x`) = (x`-x) \qquad du = dx`, u = u(x)$$

$$dv = \frac{(x`-x)}{|x`-x|} \qquad v = |x`-x| \tag{18}$$

$$= [(x`-x)|x`-x|]_{x-\delta}^{x+\delta} - \int_{x-\delta}^{x+\delta} |x`-x| dx`$$

$$= \frac{1}{2} u``(x)$$

$$L^3u = [(x`-x)^2|x`-x|]_{x-\delta}^{x+\delta} - 2\int_{x-\delta}^{x+\delta}|x`-x|(x`-x)dx`=$$

$$u(x) = (x`-x) \qquad dv = |x`-x|$$

$$du = dx` \qquad v = \frac{|x`-x|^2}{2} \tag{19}$$

$$= [(x`-x)^2|x`-x|]_{x-\delta}^{x+\delta} - [(x`-x)|x`-x|]_{x-\delta}^{x+\delta} + \int_{x-\delta}^{x+\delta}|x`-x|^2 \, dx`$$

$$= 0$$

$$\vdots$$

Using eq(17),(18),(19) in eq(14) we have:

$$L_\delta u(x) = \frac{1}{2}u``(x) + \frac{1}{48}\delta^2 u````(x) + \cdots \tag{20}$$

It follows $L_\delta^k u(x) = 0$ if k is odd since then the integrand is an odd function in $x`-x$. Which we can use to determine the right-hand side of (20) for polynomial exact solutions $u(x)$.

Application[9]

The wave equation $u(x,t) = \cos(x)\cos(t)$ which represent the exact solution of wave equation $u_{tt} - u_{xx} = 0$ [7].prove $u(x,t)$ is solution of the model in eq(12)

Solve:

we prove $u(x,t)$ is the solution of the model by mathematical analysis:

We apply $u(x,t)$ in eq(12)we have

$$L_\delta = \frac{1}{\delta^2}\int_{x-\delta}^{x+\delta}\frac{\cos(x`)\cos(t) - \cos(x)\cos(t)}{|x`-x|}dx`$$

Using eq(20) we have;

$$-L_\delta = \frac{1}{2}(-\cos(x)\cos(t)) + \frac{1}{48}(\cos(x)\cos(t))\delta^2 + \ldots$$

We note the above solutions , the derivatives of $u(x,t)$ is frequency, we cannot determined the convergence of the solutions, we take

$$|-L_\delta| = \left|-\frac{1}{2}\cos(x)\cos(t) + \frac{1}{48}\cos(x)\cos(t)\delta^2 - \ldots\right| = \cos(x)\cos(t)(\frac{1}{2} + \frac{1}{48}\delta^2 + \ldots)$$

since δ convergence to 0

then $\left(\frac{1}{2} + \frac{1}{48}\delta^2 + \ldots\right)$ is convergence of 0

then $\cos(x)\cos(t)\left(\frac{1}{2} + \frac{1}{48}\delta^2 + \ldots\right)$ is convergence

By convergence absolutely, if $|-L_\delta|$ is convergence then $-L_\delta$ is convergence.

The convergence of the solutions series dependent on the horizon δ ,the horizon δ control on convergence of the solution.

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