Certain Types of Compact Spaces

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Abstract

In this paper , we used the concept of generalized closed (g-closed) and generalized compact (g-compact) sets to construct a new types of compact spaces and functions which are compactly generalized closed space (cgc-space) , generalized compactly generalized closed space and generalized coercive function (g-coercive) and investigate the properties of these concepts .

Keywords: g-open , g-closed , cgc-space , gcgc-space and g-coercive function .

Introduction

This concept of generalized closed (g-closed) set was introduced by Levin N. [1] and studied its properties. Selvarani S. [2] gave the definition of g-neighborhood of a point $x \in X$, gT_2 -space and g-compact space . The generalized closure of $A \subseteq X$ is the intersection of all g-closed sets which contain A and denoted by $gcl(A)$ [1] . In [4] Balachandran K. , Sundaram P. and Maki H. introduced the certain types of continuous functions. Finally in [3] Ali J. H. and Mohammed J. A. defined certain type of compact functions. We use T_{ind} to denote the indiscrete topology on a non-empty sets X and T_U to denote the usual topology on the set of real numbers R. Throughout this paper (X,T) and (Y,T) (or simply X and Y) represent to non-empty topological spaces on which no separation axiom are assumed , unless otherwise mentioned .

1. Basic Definitions and Notations:

1.1. Definition [1]:

A subset A of a topological space X is called generalized closed (for brief g-closed) set if $cl(A) \subseteq U$ for every open set U in X contains A . The complement of g-closed set is called g-open set.

1.2. Example:

Let $X = \{1, 2, 3\}$ with $T = T_{\text{ind}}$, then $A = \{1\}$ is g-closed set.

1.3. Example:

Let $X = R$, $T = T_U$, then $A = (a, b)$ is not g-closed set.

1.4. Remark [1]:

(i) Every closed set is g-closed.

(ii) Every open set is g-open.

The converse of (i , ii) in remark (1.4) is not true in general as the following example shows:

1.5. Example:

In example (1.2) , $A = \{1\}$ is g-closed set but not closed and $B = \{2,3\}$ is g-open but its not open .

1.6. Theorem [1]:

A subset A of a topological space X is g-closed set if and only if $cl(A)$-A contains no non-empty closed set.

1.7. Theorem [1]:

A subset A of a topological space X is g-open if and only if $F \subseteq int(A)$, for every closed set F in X contained in A.

1.8. Theorem [1]:

Let X be a topological space, Y is a closed (open) set in X . Then:

(i) If B is g-closed (g-open) set in X then $B \cap Y$ is g-closed (g-open) set in X .

(ii) If B is g-closed (g-open) set in X then $B \cap Y$ is g-closed (g-open) set in Y.

1.9. Theorem [1]:

Let X be a topological space and $B \subseteq Y \subseteq X$. Then:

(i) if B is g-closed (g-open) set in Y and Y is g-closed (g-open) set in X , then B is g-closed (g-open) set in X .

(ii) if B is g-closed (g-open) set in X then B is g-closed (g-open) in Y .

Note that if B is g-closed (g-open) in Y then B not necessary be g-closed (g-open) set in X as the following example shows:

1.10. Example:

Let $X = R$ with $T = T_U$ and $Y = \{1 , 2\}$, then $B = \{1\}$ is g-open set in Y, but B is not g-open in R .

1.11. Definition [2]:

Let X be a topological space and $A \subseteq X$. A generalized neighborhood of A (for brief g-neighborhood) is any subset of X which contains g-open set containing A . The family of all g-neighborhoods of a subset A of X denoted by $\mathcal{N}_g(A)$ and the family of all g-neighborhoods of $x \in X$ denoted by $\mathcal{N}_g(x)$.

1.12. Definition [3]:

A topological space X is called generalized Hausdorff (for brief gT_2) if for any two distinct points $x, y \in X$ there are disjoint g-open sets U , V of X such that $x \in U$ and $y \in V$.

1.13. Remark [3]:

Every T_2-space is gT_2-space . But the converse is not true in general. In example (1.2) , X is gT_2-space. But X is not T_2-space.

1.14. Remark [2]:

The intersection of two g-closed sets need not be g-closed and the union of two g-open sets need not be g-open as the following example shows:

1.15. Example:

Let $X = \{a , b , c\}$ and $T = \{\emptyset , X , \{a\}\}$ be a topology on X , then $\{a , b\}$ and $\{a , c\}$ are g-closed sets in X , but $\{a , b\} \cap \{a , c\} = \{a\}$ is not g-closed set and $\{b\}, \{c\}$ are g-open sets but $\{b\} \cup \{c\} = \{b, c\}$ is not g-open .

1.16. Definition [2]:

A topological space X is called generalized multiplicative space (IG-space) if arbitrary intersection of g-closed sets of X is g-closed set .

1.17. Remark [2]:

(i) $gcl(A)$ need not be g-closed, since the intersection of g-closed sets is not to be g-closed .

(ii) $x \in gcl(A)$ if and only if for every g-open set U containing x , $U \cap A \neq \emptyset$.

(iii) If X be an IG-space, then $gcl(A)$ is g-closed set.

(iv) Every T_1-space is an IG-space.

1.18. Definition [4]:

Let $f: X \to Y$ be a function from a topological space X into a topological space Y, then f is called:

(i) generalized continuous (g-continuous) function if $f^{-1}(A)$ is g-closed set in X for every closed set A in Y .

(ii) generalized irresolute continuous (gI-continuous) function if $f^{-1}(A)$ is g-closed set in X for every g-closed set A in Y .

1.19. Definition [4]:

A function $f: X \to Y$ is called:

(i) generalized closed (g-closed) if $f(B)$ is g-closed set in Y for every closed set B in X .

(ii) generalized irresolute closed (gI-closed) function if $f(B)$ is g-closed set in Y for every g-closed set B in X .

1.20. Definition [4]:

A function $f: X \to Y$ is called:

(i) generalized open (g-open) function if $f(B)$ is g-open set in Y for every open set B in X .

(ii) generalized irresolute open (gI-open) function if $f(B)$ is g-open set in Y for every g-open set B in X .

1.21. Definition [3]:

A topological space X is called generalized compact (g-compact) space if every g-open cover of X has finite subcover.

1.22. Remark [5]:

Every g-compact space is compact. The converse is not true in general as the following example shows:

1.23. Example [5]:

Let $X = \{x\} \cup \{ x_i : i \in I \}$, I uncountable, $T = \{\emptyset, X , \{x\}\}$ be a topology on X . Then X is compact but is not g-compact, since $\{\{x , x_i\}: i \in I\}$ is g-open cover of X and has no finite subcover.

1.24. Theorem [2] ,[3],[5]:

(i) Every g-closed subset of g-compact space is g-compact.
(ii) The intersection of g-compact subset with g-closed subset is g-compact.
(iii) Every g-compact subspace of gT_2-space is g-closed.
(iv) Every finite subset is g-compact.
(v) Every T_1 compact space is g-compact.

1.25. Theorem [3]:

(i) Let X be a topological space and F is g-closed subset of X . Then $F \cap K$ is g-compact in F for every g-compact set K in X .

(ii) Let Y be a g-open set of a topological X and $K \subseteq Y$, then K is g-compact set in Y if and only if K is g-compact set in X .

1.26. Theorem [3]:

(i) Let f be gI-continuous function from g-compact space X onto a topological space Y , then Y is g-compact space.
(ii) Let $f: X \rightarrow Y$ be gI-continuous function , then the image $f(A)$ of any g-compact set A in X is g-compact set in Y .
(iii) Let f be gI-continuous function from g-compact space X into a gT_2-space Y is gI-closed.

1.27. Definition [3]:

Let $f: X \rightarrow Y$ be a function , then f is called generalized irresolute compact (gI-compact) if $f^{-1}(K)$ is g-compact set in X for every g-compact set K in Y .

1.28. Definition [6]:

A set D is called a directed if there is a relation $\leq$ on D satisfying:
(i) $d \leq d$ for each $d \in D$.
(ii) If $d_1 \leq d_2$ and $d_2 \leq d_3$ then $d_1 \leq d_3$.
(iii) If $d_1, d_2 \in D$, there is some $d_3 \in D$ with $d_1 \leq d_3$ and $d_2 \leq d_3$.

1.29. Definition [7]:

A net in a set X is a function $\chi: D \rightarrow X$, where D is directed set. The point $\chi(d)$ is usually denoted by χ_d .

1.30. Definition [7]:

A subnet of a net $\chi: D \rightarrow X$ is the composition $\chi o \varphi$, where $\varphi: M \rightarrow D$ and M is directed set, such that :
(i) $\varphi(m_1) \leq \varphi(m_2)$, where $m_1 \leq m_2$.
(ii) For all $d \in D$ there is some $m \in M$ such that $d \leq \varphi(m)$ for $m \in M$. The point $\chi o \varphi(m)$ is often written χ_{dm} .

1.31. Definition [7]:

Let $(\chi_d)_{d\in D}$ be a net in a topological space X and $A \subseteq X$, $x \in X$ then:

(i) $(\chi_d)_{d\in D}$ is eventually in A if there is $d_0 \in D$ such that $\chi_d \in A$ for all $d \geq d_0$.

(ii) $(\chi_d)_{d\in D}$ is frequently in A if for all $d \in D$ there is $d_0 \in D$ with $d \geq d_0$ such that $\chi_{d_0} \in A$.

1.32. Definition [5]:

Let $(\chi_d)_{d\in D}$ be a net in a topological space X , $x \in X$. Then $(\chi_d)_{d\in D}$ is said to be generalized converges to a point x (for brief g-converges) if $(\chi_d)_{d\in D}$ eventually in every g-neighborhood of x (written $\chi_d \overset{g}{\rightarrow} x$) . A point x is called generalized limit point (for brief g-limit point) of $(\chi_d)_{d\in D}$.

1.33. Theorem:

Let X be a topological space and $A \subseteq X$, $x \in X$. Then $x \in gcl(A)$ if and only if there is a net $(\chi_d)_{d\in D}$ in A such that $\chi_d \overset{g}{\rightarrow} x$.

Proof:

Suppose that there is a net $(\chi_d)_{d\in D}$ in A such that $\chi_d \overset{g}{\rightarrow} x$. To prove that $x \in gcl(A)$. Let $U \in \mathcal{N}_g(x)$, since $\chi_d \overset{g}{\rightarrow} x$, there is $d_0 \in D$ with $\chi_d \in U$ for all $d \geq d_0$. But $\chi_d \in U$ for all $d \in D$. So $A \cap U \neq \emptyset$ for all $U \in \mathcal{N}_g(x)$. By remark (1.17.ii) , $x \in gcl(A)$.

Conversely:

Suppose that $x \in gcl(A)$. To prove that there is a net $(\chi_d)_{d\in D}$ in A such that $\chi_d \overset{g}{\rightarrow} x$.
Since $x \in gcl(A)$, by remark (1.17.ii) , $A \cap U \neq \emptyset$ for all $U \in \mathcal{N}_g(x)$. Then $D = \mathcal{N}_g(x)$ is directed set by inclusion. Since $\cap U \neq \emptyset$ $\forall$ $U \in \mathcal{N}_g(x)$, there is $\chi_U \in A \cap U$.
Define $\chi: D \rightarrow A$ by $\chi(U) = \chi_U$ for all $U \in \mathcal{N}_g(x)$. Hence $(\chi_U)_{U\in\mathcal{N}_g(x)}$ is a net in A . To prove that $\chi_U \overset{g}{\rightarrow} x$. Let $U \in \mathcal{N}_g(x)$ to find $d_0 \in D$ such that $\chi_d \in U$ for all $d \geq d_0$. Let $d_0 = U$, then for all $d \geq d_0$ we have $d = V \in \mathcal{N}_g(x)$ i.e., $V \geq U \Leftrightarrow V \subseteq U$.
$\chi_d = \chi(d) = \chi(V) = \chi_V \in V \cap A \subseteq V \subseteq U$, then $\chi_V \in U$ for all $d \geq d_0$. Thus $\chi_U \overset{g}{\rightarrow} x$.

1.34. Corollary:

Let X be a topological space and $A \subseteq X$, $x \in X$. Then $x \in gcl(A)$ if and only if there is a net $(\chi_d)_{d\in D}$ in A such that $\chi_d \overset{g}{\propto} x$.

1.35. Theorem [8]:

Let X be a T_2-space. Then X is g-compact if and only if every net in X has a g-cluster point in X .

1.36. Remark [7]:

Let $f: X \to Y$ be a function from a set X into a set Y , then:

(i) If $(\chi_d)_{d\in D}$ is a net in X , then $\{f(\chi_d)\}_{d\in D}$ is a net in Y .

(ii) If f is onto and $(y_d)_{d\in D}$ be a net in Y , then there is a net $(\chi_d)_{d\in D}$ in X such that $f(\chi_d) = y_d$, for each $d \in D$.

1.37. Theorem:

Let X and Y be topological spaces . A function $f: X \to Y$ is g-continuous if and only if whenever $(\chi_d)_{d\in D}$ is a net in X such that $\chi_d \overset{g}{\to} x$, then $f(\chi_d) \to f(x)$ in Y .

Proof: Clear.

1.38. Corollary:

Let X and Y be topological spaces . A function $f: X \to Y$ is gI-continuous if and only if whenever $(\chi_d)_{d\in D}$ is a net in X such that $\chi_d \overset{g}{\to} x$, then $f(\chi_d) \overset{g}{\to} f(x)$ in Y .

Proof:

Suppose that $f: X \to Y$ is gI-continuous and $(\chi_d)_{d\in D}$ is a net in X such that $\chi_d \overset{g}{\to} x$. To prove that $f(\chi_d) \overset{g}{\to} f(x)$. Let $V \in \mathcal{N}_g\big(f(x)\big)$ in Y, then $f^{-1}(V) \in \mathcal{N}_g(x)$, for some $d_0 \in D$, $d \geq d_0$ implies that $\chi_d \in f^{-1}(V)$. Thus showing that $f(\chi_d) \overset{g}{\to} f(x)$, since $(\chi_d)_{d\in D}$ is eventually in each g-neighborhood of $f(x)$, then by remark (1.36.i), $\{f(\chi_d)\}$ is a net in Y which is eventually in each g-neighborhood of $f(x)$. Therefore $f(\chi_d) \overset{g}{\to} f(x)$.

Conversely:

To prove that f is gI-continuous , suppose not , then there is $V \in \mathcal{N}_g\big(f(x)\big)$ such that $f(U) \not\subset V$ for any $U \in \mathcal{N}_g(x)$. Thus for all $U \in \mathcal{N}_g(x)$ we can $\chi_U \in U$ such that $f(\chi_U) \notin V$, but $(\chi_U)_{U\in\mathcal{N}_g(x)}$ is a net in X with $\chi_U \overset{g}{\to} x$, while $\{f(\chi_U)\}_{U\in\mathcal{N}_g(x)}$ is not g-convergent to $f(x)$. This is a contradiction.

2. Compactly g-closed and g-compactly g-closed spaces:

This section is devoted to a new concept which is called compactly g-closed space and generalized compactly g-closed space. Several various examples , theorems and remarks on these concepts are proved . Furthermore theorems are stated as well as the relationships between these concepts .

2.1. Definition:

Let X be a topological space. A subset $A \subseteq X$ is called compactly generalized closed (for brief cgc-set) if $A \cap K$ is g-compact set for every g-compact set K in X .

2.2. Example:

(i) Every finite subset of a topological space is cgc-set.
(ii) Every subset of indiscrete space is cgc-set.

2.3. Theorem:

Every g-closed subset of a topological space is cgc-set.

Proof:

Let A be a g-closed subset of a topological space X and K be a compact subset of X , by Theorem (1.24.ii) , $A \cap K$ is g-compact set. Thus A is cgc-set.

The converse of theorem (2.3) need not true in general as the following example shows:

2.4. Example:

Let $X = \{a , b, c\}$ and $T = \{\emptyset, X, \{a, b\}\}$ be a topology on X , then $A = \{a, b\}$ is cgc-set but it is not g-closed set.

2.5. Theorem:

Let X be a T_2-space and $A \subseteq X$. Then A is cgc-set if and only if it is g-closed set .

Proof:

Let A be a cgc-set in X and $x \in gcl(A)$. By theorem (1.33) , there is a net $(\chi_d)_{d \in D}$ in A such that $\chi_d \xrightarrow{g} x$. Then $K = \{\chi_d, x\}$ is g-compact set . Since A is cgc-set , then $A \cap K$ is g-compact set in X . But X is a T_2 , then $A \cap K$ is g-closed . Since $\chi_d \xrightarrow{g} x$ and $\chi_d \in A \cap K$, then by theorem (1.33) , $x \in A \cap K$, hence $x \in A$. Thus A is g-closed set.
Conversely: By using Theorem (2.3) .

2.6. Theorem:

Let $f: X \to Y$ is a bijective, gI-continuous , gI-compact function and $A \subseteq X$. Then A is cgc-set in X if and only if $f(A)$ is cgc-set in Y.

Proof:

Let A be a cgc-set in X and let K be a g-compact set in Y. Since f be a gI-compact, then $f^{-1}(K)$ is g-compact set in X. Thus $A \cap f^{-1}(K)$ is g-compact set in X. By theorem (1.26.ii), $f\left(A \cap f^{-1}(K)\right)$ is g-compact set in Y. But $f\left(A \cap f^{-1}(K)\right) = f(A) \cap K$ is g-compact set in Y. Hence $f(A)$ is cgc-set in Y.

Conversely:

Let $f(A)$ be a cgc-set in Y. To prove that A is cgc-set in . Let K be a g-compact set in X. Since f be a gI-continuous, then by theorem (1.26.ii), $f(K)$ is g-compact set in Y. Thus $f(A) \cap f(K)$ is g-compact set in Y, thus $f^{-1}\left(f(A) \cap f(K)\right)$ is g-compact set in X. (since f gI-compact). But $f^{-1}\left(f(A) \cap f(K)\right) = A \cap K$. Thus A is cgc-set in X.

2.7. Theorem:

Let B be a g-open subset of a topological space X. Then B is cgc-set in X if and only if the inclusion function $i: B \to X$ is gI-compact.

Proof:

Suppose that B be a cgc-set and K be a g-compact set in X. Then $B \cap K$ is g-compact set in X, by theorem (1.25.ii), $B \cap K$ is g-compact set in B. But $B \cap K = i^{-1}(K)$, then $i^{-1}(K)$ is g-compact set in B. Thus $i: B \to X$ is gI-compact.

Conversely:

Let K be a g-compact set in X, since $i: B \to X$ is gI-compact. Then $i^{-1}(K) = B \cap K$ is g-compact set in B, thus by theorem (1.25.ii), $B \cap K$ is g-compact set in X for every g-compact set K in X, Therefore B is cgc-set in X.

2.8. Definition:

A subset A of a topological space X is said to be generalized compact generalized closed set (for brief gcgc-set), if $A \cap K$ is g-closed set in X for every g-compact set K in X.

2.9. Example:

Every subset of a discrete space is gcgc-set.

2.10. Remark:

Not every set of a topological space is gcgc-set as the example (2.4) shows.

2.11. Theorem:

Every gcgc-set in a topological space is gcg-set.

Proof:

Let A be a gcgc-set of a topological space X and let K be a g-compact subset of X. Then $A \cap K$ is g-closed set in X . Since $A \cap K \subseteq K$, then by remark (1.24.i) , $A \cap K$ is g-compact set . Therefore A is cgc-set in X .

2.12. Theorem:

Let X be a T_2-space and $A \subseteq X$, the following statements are equivalent:

(i) A is cgc-set.

(ii) A is gcgc-set.

(iii) A is g-closed set.

Proof:

(**i** $\Longrightarrow$ **ii**) Let A is cgc-set in X and let K be a g-compact set in X . Then $A \cap K$ is g-compact set in X . Since X is a T_2-space , then by theorem (1.24.iii) , $A \cap K$ is g-closed set in X . Thus A is gcgc-set in X .

(**ii** $\Rightarrow$ **i**) By using theorem (2.11) .

(**iii** $\Rightarrow$ **i**) By using theorem (2.3) .

2.13. Remark:

If X is not T_2-space , then it is not necessary that cgc-set is gcgc-set as the following example shows:

Let $X = \{a, b, c\}$ and $T = \{\ U \subseteq X : a \in U\ \} \cup \{\emptyset\}$ be a topology on X , clear that (X, T) is not T_2-space . Since $\{a, b\}, \{b\} \subset X$ and $\{b\}$ is g-compact set in X and $\{a, b\} \cap \{b\} = \{b\}$ is g-closed but $\{a, b\}$ is not g-closed set.

Recall that a bijective function $f: X \rightarrow Y$ is called generalized irresolute homeomorphism (gI-homeomorphism) if f and f^{-1} are gI-continuous [7].

2.14. Theorem [9]:

A bijection function $f: X \rightarrow Y$ is gI-homeomorphism if f is gI-continuous and gI-open (gI-closed) function.

2.15. Theorem:

The following conditions on a Hausdorff space Y are equivalent:

(i) The only g-open subset of Y which is gcgc-set is the whole space and the empty set.

(ii) Every gI-open , gI-continuous and gI-compact function from a topological space X into Y is onto .

(iii) Every one to one , gI-open , gI-continuous and gI-compact function from a topological space X into Y is gI-homeomorphism.

Proof:

(**i** ⇒ **ii**) Let $f: X \rightarrow Y$ be a gI-open , gI-continuous and gI-compact function . Since X is non - empty g-open set , then $f(X)$ is non-empty g-open set in Y. To prove $f(X)$ is gcgc-set in Y. Let K be a g-compact set in Y then $f^{-1}(K)$ is g-compact set in X , since f is gI-compact. Thus by theorem (1.26.ii) , $f(f^{-1}(K))$ is g-compact set in Y. By theorem (1.24.iii) , $f(f^{-1}(K))$ is g-closed set in Y. Since $f(X) \cap K = f\big(f^{-1}(K)\big)$, then $f(X) \cap K$ is g-closed set in Y. So $f(X)$ is gcgc-set. But $f(X) \neq \emptyset$, then $f(X) = Y$. Thus f is onto.

(**ii** ⇒ **iii**)Let $f: X \rightarrow Y$ be an one to one, gI-open, gI-continuous and gI-compact function. Then by (ii) , f is onto and one to one , hence it is bijection . Then by theorem (2.14) , f is gI-homeomorphism .

(**iii** ⇒ **i**) Let A be a non-empty g-open subset of Y which is gcgc-set. Then by theorem (2.11) , A is cgc-set , since A is g-open . Then by theorem (2.7) , the inclusion function $i: A \rightarrow Y$ is gI-compact. To prove $i: A \rightarrow Y$ is gI-continuous, let B is g-open set in Y, then $A \cap B$ is g-open set. But $A \cap B = i^{-1}(B)$ is g-open set in A . Thus , the inclusion function is gI-continuous, by (iii) , the inclusion function is gI-homeomorphism . Thus $A = Y$, this complete proof .

2.16. Definition:

A topological space X is said to be compactly generalized closed space (for brief cgc-space) if every cgc-set of X is g-closed.

2.17. Example:

(i) Every indiscrete space is cgc-space.
(ii) Every T_2-space is cgc-space.

2.18. Remark:

The example in remark (2.13) shows that not every topological space is cgc-space.

2.19. Theorem:

Let X be a topological space and Y is cgc-space. Then every gI-continuous and gI-compact onto function $f: X \rightarrow Y$ is gI-closed.

Proof:

Let F be a g-closed subset of X. To prove that $f(F)$ is g-closed subset of Y. Let K be a g-compact subset of Y . Since f is gI-compact, then $f^{-1}(K)$ is g-compact set in X.
By remark (1.24.ii) , $F \cap f^{-1}(K)$ is g-compact set in X .
Since f is gI-continuous, then by theorem (1.26.ii), $f(F \cap f^{-1}(K))$ is g-compact set of Y. But $f\big(F \cap f^{-1}(K)\big) = f(F) \cap K$, thus $f(F) \cap K$ is g-compact set of Y. Hence $f(F)$ is cgc-set in Y. Since Y is cgc-space, then $f(F)$ is g-closed set in Y . Thus f is gI-closed function.

2.20. Definition:

A topological space X is said to be generalized compactly generalized closed (for brief gcgc-space) if every gcgc-set of X is g-closed.

2.21. Example:

(i) Every T_2-space is gcgc-space.
(ii) Every indiscrete space is gcgc-space.

2.22. Theorem:

Let X be a T_2-space . Then cgc-space and gcgc-space are equivalent.

Proof: By using theorem (2.12) .

2.23. Definition:

Let X and Y be topological spaces. A function $f: X \to Y$ is called generalized coercive (for brief g-coercive) if for every g-compact subset B of Y there is g-compact subset A of X such that $f(X \setminus A) \subseteq (Y \setminus B)$.

2.24. Example:

The identity function of any topological space is g-coercive.

2.25. Theorem:

If $f: X \to Y$ is a function, such that X is g-compact space , then f is g-coercive.

Proof:

Let B be a g-compact subset of Y . Since X is g-compact space. Then $(X \setminus X) = f(\emptyset) = \emptyset \subseteq f(Y \setminus B)$. Thus f is g-coercive function.

2.26. Theorem:

Let X and Y be T_2-spaces and $f: X \to Y$ is gI-continuous function. Then f is g-coercive if and only if f is gI-compact .

Proof:

Suppose that f is g-coercive and let B be a g-compact subset of Y. To prove that f is g-compact , since Y is T_2-space then by (1.24.iii) , B is g-closed but f is gI-continuous . Then $f^{-1}(B)$ is g-closed subset of X . Since f is g-coercive function , then there is a g-compact set A in X such that $f(X \setminus A) \subseteq (Y \setminus B)$.

since $f^{-1}(B)$ is g-closed, then by corollary (1.34) , every net in $f^{-1}(B)$ has g-cluster in itself itself . Then by theorem (1.35) , $f^{-1}(B)$ is g-compact subset in X . Therefore f is gI-compact function.

Conversely: By using theorem (2.25) .

2.27. Theorem:

Let X and Y be topological spaces and $f: X \to Y$ be a function. Then:

(i) If $f: X \to Y$ be a g-coercive function with F is g-closed and open subset of X , then the restriction function $f_{/F}: F \to Y$ is g-coercive .

(ii) If X is g-compact and F is g-closed subset of X , then $f_{/F}: F \to Y$ is g-coercive function.

Proof:

(i) Let B be a g-compact subset of Y , since f is a g-coercive . Then there is a g-compact subset A of X such that $f(X \setminus A) \subseteq (Y \setminus B)$.

Since F is g-closed subset of X , then by theorem (1.24.ii) , $F \cap A$ is g-compact set in X. Since F is open in X ,by theorem (1.25.ii), $F \cap A$ is g-compact set in F.

Since $f_{/F}(F \cap A) = f(F \setminus A)$ and $F \setminus A \subseteq X \setminus A$, then $f(F \setminus A) \subseteq f(X \setminus A)$.

Thus $f_{/F}(F \setminus F \cap A) \subseteq Y \setminus B$, hence $f_{/F}: F \to Y$ is g-coercive.

(ii) By using theorem (2.25) and (i) .

2.28. Theorem:

A composition of two g-coercive functions is g-coercive.

Proof:

Let $f: X \to Y$ and $h: Y \to Z$ be a g-coercive functions . Let C is a g-compact subset of Z , then there is a g-compact subset B of Y such that $h(Y \setminus B) \subseteq Z \setminus C$.

Since f is a g-coercive , then there is a g-compact subset A of X such that $(X \setminus A) \subseteq Y \setminus B$. So $h(f(X \setminus A)) \subseteq h(Y \setminus B)$, but $h(Y \setminus B) \subseteq Z \setminus C$. Hence $h(f(X \setminus A)) = hof(X \setminus A) \subseteq Z \setminus C$, therefore hof is g-coercive function.

2.29. Theorem:

If $f: X \to Y$ is bijective , gI-compact and $h: Y \to Z$ is a g-coercive function, then hof is g-coercive function .

Proof:

Let C be a g-compact subset of Z , then there is a g-compact subset B of Y such that $h(Y \setminus B) \subseteq Z \setminus C$. Put $A = f^{-1}(B)$, since f is gI-compact then A is a g-compact subset of X.

Thus $hof(X \setminus A) = h(f(X \cap A^c)) = h(f(X) \cap f(A^c))$.

Since f is a bijective, then $hof(X \setminus A) = h(f(X) \cap f(A^c)) = h(Y \setminus f(f^{-1}(B))^c = h(Y \cap B^c)$ $= h(Y \setminus B) \subseteq Z \setminus C$. Thus hof is g-coercive function.

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أنماط معينة من الفضاءات المرصوصة

من قبل

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المستخلص :

في هذا البحث أستخدمنا مفهومي المجموعات المغلقة المعممة (المغلقة-g) والمرصوصة المعممة (المرصوصة-g) لأنشاء أنواع جديدة من الفضاءات المرصوصة والدوال أسميناها الفضاءات المرصوصة المغلقة المعممة (الفضاءات -cgc) والفضاءات المعممة المرصوصة المعممة (الفضاءات-gcgc) والدوال الأضطرارية المعممة (الأضطرارية -g) ودرسنا خواص هذه المفاهيم .