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Asymptotic solution for linear quadratic optimal control problems with quadratic small parameter

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Abstract:

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Keywords: asymptotic, singularly perturbed , linear-quadratic optimal , boundary function, small parameter.

Mathematics subject classification :34E15

Asymptotic solution for linear quadratic optimal control problems with quadratic small parameter

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I.Introduction:

The theory of optimal control system began develop in the 1960s due practical needs and general inters to control problem. Most papers devoted to problems of optimal control with a small parameter. Asymptotic analysis of solution is based on construction of expansion of solution by definition a series of boundary function and postulated this expansion into condition .

Vasilive in [13] A study optimal control system of perturbed linear, quadratic problems with one small parameter □ and he is not using the boundary-layer function method. Melnik in [10] study asymptotics of solutions of discontinuous singularly Perturbed problems and he finds the solution by divided the interval of solutions to several subintervals such that the problem is continuous on that subinterval.

In this paper, we are using a direct scheme for a boundary functions for singularly perturbed linearly quadratic problems of the optimal controls with quadratic small parameter . The asymptotic of the solution containing the boundary-layer functions of four types is constructed.

II.view Problem:

Consider minimum functional problem:

$$J_\epsilon(u) = \frac{1}{2}\int_0^T [< X, W(t)X > + < u, R(t)u >]dt \dots\dots\dots\dots\dots\dots\dots.(1)$$

On the system:

$$\left.\begin{aligned} \dot{x} &= a_1(t)x + b_1(t)y + c_1(t)z + d_1(t)u \\ \epsilon\dot{y} &= a_2(t)x + b_2(t)y + c_2(t)z + d_2(t)u \\ \epsilon^2\dot{z} &= a_3(t)x + b_3(t)y + c_3(t)z + d_3(t)u \end{aligned}\right\} \dots\dots\dots..(2)$$

$$\left.\begin{array}{l} x(0) = x_0 \\ y(0) = y_0 \\ z(0) = z_0 \end{array}\right\} \dots\dots\dots\dots\dots\dots\dots (3)$$

Such that $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and W(t),R(t),

$a_i(t,\epsilon), b_i(t,\epsilon), c_i(t,\epsilon), d_i(t,\epsilon), i = \overline{1,3}$ are smooth on the interval [0,T], $\epsilon \geq 0$ and $W(t,\epsilon), R(t,\epsilon)$ are symmetric matrices.

We will use the following matrices:

$$W = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{pmatrix}, D = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

We will work under the following condition:

- The matrices c_3 , $(b_2 - c_2 c_3^{-1} b_3)$ are stable, i.e. the real parts of their eigenvalues are negative.

III. Construction of asymptotic expansion :

The following method is called direct method [3] by using a boundary functions [9].

The solution of problem (1-3) by this method will be written :

$$x(t,\epsilon) = \bar{x}(t,\epsilon) + \pi_{1x}(\tau_1,\epsilon) + \pi_{2x}(\tau_2,\epsilon) + Q_{1x}(\eta_1,\epsilon) + Q_{2x}(\eta_2,\epsilon)$$

$$y(t,\epsilon) = \bar{y}(t,\epsilon) + \pi_{1y}(\tau_1,\epsilon) + \pi_{2y}(\tau_2,\epsilon) + Q_{1y}(\eta_1,\epsilon) + Q_{2y}(\eta_2,\epsilon)$$

$$z(t,\epsilon) = \bar{z}(t,\epsilon) + \pi_{1z}(\tau_1,\epsilon) + \pi_{2z}(\tau_2,\epsilon) + Q_{1z}(\eta_1,\epsilon) + Q_{2z}(\eta_2,\epsilon) \dots\dots\dots (4)$$

$$J_\epsilon(u) = \sum_{i=0}^{\infty} \epsilon^i J_i$$

$\bar{x}(t,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \bar{x}_i(t)$, $\pi_{1x}(\tau_1,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \pi_{1x}^i(\tau_1)$, $\pi_{2x}(\tau_2,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \pi_{2x}^i(\tau_2)$

$Q_{1x}(\tau_1,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i Q_{1x}^i(\tau_1)$, $Q_{2x}(\tau_2,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i Q_{2x}^i(\tau_2)$

$\bar{y}(t,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \bar{y}_i(t)$, $\pi_{1y}(\tau_1,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \pi_{1y}^i(\tau_1)$, $\pi_{2y}(\tau_2,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \pi_{2y}^i(\tau_2)$

$Q_{1y}(\tau_1,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i Q_{1y}^i(\tau_1)$, $Q_{2y}(\tau_2,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i Q_{2y}^i(\tau_2)$

$\bar{z}(t,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \bar{z}_i(t)$, $\pi_{1z}(\tau_1,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \pi_{1z}^i(\tau_1)$, $\pi_{2z}(\tau_2,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i \pi_{2z}^i(\tau_2)$

$Q_{1z}(\tau_1,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i Q_{1z}^i(\tau_1)$, $Q_{2z}(\tau_2,\epsilon) = \sum_{i=0}^{\infty} \epsilon^i Q_{2z}^i(\tau_2)$

$$\tau_1 = \frac{t}{\epsilon}, \tau_2 = \frac{t}{\epsilon^2}, \eta_1 = \frac{t-T}{\epsilon}, \eta_1 = \frac{t-T}{\epsilon^2}$$

Such that:

$\pi_{1x}^i, \pi_{2x}^i, \pi_{1y}^i, \pi_{2y}^i, \pi_{1z}^i, \pi_{2z}^i$ are a boundary functions in a neighborhoods of t=0 and it satisfies the following equality :

$$\left\| \pi_{jg}^i(\tau_j) \right\| \leq c e^{-\omega \tau_i}, \quad j = 1,2, \quad g = x,y,z \dots\dots\dots\dots (5)$$

$Q_{1x}^i, Q_{2x}^i, Q_{1y}^i, Q_{2y}^i, Q_{1z}^i, Q_{2z}^i$ are a boundary functions in a neighborhoods of t=0 and it satisfies the following equality :

$$\left\| Q_{jg}^i(\eta_j) \right\| \leq c e^{-\omega \eta_i}, \quad j = 1,2, g = x,y,z \dots\dots\dots\dots (6)$$

Where c,ω are positive constants.

Now by substituting the expansion (4) in the equations 1-3 and equating coefficients which the same power of ϵ we obtain:

$$\bar{x}_0(0) = x_0 \dots \dots \dots \dots \dots \dots .(7)$$

$$\bar{x}_1(0) + \pi_{1x}^1(0) = 0 \dots \dots \dots \dots \dots .(8)$$

$$\bar{x}_j(0) + \pi_{1x}^j(0) + \pi_{2x}^j(0) = 0\,, \quad j \geq 2 \dots \dots \dots \dots \dots .(9)$$

$$\bar{y}_0(0) + \pi_{1y}^0(0) = y_0 \dots \dots \dots \dots \dots .(10)$$

$$\bar{y}_j(0) + \pi_{1y}^j(0) + \pi_{2y}^j(0) = 0\,, \quad j \geq 1 \dots \dots \dots \dots \dots .(11)$$

$$\bar{z}_0(0) + \pi_{1z}^0(0) + \pi_{2z}^0(0) = z_0 \dots \dots \dots \dots \dots .(12)$$

$$\bar{z}_j(0) + \pi_{1z}^j(0) + \pi_{2z}^j(0) = 0\,, \quad j \geq 1 \dots \dots \dots \dots \dots .(13)$$

By considering (5) we obtain :

$$\pi_{1x}^0(\tau_1) = Q_{1x}^0(\eta_1) = \pi_{2x}^0(\tau_2) = Q_{2x}^0(\eta_2) = \pi_{2x}^1(\tau_2) = 0$$

$$Q_{2x}^1(\eta_2) = \pi_{1y}^0(\tau_1) = Q_{1y}^0(\eta_1) = 0 \dots \dots (14)$$

In the following statements we construction the minimize functional J_0 , at $\epsilon = 0$ and from 1-3 we have :

$$\bar{J}_0(\bar{u}_0) = \frac{1}{2}\int_0^T [< \bar{X}_0(t), W_0(t)\bar{X}_0(t) > + < \bar{u}_0(t), R_0(t)\bar{u}_0(t) >] dt \dots \dots \dots \dots \dots \dots \dots .(15)$$

$$\frac{d\bar{x}_0}{dt} = a_1(t)\bar{x}_0 + b_1(t)\bar{y}_0 + c_1(t)\bar{z}_0 + d_1(t)\bar{u}_0\,, \bar{x}_0(0) = x_0$$

$$0 = a_2(t)\bar{x}_0 + b_2(t)\bar{y}_0 + c_2(t)\bar{z}_0 + d_2(t)\bar{u}_0\,, \bar{y}_0(0) = y_0$$

$$0 = a_3(t)\bar{x}_0 + b_3(t)\bar{y}_0 + c_3(t)\bar{z}_0 + d_3(t)\bar{u}_0\,, \bar{z}_0(0) = z_0 \dots \dots .(16)$$

Now we use the Hamilton method for solving the problem (15-16):

$$\bar{X}_0(t) = \begin{pmatrix} \bar{x}_0(t) \\ \bar{y}_0(t) \\ \bar{z}_0(t) \end{pmatrix}$$

$$\bar{\Psi}_0 = \begin{pmatrix} \bar{p}_{01}(t) \\ \bar{p}_{02}(t) \\ \bar{p}_{03}(t) \end{pmatrix}$$

$$\bar{H}_0(t) = \bar{p}_{01}(t)(a_1(t)\bar{x}_0 + b_1(t)\bar{y}_0 + c_1(t)\bar{z}_0 + d_1(t)\bar{u}_0) + \bar{p}_{02}(t)(a_2(t)\bar{x}_0 + b_2(t)\bar{y}_0 + c_2(t)\bar{z}_0 + d_2(t)\bar{u}_0) + \bar{p}_{03}(t)(a_3(t)\bar{x}_0 + b_3(t)\bar{y}_0 + c_3(t)\bar{z}_0 + d_3(t)\bar{u}_0) - \frac{1}{2} < \bar{X}_0(t), W_0(t)\bar{X}_0(t) > - \frac{1}{2} < \bar{u}_0(t), R_0(t)\bar{u}_0(t) >,$$

By conditions of optimal control we obtain:

$$\frac{\partial \bar{H}_0}{\partial \bar{y}_0} = 0, \frac{\partial \bar{H}_0}{\partial \bar{z}_0} = 0, \frac{\partial \bar{H}_0}{\partial \bar{u}_0} = 0 \dots \dots \dots \dots \dots ..(18)$$

$$\frac{\partial \bar{H}_0}{\partial \bar{x}_0} = -\frac{d\bar{p}_{01}(t)}{dt}$$

$$-\frac{d\bar{p}_{01}(t)}{dt} = a_1\bar{p}_{01}(t) + a_2\bar{p}_{02}(t) + a_3\bar{p}_{03}(t) - w_{11}^0(t)\bar{x}_0(t) - w_{12}^0(t)\bar{y}_0(t) - w_{13}^0(t)\bar{z}_0(t)$$

$$\bar{p}_{01}(T) = 0 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots ..(19)$$

From (18) we have :

$$\bar{p}_{03}(t) = (c_3(t)^{Tr})^{-1}[-\bar{p}_{01}(t)^{Tr}c_1 - \bar{p}_{02}(t)^{Tr}c_2 + w_{13}(t)\bar{x}_0(t) + w_{23}(t)\bar{y}_0(t) + w_{33}(t)\bar{z}_0(t)] \dots \dots \dots \dots \dots \dots ..(20)$$

$\bar{p}_{02}(t)$
$= [(b_2(t)$
$- b_3(t)(c_3(t))^{-1}c_2(t))^{Tr}]^{-1}[\bar{p}_{01}(t)^{Tr}[-b_2(t)$
$+ b_3(t)(c_3(t))^{-1}c_2(t)]$
$- b_3(t)^{Tr}\big(c_3(t)\big)^{-1}\big(w_{13}(t)\bar{x}_0(t) + w_{23}(t)\bar{y}_0(t)$
$+ w_{33}(t)\bar{z}_0(t)\big) + w_{12}(t)\bar{x}_0(t) + w_{22}(t)\bar{y}_0(t)$
$+ w_{32}(t)\bar{z}_0(t)\big]$ ……….(21)

Proposition 3.1(sufficient condition):

The control :

$$\bar{u}_{0*} = (R_0(t))^{-1}[d_1^{Tr}(t)\bar{p}_{01} + d_2^{Tr}(t)\bar{p}_{02} + d_3^{Tr}(t)\bar{p}_{03}] \text{ ………..(22)}$$

is optimal control for problems 15-16.

Proof:

We must proof: $J_0(\bar{u}_{0*}) \le J_0(\bar{u}_0)$, $\forall$ $\bar{u}_0$

i.e. $J_0(\bar{u}_0) - J_0(\bar{u}_{0*}) \ge 0$

We note that:

$$J_0(\bar{u}_0) - J_0(\bar{u}_{0*}) = \frac{1}{2}\int_0^T [< \begin{pmatrix} \bar{x}_0 - \bar{x}_{0*} \\ \bar{y}_0 - \bar{y}_{0*} \\ \bar{z}_0 - \bar{z}_{0*} \end{pmatrix}, W_0(t)\begin{pmatrix} \bar{x}_0 - \bar{x}_{0*} \\ \bar{y}_0 - \bar{y}_{0*} \\ \bar{z}_0 - \bar{z}_{0*} \end{pmatrix} > + < \bar{u}_0 - \bar{u}_{0*}, R_0(t)(\bar{u}_0 - \bar{u}_{0*}) >]dt + \Delta \text{ ……………(23)}$$

Such that:

$$\Delta = \int_0^T [< W_0(t)\begin{pmatrix} \bar{x}_0 - \bar{x}_{0*} \\ \bar{y}_0 - \bar{y}_{0*} \\ \bar{z}_0 - \bar{z}_{0*} \end{pmatrix}, \begin{pmatrix} \bar{x}_{0*} \\ \bar{y}_{0*} \\ \bar{z}_{0*} \end{pmatrix} > + < R_0(t)(\bar{u}_0 - \bar{u}_{0*}), \bar{u}_{0*} >]dt$$

By using equations 16,19,20,21,22 we have:

$$\Delta = \int_0^T [< \begin{pmatrix} \dot{\bar{p}}_{01} + a_1\bar{p}_{01} + a_2\bar{p}_{02} + a_3\bar{p}_{03} \\ b_1\bar{p}_{01} + b_2\bar{p}_{02} + b_3\bar{p}_{03} \\ c_1\bar{p}_{01} + c_2\bar{p}_{02} + c_3\bar{p}_{03} \end{pmatrix}, \begin{pmatrix} \bar{x}_0 - \bar{x}_{0*} \\ \bar{y}_0 - \bar{y}_{0*} \\ \bar{z}_0 - \bar{z}_{0*} \end{pmatrix} > + < d_1^{Tr}(t)\bar{p}_{01} + d_2^{Tr}(t)\bar{p}_{02} + d_3^{Tr}(t)\bar{p}_{03}, (\bar{u}_0 - \bar{u}_{0*}) >]dt$$

$$\Delta = \int_0^T [\begin{pmatrix} \dot{\bar{p}}_{01} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \bar{x}_0 - \bar{x}_{0*} \\ \bar{y}_0 - \bar{y}_{0*} \\ \bar{z}_0 - \bar{z}_{0*} \end{pmatrix} > + < \begin{pmatrix} a_1\bar{p}_{01} + a_2\bar{p}_{02} + a_3\bar{p}_{03} \\ b_1\bar{p}_{01} + b_2\bar{p}_{02} + b_3\bar{p}_{03} \\ c_1\bar{p}_{01} + c_2\bar{p}_{02} + c_3\bar{p}_{03} \end{pmatrix}, \begin{pmatrix} \bar{x}_0 - \bar{x}_{0*} \\ \bar{y}_0 - \bar{y}_{0*} \\ \bar{z}_0 - \bar{z}_{0*} \end{pmatrix} > + < \bar{p}_{01}, d_1(\bar{u}_0 - \bar{u}_{0*}) > + < \bar{p}_{02}, d_2(\bar{u}_0 - \bar{u}_{0*}) > + < \bar{p}_{03}, d_3(\bar{u}_0 - \bar{u}_{0*}) >]dt$$

$$\Delta = \int_0^T [\begin{pmatrix} \dot{\bar{p}}_{01} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \bar{x}_0 - \bar{x}_{0*} \\ \bar{y}_0 - \bar{y}_{0*} \\ \bar{z}_0 - \bar{z}_{0*} \end{pmatrix} > + < \bar{p}_{01}, \dot{\bar{x}}_0 - \dot{\bar{x}}_{0*} > + < \bar{p}_{02}, 0 > + < \bar{p}_{03}, 0 >]dt$$

By (19) we obtain $\Delta = 0$.

Since the first part of (23) is positive for all $t \in [0, T]$, thus : $J_0(\bar{u}_0) - J_0(\bar{u}_{0*}) \ge 0 \rightarrow$
$J_0(\bar{u}_{0*}) \le J_0(\bar{u}_0)$ $\forall$ $control$ $\bar{u}_0$

Therefore $\bar{u}_{0*}$ is optimal control for the problem 15-16.

Proposition 3.2 (necessary condition):

The optimal control for problem 15-16 is :

$$\bar{u}_{0*} = (R_0(t))^{-1}[d_1^{Tr}(t)\bar{p}_{01} + d_2^{Tr}(t)\bar{p}_{02} + d_3^{Tr}(t)\bar{p}_{03}]$$

Proof:

From (16) we have :

$$\bar{y}_0 = \frac{-(c_3b_2 - c_2b_3)^{-1}}{k_1}[\frac{(c_3a_2 - c_2a_3)}{h_1}\bar{x}_0 + \frac{(c_3d_2 - c_2d_3)}{h_2}\bar{u}_0]$$

$$\bar{z}_0 = \frac{-(b_3c_2 - b_2c_3)^{-1}}{k_2}\left[\frac{(b_3a_2 - b_2a_3)}{h_3}\bar{x}_0 + \frac{(b_3d_2 - b_2d_3)}{h_4}\bar{u}_0\right]$$

By substation $\bar{y}_0, \bar{z}_0$ in the equation (15) we have:

$$J_0(\bar{u}_0) = \frac{1}{2}\int_0^T [< \begin{pmatrix} \bar{x}_0 \\ -k_1^{-1}(a_1\bar{x}_0 + a_2\bar{u}_0) \\ -k_2^{-1}(a_3\bar{x}_0 + a_4\bar{u}_0) \end{pmatrix}, W_0(t)\begin{pmatrix} \bar{x}_0 \\ -k_1^{-1}(a_1\bar{x}_0 + a_2\bar{u}_0) \\ -k_2^{-1}(a_3\bar{x}_0 + a_4\bar{u}_0) \end{pmatrix} > + < \bar{u}_0, R_0(t)\bar{u}_0) >]dt +$$

and the system:

$$\dot{\bar{x}}_0 = (a_1 - b_1h_1k_1^{-1} - c_1h_3k_2^{-1})\bar{x}_0 + (d_1 - b_1h_2k_1^{-1} - c_1h_4k_2^{-1})\bar{u}_0 , \bar{x}_0(0) = x_0$$

Such that:

$$k_1 = c_3b_2 - c_2b_3, k_2 = -k_1$$

$$h_1 = c_3a_2 - c_2a_3 , h_2 = c_3d_2 - c_2d_3 , h_3 = b_3a_2 - b_2a_3, h_4 = b_3d_2 - b_2d_3$$

The necessary condition of optimal control follows from [5]

Example 3.3:

Consider the problem:

$$Min\, J(u) = \frac{1}{2}\int_0^1 (x^2 + y^2 + u^2)dt \ldots\ldots (1)$$

$$\dot{x}(t,\varepsilon) = 3x(t,\varepsilon) - y(t,\varepsilon),$$

$$\varepsilon\dot{y}(t,\varepsilon) = 5x(t,\varepsilon) - 3y(t,\varepsilon),$$

$$\varepsilon^2\dot{z}(t,\varepsilon) = 2z(t,\varepsilon) + u(t,\varepsilon), \quad t\epsilon[0,1] \ldots\ldots (2)$$

$$x(0,\varepsilon) = 1 \ \ , y(0,\varepsilon) = 1 \ \ , z(0,\varepsilon) = 1 \ldots\ldots (3)$$

The Hamilton function will be:

$$H = P(3x - y) + \emptyset(5x - 3y) + \psi(2z + u) - \frac{1}{2}x^2 - \frac{1}{2}y^2 - \frac{1}{2}u^2$$

Thus by Maximum principle we have a system:

$$\left.\begin{array}{c} \dot{P} = -3P - 5\emptyset + x, \\ \varepsilon\dot{\emptyset} = P + 3\emptyset + y , \\ \varepsilon^2\dot{\psi} = -2\psi , \\ u = \psi. \end{array}\right\} \ldots\ldots\ldots (4)$$

$$P(1) = 0 \ , \phi(1) = 0 \ , \psi(1) = 0 \ldots\ldots (5)$$

Now we solve the systems (2-3) and (4-5) together, we will have the exact solution:

$$x(t,\varepsilon) = c_1e^{(a+b)t} + c_2e^{(a-b)t}$$

$$y(t,\varepsilon) = c_1(3 - a - b)e^{(a+b)t} + c_2(3 - a + b)e^{(a-b)t}$$

$$z(t,\varepsilon) = \frac{3}{2}e^{\frac{2}{\varepsilon^2}t} - \frac{1}{2}$$

Such that:

$$c_1 = \frac{3 - a + b}{2b} \ , c_2 = 1 - c_1 \ , a = \frac{3\varepsilon - 3}{2\varepsilon} \ ,$$

$$b = \frac{\sqrt{\frac{9}{\varepsilon^2} - \frac{2}{\varepsilon} + 9}}{2}$$

Now , let $\varepsilon = 0$ and we will find the solution in this case:

$$\dot{\bar{x}}_0 = 3\bar{x}_0 - \bar{y}_0, 0 = 5\bar{x}_0 - 3\bar{y}_0, 0 = 2\bar{z}_0 + \bar{u}_0.$$

$$\frac{-4}{3}\bar{P}_0 = \dot{\bar{P}}_0, \bar{y}_0 = \vec{\emptyset}_0 , \bar{u}_0 = 0$$

Thus the solution is:

$$\bar{x}_0 = e^{\frac{4}{3}t} \ , \bar{y}_0 = \frac{5}{3}e^{\frac{4}{3}t} \ , \bar{z}_0 = 0 = \bar{u}_0$$

Finally we find asymptotically solution:

$$\tilde{x}_0(t) = e^{\frac{4}{3}t}, \tilde{y}_0(t) = \frac{5}{3}e^{\frac{4}{3}t} + e^{\frac{-3}{\varepsilon}t}, \tilde{z}_0(t) = \frac{-1}{2}e^{\frac{-2}{\varepsilon}t} + \left(1+\frac{1}{\varepsilon^2}\right)e^{\frac{-2}{\varepsilon^2}t}, \tilde{u}_0(t) = e^{\frac{-2}{\varepsilon}t} + e^{\frac{-2}{\varepsilon^2}t}$$

For compare between the solutions we take $\varepsilon = 0.1, \varepsilon = 0.01, \varepsilon = 0.05$ and composition the following table:

ε	J(u)	$J(\bar{u}_0)$	$J(\tilde{u}_0)$
0.1	10.8594	9.485941	9.5707
0.01	9.6211	9.485941	9.4937
0.05	10.1674	9.485941	9.5263

Table(1.1)

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الحل التناظري لمسائل التحكم الأمثل التربيعية الخطية مع المعلمة الصغيرة التربيعية

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<u>الملخص:</u>

في هذا البحث قمنا ببناء توسيع حل متناظر لمسائل الحل الامثل التربيعي الخطي لمعادلات الاضطراب المنفردة. وتستند هذه الصيغة على التعويض المباشر عن توسع مقارب مفترض لنوع من الدوال المحدودة وايجاد شروط المسألة لايجاد حدود التوسيع. تم إثبات التوسيع عند ($\varepsilon = 0$) للحل المتناظر. في مسائل الرياضيات الفيزياوية نقول إن المسائل التربيعية الخطية المنفردة هي تمثل السرعة "سريعة وبطيئة"، في هذا البحث ناقشنا مفهوم "أبطأ سرعة" من خلال اتخاذ المعلمة الصغيرة التربيعية في مسائل الاضطراب الخطي التربيعي المنفردة.