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Mathematical Model for the Concentration of Pollution and Dissolved Oxygen in the Diwaniya River (Iraq)

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Abstract

A mathematical model provides the ability to predict the contaminant concentration levels of a river. We present a simple mathematical model for river pollution. The model consists of a pair of coupled reaction diffusion-advection equations for the pollutant and dissolved oxygen concentrations, respectively. We consider the steady state case in one spatial dimension. For simplified cases the model is solved analytically by considering the case of zero dispersion , that's mean(D_p=0 and D_x=0). The study represent the first attempt for the researchers to study the problem of pollution, and we think that this mathematical analysis would provide a better planning for the water or river controlling.

Introduction

Water pollution from human activities, either industrial or domestic, is a major problem in many countries. Every year, approximately 25 million persons die as a result of water pollution. When assessing the quality of water in a river , there are many factors to be considered: the level of dissolved oxygen; the presence of nitrates, chlorides, phosphates; the level of suspended solids; environmental hormones; chemical oxygen demand ,such as heavy metals , and the presence of bacteria. Pollutants from agricultural operations can be a significant contributor to the impairment of surface and groundwater quality [1].

Mathematical water quality models date back to the well-known model in 1925 of Streeter and Phelps; where they described the balance of dissolved oxygen in rivers[1].

S.C. Chapra in 1997 [1] stated the standard equations of the water pollution by using advection-diffusion equations for the pollutant and dissolved oxygen concentrations, these equation are stated and used by many researchers dependent on the conditions of their work , then developed this equation by S. A. Socolofsky and et.al [2]. W.Ruch and et.al in 1998 [3] studied mathematically the problems of surface water pollution, such as eutrophication, acute and chronic toxicity.

B. pimpuncha and et.al in 2009 [1] presented a simple mathematical model for river pollution and investigate the effect of aeration on the degradation of pollutant, their model consists of a pair of coupled reaction-diffusion-advection equations for the pollutant and dissolved oxygen concentrations Advection-diffusion equation: is the partial differential equation that governs the motion of a conserved scalar field as it is advected by a known velocity vector field. It is derived using the scalar fields conservation law. In chemistry, engineering and earth sciences, advection is a transport mechanism of a substance or conserved property by a fluid due to the fluids bulk motion. An example of advection is the transport of pollutants or silt in a river by bulk water flow downstream[2].

An observed law stating that the rate at which one substance diffuses through another is directly proportional to the concentration gradient of the diffusing substance[4]. Ficks' law[3]. relates the diffusive flux to the concentration under the assumption of steady state. It postulates that the flux goes from regions of high concentration to regions of low concentration, with a magnitude that is proportional to the concentration gradient (spatial derivative). In one (spatial) dimension [2]:

$$J_x = uC + q_x \tag{1.1}$$

$$uC - D\frac{\partial C}{\partial x} \tag{1.2}$$

Where J_x is the diffusion density, u is the mean flow velocity, D is the diffusion coefficient $C$is the concentration , $q$is the rate of pollutant addition along the river, x is the position.

The standard advection-diffusion-equation may be written as follows:

$$\frac{\partial C}{\partial t} + \frac{\partial uC}{\partial x} = D\frac{\partial^2 C}{\partial^2 x} \tag{1.3}$$

Table 1: **r**efer to variables and parameter values that supplied from the directorate of environment in Diwnaiya

Symbol	Definition	Value
L	is the polluted length of river (m)	15786
D_x	is the dispersion coefficient of dissolved oxygen in the x direction ($m^2 day^{-1}$)	–
D_p	is the dispersion coefficient of pollutant in the x direction ($m^2 day^{-1}$)	–
A	is the cross-section area of the river(m^2)	39.266
α	is the mass transfer of oxygen from air to water($m^2 day^{-1}$)	16.50*
S	is the saturated oxygen concentration (kg m^{-3})	< 5
Z	is the mass transfer of solid (solute) to the water of the river in (m^2day)	76.054
v	is the water velocity in the x- direction (m day)	0.7
k	is the degradation rate coefficient (day^{-1})	8.27*
q	Is the rate of pollutant addition along the river (kg $m^{-1} day^{-1}$)	0.098

1) * references [1].

Mathematical Model

For our model we assumed that the advection–diffusion-equation may be a good approximation to model of river pollution. We assumed that the river (Shat Al-Diwanya In Iraq) has a uniform cross-sectional area, there fore , we are assumed that the river to be linear or one-dimensional. That's mean A one–dimensional river cross-section with arbitrary interior and endpoints at (x=0) and (x=L) .

These equations account expansion of the pollutant and the dissolved oxygen concentrations, respectively. It's given by:

$$\frac{\partial (AP_s(x,t))}{\partial t} = D_p\frac{\partial^2 (AP_s(x,t))}{\partial x^2} - \frac{\partial (\upsilon AP_s(x,t))}{\partial x} + Z\int P_s(x,t)dx \tag{1.4}$$

$$\frac{\partial(AX_s(x,t))}{\partial t}=D_X\frac{\partial^2(AX_s(x,t))}{\partial x^2}-\frac{\partial(\upsilon AX_s(x,t))}{\partial x}+\alpha\int(S-X_s(x,t)) \tag{1.5}$$

The first equation includes the mass transferred of solid (solute) to the water (river) in m^2/day, and it's the concentration of pollution. The second equation is a mass balance for dissolved oxygen, with addition through the surface at a rate proportional to the degree of saturation of dissolved oxygen *(S - X)*, and consumption during the oxidation of the pollution. We can write equations (1.4) and (1.5) as:

$$\frac{\partial(AP_s(x,t))}{\partial t}=D_p\frac{\partial^2(AP_s(x,t))}{\partial x^2}-\frac{\partial(\upsilon AP_s(x,t))}{\partial x}+ZP_s(x,t) \tag{1.6}$$

$$\frac{\partial(AX_s(x,t))}{\partial t}=D_X\frac{\partial^2(AX_s(x,t))}{\partial x^2}-\frac{\partial(\upsilon AX_s(x,t))}{\partial x}+\alpha(S-X_s(x,t)) \tag{1.7}$$

For these the only variation is with the distance downstream on the river and so we write $P_s(x,t)=P_s(x)$ and $X_s(x,t)=X_s(x)$ .To simplify the equations , we will consider only the steady–state solutions.

Analytic Steady-State Solutions for Special Case (Zero Dispersion)

We begin by considering the case when the dispersion can be taken to be $D_P = 0, D_X = 0$ For this case the equations (1.6) and (1.7) becomes as:

$$-\frac{d(\upsilon AP_s(x))}{dx}+ZP_s(x)=0 \tag{1.8}$$

$$-\frac{d(\upsilon AX_s(x))}{dx}+\alpha(S-X_s(x))=0 \tag{1.9}$$

with boundary conditions $P_s(0)=\frac{q}{kA}$, $X_s(0)=S+\frac{q}{kA}$. For this case there is no pollution upstream because of the absence of dispersion. By solving equation (1.8), we get:

$$\frac{d(\upsilon AP_s(x))}{dx}-ZP_s(x)=0 \tag{1.10}$$

$$\upsilon A\frac{d(P_s(x))}{dx}-ZP_s(x)=0$$

$$\frac{d(P_s(x))}{dx}-\frac{Z}{\upsilon A}P_s(x)=0 \tag{1.11}$$

Then, by solving the equation (1.11), we can find the integral operator of this equation as:

$$\frac{d(P_s(x))}{dx}-\frac{Z}{\upsilon A}P_s(x)=0$$

$$\frac{d(P_s(x))}{dx}=\frac{Z}{\upsilon A}P_s(x)$$

$$\frac{d(P_s(x))}{P_s x}=\frac{Z}{\upsilon A}dx$$

$$\log P_s(x)=\int\frac{z}{\upsilon A}dx+C \text{ ..., where } C=\frac{q}{kA}$$

$$=\frac{z}{\upsilon A}x+\frac{q}{kA}$$

$$P_s(x)=e^{\frac{Z}{\upsilon A}x+\frac{q}{kA}}$$

$$=Me^{\frac{Z}{\upsilon A}x} \text{ ... , such that } M=e^{\frac{q}{kA}}$$

$$P_s(x)e^{\int -\frac{z}{\upsilon A}dx} = \int 0 e^{\int -\frac{z}{\upsilon A}dx} dx + C$$

$$P_s(x)e^{-\frac{Z}{\upsilon A}x} = 0 + \frac{q}{kA}$$

We find the pollution concentration downstream of the river *Ps(x)*, such that:

$$P_s(x) = \frac{q}{kA} e^{\frac{Z}{\upsilon A}x} \tag{1.12}$$

Which represent the pollutant concentration downstream of the river, these result tends to that the limit of these equation as:

if $x = 0$, then $P_s(x) = P_s(0) = \frac{q}{kA}$

and if x = ∞, then $P_s(x) = P_s(\infty) = \frac{q}{kA} e^{\infty}$

we believe that results are close to the truth because there is no government programming. For treatment or decreasing the high pollutant in the river.

Now to find the dissolved oxygen concentration by solving equation (1.9), we get:

$$\frac{d(\upsilon A X_s(x))}{dx} - \alpha(S - X_s(x)) = 0 \tag{1.13}$$

$$\upsilon A \frac{d(X_s(x))}{dx} + \alpha X_s(x) - \alpha S = 0$$

$$\upsilon A \frac{d(X_s(x))}{dx} + \alpha X_s(x) = \alpha S$$

$$\frac{d(X_s(x))}{dx} + \frac{\alpha}{\upsilon A} X_s(x) \frac{\alpha S}{\upsilon A} \tag{1.14}$$

Then, by solving the equation (1.14), we can find the integral operator of this equation as:

$$\frac{d(X_s(x))}{dx} + \frac{\alpha}{\upsilon A} X_s(x) = 0$$

$$\frac{d(X_s(x))}{dx} - \frac{\alpha}{\upsilon A} X_s(x)$$

$\log X_s(x) = \int -\frac{\alpha}{\upsilon A} dx + C$ ..., where $C = \frac{q}{kA}$

$$= -\frac{\alpha}{\upsilon A} x + \frac{q}{kA}$$

$$X_s(x) = e^{-\frac{\alpha}{\upsilon A}x + \frac{q}{kA}}$$

$= g e^{-\frac{\alpha}{\upsilon A}x}$ ..., such that $g = e^{\frac{q}{kA}}$

$$X_s(x) e^{\int \frac{\alpha}{\upsilon A}dx} = \int \frac{\alpha S}{\upsilon A} e^{\int \frac{\alpha}{\upsilon A}dx} dx + \frac{q}{kA}$$

$$X_s(x) e^{\frac{\alpha}{\upsilon A}x} = \int \frac{\alpha S}{\upsilon A} e^{\frac{\alpha}{\upsilon A}x} dx + \frac{q}{kA}$$

$$X_s(x)^{\frac{\alpha}{e\upsilon A}x} = S e^{\frac{\alpha}{\upsilon A}x} + \frac{q}{kA}$$

Hence the dissolved oxygen concentration is:

$$X_s(X) = \frac{q}{kA} e^{-\frac{\alpha}{\upsilon A}x} + S \tag{1.15}$$

Which represent the dissolved oxygen concentration downstream of the river, these result tends to that the limit of these equation as:

if x = 0 , then $X_s(0) = \frac{q}{kA} + S$

if x = ∞ , then $X_s(\infty) = \frac{q}{kA} e^{-\infty} + S = S$

Which means that the downstream of river dissolved oxygen being constant and its value consistent with the saturated value of S. these mathematical results consisted with the actual results, and these results may be attributed to the controlling of adding of pollutant that reduces the dissolved oxygen that necessary for survival life

Conclusion

We have presented a mathematical model for the river pollution by considering couple of non linear partial differential equations. In this paper , we studied the simplified case by analytical steady state solution for the zero dispersion. The model can also be used to predict the level of pollution concentration and dissolved oxygen concentration depending on the data that supplied from the environmental directorate of Al-Diwaniya. We found that the concentration of pollution and dissolved oxygen level remain within the critical value of these parameters and approximately consistent with the values that measured for different stations of AL-Diwaniya city, and we believe that there is no government programming. For treatment or decreasing the high pollutant in the river.

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