

Some Properties of Operator Topology Space that associated with Alexandroff Spaces

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Abstract

In this paper, a new concept that utilizing from theory of operator in topological space has introduced. This operator has studied T- Alexandroff space associated to a topology on X. Some properties of this operator has discussed as well.

Indexing terms/Keywords

Alexandroff space, Operator topological space.

Academic Discipline and Sub-Disciplines

Topology

1. INTRODUCTION

Alexandroff spaces are classes of the topological spaces which satisfy the property that each point contained in smallest open set. Equivalently an arbitrary intersection of open sets is open. This subject was first studied by Alexandroff in 1937 [1]. There are many properties of this are proved such as a subspace Y of an Alexandroff space X is an Alexandroff space [4]. This paper will provide new types of Alexandroff spaces. T- Alexandroff space, T*- Alexandroff space and θ - Alexandroff where T is an operator associated with the topology T on X will not only introduce but their properties will also study. A new space that is operator topological space (O. T. S.) have been defined. T is space is named good space.

1.1 Operator Topological Space:

Suppose that (X, T) be a Topological space and let $T : P(X) \longrightarrow P(X)$ be a function such that $W \subseteq T(W)$ for each $W \in T$ then T is called an operator associated with the topology T on X and the triple (X, T, T) is called a n operator topological space. A set $A \subseteq X$ is called T- open if there exist open set W such that $x \in W \subseteq T(W) \subseteq A$. If $B \subseteq X$ which is not necessary open and it satisfies: $B \subseteq T(B)$ then B is called T*-open [3]. So, the following implications are true: T-open $\longrightarrow$ T*- open $\longrightarrow$ T*- closed, where the complement of T*-open is called T*- closed.

1.2 T- Alexandroff and T*- Alexandroff Spaces:

The concept of semi- Alexandroff space have been introduced as follows: A topological space (X, T) is called semi- Alexandroff if any intersection of open sets is semi-open [2].

Definition 1.2.1

An O. T. S. is called T*- Alexandroff if any intersection of open sets is T*-open.

Remark 1.2.2

Let (X, T, T) be an (O. T. S.) where $T : P(X) \longrightarrow P(X)$ is defined as follows : $T(A) = \text{cl}(\text{int } A)$ then T*-open sets are exactly the semi-open sets and T*- Alexandroff space will be semi- Alexandroff space.

Definition 1.2.3

Let (X, T, T) be an (O. T. S.) then X is called T- Alexandroff space if any intersection of T-open sets is open..

Remark 1.2.4

Let (X, T, T) be an (O. T. S.) where $T : P(X) \longrightarrow P(X)$ is defined as follows : $T(A) = \text{cl}(A)$ then T-open sets are exactly the θ -open sets and T- Alexandroff space will be θ - Alexandroff space.

Remark 1.2.5

- i) Alexandroff space is T-Alexandroff .
- ii) Alexandroff space is T^* -Alexandroff .
- iii) The concepts of T- Alexandroff and T^* - Alexandroff are independent.

Definition 1.2.6

Let (X, T, T) , (Y, σ, L) are O. T. S. and let $f : (X, T, T) \longrightarrow (Y, \sigma, L)$ is called $(T, L)^*$ Homeomorphism. This means:

- 1) f is continuous.
- 2) f is one to one and onto.
- 3) f is $(T, L)^*$ -open (that is the image of T^* -open set in X is L^* -open in Y).

Definition 1.2.7

A property of O. T. S. is called $(T, L)^*$ -topological property if it is preserved by $(T, L)^*$ - Homeomorphism.

Theorem 1.2.8

The property " T^* Alexandroff" is a $(T, L)^*$ - topological property.

Proof :

Let $f : (X, T, T) \longrightarrow (Y, \sigma, L)$ be a $(T, L)^*$ Homeomorphism. And suppose (X, T, T) is T^* - Alexandroff, we will prove that (Y, σ, L) is L^* - Alexandroff.

Let $F = \{ W_\alpha : \alpha \in \Omega \}$ be any family of σ -open sets in Y . Now $F^{-1} = \{ f^{-1}(W_\alpha) : \alpha \in \Omega \}$ is family of T-open sets in X . since X is T^* - Alexandroff then $\bigcap_{\alpha \in \Omega} f^{-1}(W_\alpha)$ is T^* -open, hence $f(\bigcap_{\alpha \in \Omega} f^{-1}(W_\alpha))$ is L^* -open.

Now, $f(\bigcap_{\alpha \in \Omega} f^{-1}(W_\alpha)) = \bigcap_{\alpha \in \Omega} f(f^{-1}(W_\alpha)) = \bigcap_{\alpha \in \Omega} W_\alpha$. Hence $\bigcap_{\alpha \in \Omega} W_\alpha$ is L^* - open and Y is L^* - Alexandroff.

The converse of this theorem is not correct as the following example that show that not every T^* - Alexandroff is Alexandroff.

Example 1.2.9

Let X be the real line with the topology $T = \{ X, \emptyset, \{0\} \} \cup \{ G \subseteq X : 0 \in G \text{ and } G^c \text{ is finite} \}$. Define $T : P(X) \longrightarrow P(X)$ as follows : $T(A) = \text{cl}(\text{int } A)$ hence T^* -open sets are semi-open sets and T^* - Alexandroff is semi-Alexandroff.

It is clear that $\{0\}$ is open and dense in (X, T, T) and so (X, T) is semi- Alexandroff (that is (X, T, T) is T^* - Alexandroff) [2]. In the other hand, the intersection of all open sets $\{x\}^c$, where x is irrational is not open, thus (X, T, T) is not Alexandroff.

Remark 1.2.10

Example (1.2.9) shows that a close subsets of T^* -Alexandroff spaces does not need to be T^* - Alexandroff. Consider the close subset is $\{0\}^c$. Now, $\{0\}^c$ is an infinite subset with cofinite topology, hence $\{0\}^c$ is not T^* - Alexandroff .

Definition 1.2.11

Let (X, τ, τ^*) be an O. T. S. it can be say that X is a good space if it has the following property: A singleton $\{x\}$ is T^* -open if and only if it is open.

If $T(A) = \text{cl}(\text{int } A)$ then (X, τ, τ^*) is a good space because: a singleton $\{x\}$ is semi-open if and only if it is open.

Theorem 1.2.12

Let (X, τ, τ^*) be a good space and if X is T_1 and T^* - Alexandroff then X is discrete.

Proof :

Let $x \in X$, let $y \neq x$ then there exist an open set W_y containing x such that $y \notin W_y$ since X is T^* - Alexandroff and since $\{x\} = \bigcap_{y \neq x} W_y$ then $\{x\}$ is T^* -open and hence open. Thus X is discrete.

Remark 1.2.13

If X is discrete, then X will be T_1 and T^* - Alexandroff. Of course, every discrete space is good space.

Theorem 1.2.14

Let $f: (X, \tau) \longrightarrow (Y, \sigma, L)$ be a surjection continuous function suppose also that f is L^* -closed function (that is the image of every closed in X is L^* -closed in Y) and if X is Alexandroff, then Y is L^* - Alexandroff.

Proof :

Let $F = \{W_\alpha: \alpha \in \Omega\}$ be any family of closed sets in Y consider $F^{-1} = \{f^{-1}(W_\alpha): \alpha \in \Omega\}$, so F^{-1} is family of closed sets in X . But X is Alexandroff, hence $\bigcup_{\alpha \in \Omega} f^{-1}(W_\alpha)$ is closed in X . Now $f(\bigcup_{\alpha \in \Omega} f^{-1}(W_\alpha))$ is L^* -closed in Y but $f(\bigcup_{\alpha \in \Omega} f^{-1}(W_\alpha)) = \bigcup_{\alpha \in \Omega} f(f^{-1}(W_\alpha)) = \bigcup_{\alpha \in \Omega} W_\alpha$. Hence is L^* -closed in Y , which means that Y is L^* - Alexandroff.

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