Jordan Right Derivations on Completely Prime Γ- Rings

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Abstract

We define a Jordan right derivation on Γ-ring and show that the existence of a non-Zero Jordan right derivation on a completely prime Γ-ring implies Γ-ring is Commutative.

We show that a Jordan right derivation on completely Γ-rings is a right derivation.

الخلاصة:
قدمنا في هذا البحث تعريف اشتقاق جوردان الايمن على الحلقة من نوع كاما.
وتوصلت الى ان وجود اشتقاق جوردان الايمن غير صفري على الحلقة الاولية التامة من نوع كاما يؤدي الى ان هذه الحلقة تكون ابدالية.
وكذلك توصلنا الى ان اشتقاق جوردان الايمن على الحلقة الاولية التامة من نوع كاما يكون اشتقاق ايمن.

1-Introduction

In[2] Beresar and Vukman proved a Jordan derivation on prime ring is a derivation.In[5] Sapanc and Nakajima defined a derivation and Jordan derivation on Γ-ring and show that a Jordan derivation on acertain of completely prime Γ-ring is derivation

In this paper ,we define a Jordan right derivation on Γ-ring and we show that the existence of a nonzero Jordan right derivation on a2-torsion free Γ-ring which satisfies the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z \in M$ and $\alpha,\beta\in\Gamma$ implies a Γ-ring is commutative . Also, in the same condition, Jordan right derivation on completely prime a Γ-ring is a right derivation on a Γ-ring.

Let M and Γ be additive abelian groups. M is called a Γ-ring if for any $x,y,z\in M$ and $\alpha,\beta\in\Gamma$, the following conditions are satisfied :

(1) $x \alpha y\in M$

(2) $(x+y) \alpha z=x \alpha z+y \alpha z$

$x(\alpha+\beta)z=x \alpha z+x \beta z$

$x(\alpha(y+z))=x \alpha y+x \alpha z$

(3) $(x\ \alpha\ y)\beta\ z=x\ \alpha\ (y\ \beta\ z)$

A Γ-ring M is called prime if $a\ \Gamma\ M\ \Gamma\ b=0$ implies $a=0$ or $b=0$ and M is called completely prime if $a\ \Gamma\ b=0$ implies $a=0$ or $b=0$ $(a,b\in M)$. Since $a\ \Gamma\ b\ \Gamma\ a\ \Gamma\ b \subset a\ \Gamma\ M\Gamma\ b$, every completely prime Γ-ring is prime.

M is called 2-torsion free if $2a=0$ implies $a=0$ for all $a\in M$.

Let M be a Γ-ring and let $D:M\rightarrow M$ be an additive Map D is called a derivation if for any $a,b\in M$ and $\alpha\in\Gamma$, $D(a\ \alpha\ b)=D(a)\ \alpha\ b+a\ \alpha\ D(b)$, D is

D is called a Jordan derivation if for any $a\in M$ and $\alpha\in\Gamma$, $D(a\alpha a)=D(a)\alpha a+a\alpha D(a)$

Example:

Let R is a ring, $M=M_{1\times 2}(R)$ and $\Gamma=\{\begin{bmatrix} n.1 \\ 0 \end{bmatrix}$, $n\in Z\}$ Then M is a Γ-ring. If $d:R\rightarrow R$ is a Jordan right derivation and $N=\{(a\ a), a\in R\}$ is the subset of M then N is a Γ-ring and Map $D:N\rightarrow N$ defined by $D(a,a)=(d(a),d(a))$ is a Jordan right derivation.

2- Right Jordan Derivation

(2-1)Defination:

Let M be a Γ-ring and let $D:M\rightarrow M$ be an additive Map D is called a right derivation if for any $a,b\in M$ and $\alpha\in\Gamma$

$D(a\ \alpha\ b)=D(b)\ \alpha a+D(a)\ \alpha b$.

D is called a Jordan right derivation if for any $a\in M$ and $\alpha\in\Gamma$, $D(a\alpha a)=2D(a)\alpha a$.

(2.2) Lemma :

Let M be an arbit Γ-ring and D is a Jordan right derivation on M. Then for all $a,b\in M$ and for all $\alpha\in\Gamma$:

(i) $D(a\alpha b+b\alpha a)=2D(a)\alpha b+2D(b)\alpha a$. Specially, if M is 2-torsion free and $a\alpha b\beta c=a\beta b\alpha c$ for all $a,b,c\in M$ and $\alpha,\beta\in\Gamma$, then

(ii) $D(a\alpha b\beta a)=D(b)\alpha a\beta a+3D(a)\beta b\alpha a-D(a)\beta a\alpha b$.

(iii) $D(a\alpha b\beta c+c\alpha b\beta a)=D(b)\alpha a\beta c+D(b)\alpha c\beta a+3D(a)\beta b\alpha c+3D(c)\beta b\alpha a-D(a)\beta c\alpha b-D(c)\beta a\alpha b$.

Proof:

(i) obtained by computing

$$D((a+b)\alpha(a+b))=2D(a+b)\alpha(a+b)$$
$$=2D(a)\alpha a+2D(a)\alpha b+2D(b)\alpha a+2D(b)\alpha b$$

On other hand

$D(a+b)\alpha(a+b)=D(a\alpha a)+D(b\alpha b)+D(a\alpha b+b\alpha a)$

Therefore

$D(a\alpha b+b\alpha a)=2D(a)\alpha b+2D(b)\alpha a$

(ii) From (i), $D(a\beta b+b\beta a)=2D(b)\beta a+2D(a)\beta a$

Then replacing b by $(a\alpha b+b\alpha a)$ we have

$D(a\beta(a\beta b+b\beta a)+(a\alpha b+b\alpha a)\beta a=D(a\beta a\alpha b+a\beta b\alpha a+a\alpha b\beta a+b\alpha a\beta a)$

Since $a\alpha b\beta c=a\beta b\alpha c$ for all $a,b,c\in M$ and $\alpha,\beta\in\Gamma$, we get:

$D(a\beta(a\beta b+b\beta a)+(a\alpha b+b\alpha a)\beta a=2D(b)\alpha a\beta a+6D(a)\beta b\alpha a-2D(a)\beta a\alpha b+$

$2D(b)\alpha a\beta a+4D(a)\beta a\alpha b$

Therefore

$D(a\beta a\alpha b+b\alpha a\beta a)+2D(a\alpha b\beta a)=2D(b)\alpha a\beta a+6D(a)\beta b\alpha a-2D(a)\beta a\alpha b+$

$2D(b)\alpha a\beta a+4D(a)\beta a\alpha b$

Hence

$D(a\beta a\alpha b+b\alpha a\beta a)=2D(b)\alpha a\beta a+4D(a)\beta a\alpha b$

and $2D(a\alpha b\beta a)= 2D(b)\alpha a\beta a+6D(a)\beta b\alpha a-2D(a)\beta a\alpha b$

Since M is 2-torsion free we obtain

$D(a\alpha b\beta a)=D(b)\alpha a\beta a+3D(a)\beta b\alpha a-2D(a)\beta a\alpha b$.

(iii) Replacing a by a+c in (ii)

$D(a\alpha b\beta a)=D(b)\alpha a\beta a+3D(a)\beta b\alpha a-D(a)\beta a\alpha b$

$D((a+c)\alpha b\beta(a+c))=D(b)\alpha(a+c)\beta(a+c)+3D(a+c)\beta b\alpha(a+c)-D(a+c)\beta(a+c)\alpha b$

Therefore

$D(c(a+c)\alpha b\beta(a+c))=D(a\alpha b\beta a+c\alpha b\beta c+(a\alpha b\beta c+c\alpha b\beta a))$

$=D(b)\alpha a\beta a+D(b)\alpha a\beta c+D(b)\alpha c\beta a+D(b)\alpha c\beta c+3D(a)\beta b\alpha a+3D(a)\beta b\alpha c+$

$3D(c)\beta b\alpha a+3D(c)ab\alpha c-D(a)\beta a\alpha b-D(a)\beta c\alpha b-D(c)\beta a\alpha b-D(c)\beta c\alpha b$

Therefore

$D(a\alpha b\beta c+c\alpha b\beta a)= D(b)\alpha a\beta c+D(b)\alpha c\beta a+3D(a)\beta b\alpha c+3D(c)\beta b\alpha a-D(a)\beta c\alpha b-$

$D(c)\beta a\alpha b$.

(2.3)Lemma:

Let M is a 2-torsion free Γ-ring, D is a Jordan right derivation on M and $a\alpha b\beta c=a\beta b\alpha c$ for all $a,b,c\in M$ and $\alpha,\beta\in\Gamma$, then:

(i) $D(a)\beta(a\alpha b-b\alpha a)\alpha a=D(a)\alpha a\beta(a\alpha b-b\alpha a)$

(ii) $(D(a\alpha b)-D(b)\alpha a-D(a\alpha b)\beta(a\alpha b-b\alpha a)=0$

(iii) $D(a\alpha b-b\alpha a)\beta(a\alpha b-b\alpha a)=0$

(iv) $D(a\alpha a\beta b)=D(b)\alpha a\beta a+D(a\alpha b-b\beta a)\alpha a+D(a)\alpha(a\beta b+b\beta a)$

Proof:

(i) Replacing c by $a\alpha b$ in Lemma(2.2,iii), we have:

$D(a\alpha b)\beta(a\alpha b-b\alpha a)=D(b)\beta(a\alpha b-b\alpha a)\alpha a+D(a)a(a\alpha b-b\alpha a)$ …………(1)

Then, replacing b by $a+b$ in (1) and using (1),we get:

$D(a)\beta(aqb-b\alpha a)\alpha a+D(b)\beta(a\alpha b-b\alpha a)\alpha a+D(a)\beta(a\alpha b-b\alpha a)\alpha a+D(a)\beta(a\alpha b-b\alpha a)\alpha b$

$=2D(a)qa\beta(a\alpha b-b\alpha a)+D(b)\beta(a\alpha b-b\alpha a)\alpha a+D(a)\beta(aqb-b\alpha a)\alpha b$

Since M is a 2-torsion free, we get :

$D(a)\beta(a\alpha b-b\alpha a)\alpha a=D(a)\alpha a\beta(a\alpha b-b\alpha a)$.

(ii) Replacing a by $a+b$ in (i):

$D(a)\beta(a\alpha b-b\alpha a)\alpha a+D(a)\beta(a\alpha b-b\alpha a)\alpha b+D(b)\beta(aqb-b\alpha a)qa+D(b)\beta(a\alpha b-b\alpha a)\alpha b$

$=D(a)\alpha a\beta(a\alpha b-b\alpha a)+D(a)\alpha b\beta(a\alpha b-b\alpha a)+D(b)\alpha a\beta(a\alpha b-b\alpha a)+$

$D(b)\alpha b\beta(a\alpha b-b\alpha a)$.

Using Equation(1), we obtain:

$(D(a\alpha b)-D(b)qa-D(a)\alpha b)\beta(a\alpha b-b\alpha a)=0$ ………….(2)

(iii) Using Lemma (2.2,i) and Lemma (2.3,ii), we have:

$(D(b\alpha a)-D(b)\alpha a-D(a)\alpha b)\beta(a\alpha b-b\alpha a)=0$ …………..(3)

Taking (2) minus (3), we get:

$D(a\alpha b-b\alpha a)\beta(a\alpha b-b\alpha a)=0$

(iv) Replacing b by $b\beta a$ in Lemma(2.2,i), we obtain:

$D(a\alpha b\beta a+b\beta a\alpha a)=2D(a)qb\beta a+2D(b\beta a)\alpha a$ ……….....(4)

and replacing b by $a\beta b$ in Lemma(2.2,i), we obtain:

$D(a\alpha a\beta b+a\beta b\alpha a)=2D(a)\alpha a\beta b+2D(a\beta b)\alpha a$ ………….(5)

Taking (5) minus (4), we have:

$D(a\alpha a\beta b-b\beta a\alpha a)=2D(a\beta b-b\beta a)\alpha a+2D(a)(a\beta b-b\beta a)$ ....(6)

Replacing a by $a\beta a$ in Lemma(2.2,i), we have:

$D(a\alpha a\beta b-b\beta a\alpha a)=2D(b)\alpha a\beta a+4D(a)\beta a\alpha b$ ………….(7)

Taking (6) minus (7) and using m is 2-torsion free, we have:

$D(a\alpha a\beta b)=D(b)\alpha a\beta q+D(a\beta b-b\beta a)\alpha a+D(a)\alpha(a\beta b+b\beta a)$.

(2.4Theorem:

Let M is a completely prime and 2-torsion free Γ-ring. If there exists a non-Zero Jordan right derivation $D:M\rightarrow M$ and $a\alpha b\beta c=a\beta b\alpha c$ for all $a,b,c\in M$ and $\alpha,\beta\in\Gamma$, then M is commutative.

Proof:

From Lemma (2.3,iii).

$D(a\alpha b-b\alpha a)\beta(a\alpha b-b\alpha a)=0$ for all $\beta\in\Gamma$. Then for all $a,b\in M$ and $\alpha\in\Gamma$,

$a\alpha b-b\alpha a=0$ or $D(a\alpha b-b\alpha a)=0$. Since m is completely prime Γ-ring. If $a\alpha b-b\alpha a=0$, then M is commutative.

If $D(a\alpha b-b\alpha a)=0$, then

$2D(a\alpha b)=D(a\alpha b)+D(b\alpha a)$.

Replacing b by $a\beta b$, we obtain

$2D(a\alpha a\beta b)=2D(b)\alpha a\beta a+4D(a)\beta b\alpha a$.

Using Lemma (2.3,iv) and M is 2-torsion free, we have:

$D(a)\beta(a\alpha b-b\alpha a)=0$ for all $a,b\in M$ and $\alpha,\beta\in\Gamma$. Then, since $D\neq 0$, M is commutative.

(2.5)Corollary:

If M is completely prime ring Γ-ring with 2-torsion free and if $a\alpha b\beta c=a\beta b\alpha c$ for all $a,b,c\in M$ and $\alpha,\beta\in\Gamma$, then a Jordan right derivation on M is a right derivation on M.

Proof:

Since M is commutative and 2-torsion free, using Lemma (2.2,i), the proof is immediately shown.

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