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(Ordinal)

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U1

U2

U1 U2

(Joint Distribution)

(Symmetry)

(Marginal Homogeneity)

(Conditional

(Diagonals Parameters

(Symmetry Model)

Symmetry model)

Symmetry)

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Reyer E. B.

$$\chi^2 = N \left(\frac{F_a}{F_r F_c} - 1 \right)$$

Fieuberg , S. E.

Amara, L. A., D. B. Koch, G.G. Gilling

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Symmetry Model

$$M_{ij} = m_{ji} \quad \text{for all } i \neq j \quad \dots (1)$$

$$i = 1, 2, \dots, r \quad , \quad j = 1, 2, \dots, r$$

(i, j)

M_{ij}

.Goodman(1979)

$$M_{ij} = P_{ij} \quad , \quad P_{ij} = P_{ji} \quad \dots (2)$$

P_{ij}

$$\text{Log } M_{ij} = U + U_{1(i)} + U_{2(j)} + U_{12(ji)} \quad \dots (3)$$

$$i = 1, 2, \dots, r \quad , \quad j = 1, 2, \dots, r$$

$: U_{2(j)}$

$: U_{1(i)}$

$: U$

$$\sum_{i,j} U_{12(ij)} = \sum_i U_{1(i)} = 0$$

$$U_{12(ij)} = U_{12(ji)}$$

Conditional Symmetry Model
(Bishop, 1975)

$$F_{ijk} = \begin{cases} M_{ij} & k=1 \\ M_{ij} & k=2 \end{cases}, \quad \begin{cases} i=1,2,\dots,r \\ j=1,2,\dots,r \end{cases} \quad \dots \quad (4)$$

(Three Dimensions)

$$U_{3(k)}, k=1,2 \\ i < j$$

$$k=2, i > j$$

$$F_{ijk} \\ (r-1) \times (r-1) \times 2 \\ k=1$$

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$$\sum_{i < j} M_{ij} = \sum_{i < j} X_{ij} \quad \dots \quad (5)$$

$$\sum_{i > j} M_{ij} = \sum_{i > j} X_{ij} =$$

$$\text{Log } M_{ij} = U + U_{1(i)} + U_{2(j)} + U_{3(k)} + U_{12(i,j)} \quad \dots \quad (6)$$

$$i=1,2,\dots,r, \quad j=1,2,\dots,r, \quad k=1,2$$

Diagonal Parameters Symmetry Model
(Goodman)

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$$M_{ij} = P_{ij} \delta_k \quad i \neq j \quad k = i - j \quad \dots \quad (7)$$

$$k = i - j$$

$$(i, j)$$

$$\delta_k$$

$$P_{ij} = P_{ji}$$

$$\delta_k = 1$$

$$k = \pm 1, \pm 2, \dots, \pm r - 1$$

$$M_{ij} = P_{ij} \delta_k \quad i \neq j \quad k = 1, 2, \dots, r - 1 \quad \dots \quad (8)$$

Goodman

$$2 \times (r - k)$$

TK

$$i - j = k$$

$$i - j = -k$$

$$\text{Log } M_{ij} = U + U_{1(i)} + U_{2(j)} \quad \dots \quad (9)$$

$$i=1,2, \quad j=1,2,\dots,r-k$$

$$\delta_k = 1$$

$$\delta_k = \delta$$

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$$X_{ij}$$

$$M_{ij}$$

$$\text{Log } L(X_{ij}, M_{ij}) = \sum_{i,j} X_{ij} - \log M_{ij} - \sum_{i,j} M_{ij} - \sum_{i,j} (X_{ij})! \quad \dots \quad (10)$$

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$$\sum_{i,j} \hat{M}_{ij} = N$$

$$\sum_{i,j} X_{ij} \log M_{ij} = Nu + \sum_{i,j} (X_{i.} + X_{.j}) U_1(i) + \sum_{i,j} \frac{X_{ij} + X_{ji}}{2} U_{12}(i,j) \quad \dots (11)$$

$$\hat{M}_{ij} = \begin{cases} \frac{X_{ij} + X_{ji}}{2} & i \neq j \\ X_{ij} & i = j \end{cases} \quad \dots (12)$$

$$[\sum_{i,j} X_{ij} \log M_{ij} = un + \sum_i (X'_i u_1(i) + \sum_j X'_{.j} \cdot u_2(j) + \sum_j X'_{.k} \cdot u_3(k) + \sum_{i,j} X_{ij} U_{12}(i,j)] \quad \dots (11)$$

$$\hat{M}_{ij} = \frac{X'_{ij} \cdot X'_{..k}}{n} \quad \dots (14)$$

$$n = \sum_{i \neq j} X_{ij} \quad , \quad X_{ijk} \quad . \quad X_{ijk}$$

$$\sum_{i,j} X_{ij} \log M_{ij} = n^k U + \sum_i X_i^k + \sum_j X_{.j}^k$$

$$\hat{M}_{ij} = \frac{X_i^k \cdot X_{.j}^k}{n^k} \quad \dots (15)$$

$$\begin{matrix} X_{.j}^k & TK & (i) & X_i^k \\ . TK & & n & TK & (j) & X_i^k \\ & & & & \delta k (k=1,2,\dots,r-2) & \\ & & & & i-j = -k & i-j = k \end{matrix}$$

$$\hat{\delta k} = \frac{X_{1.}^k}{X_{2.}^k}$$

$$\hat{\delta k} = \frac{X_{r1}^k}{X_{r2}^k}$$

$$X_{2.}^k \quad TK \quad () \quad X_{1.}^k$$

$$y_2, y_1 \quad .TK \quad ()$$

$$\begin{matrix} . \delta k < 1 & y_2 & y_1 \\ . \delta k > 1 & y_2 & y_1 \end{matrix}$$

$\delta k = 1$

y_2

y_1

(Bishop, 1975)

Pearson's χ^2

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$$\chi^2 = \sum_i \frac{(X_i - M_i)^2}{M_i}$$

Likelihood ratio statistic G^2

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$$G^2 = -2 \sum_i X_i \log_e \frac{M_i}{X_i}$$

G^2

G^2

G^2

\hat{M}_i

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R/ L	1	2	3	4	5	Total
1	39	36	11	6	9	101
2	25	30	18	13	18	104
3	14	17	17	12	19	79
4	5	9	9	13	21	57
5	9	15	18	21	96	159
Total	92	107	73	65	163	500

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R/L	1	2	3	4	5	Total
1	39	30.5	12.5	5.5	9	96.5
2		30	17.5	11	16.5	105.5
3			17	10.5	18.5	76
4				13	21	61
5					96	161
						500

$G^2 = 1.8627$ $\chi^2 = 3.9421$

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:() McNamara

$$\chi^2 = \frac{(b-c)^2}{b+c}, \quad b = \sum_{i>j} X_{ij}, \quad c = \sum_{i<j} X_{ij}$$

$\chi^2 = 5.14$

0.01

χ^2

(Three Dimension)

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R/L	1	2	3	4	Total
2	25				25
3	14	17			31
4	5	9	9		23
5	9	15	18	21	63
Total	53	41	27	21	142

L/R	1	2	3	4	Total
2	36				36
3	11	18			29
4	6	13	12		31
5	9	18	19	21	67
Total	62	49	31	21	163

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R/L	1	2	3	4	Total
2	28.4				28.4
3	11.6	16.3			27.9
4	5.1	10.2	9.7		25
5	8.4	15.4	17.2	9.8	50.8
Total	53.5	41.9	26.9	9.8	132.1

L/R	1	2	3	4	Total
2	32.6				32.6
3	13.4	18.7			32.1
4	5.9	11.8	11.2		28.9
5	9.6	17.6	18.7	11.2	57.1
Total	61.5	48.1	29.9	11.2	150.7

() $G^2 = (10.25)$ () $\chi^2 = (3.37)$

$$. \delta = 142/163 = .8711$$

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I_1, I_2, I_3, I_4, I_5

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K					Total
+1	25	17	9	21	72
-1	36	18	12	21	87
+2	14	9	18		41
-2	11	13	19		43
+3	5	15			20
-3	6	18			24
+4	9				9
-4	9				9
					305

$k = 1, 2, 3, 4$ δk

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$$\hat{\delta}_1 = 72/87 = 0.8275 < 1$$

$$\hat{\delta}_2 = 41/43 = 0.9534 < 1$$

$$\hat{\delta}_3 = 20/24 = 0.8333 < 1$$

$$\hat{\delta}_4 = 9/9 = 1$$

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$k = 1, 2, 3, 4$

TK

G^2

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R/L	1	2	3	4	5	Total
1	39	33.8	12.8	6	9	100.6
2	27.36	30	19.15	11.3	18	105.81

3	12.2	15.84	17	11.5	18.8	75.44
4	5	10.7	9.5	13	22.9	61.09
5	9	15	18.05	19.1	96	157.06
Total	92.56	105.34	76.5	60.8	164.8	500

K	χ^2	G^2
1	1.9406	0.7583
2	0	0
	1.9406	0.7583

() χ^2 () $G^2 = 0.7583$ $\chi^2 = 1.9406$
 $\hat{\gamma} < 1$ $i \neq j$ $M_{ij} > M_{ji}$

$$1 = \hat{\gamma}_2, \hat{\gamma}_2 > 1$$

$$\sum_{i=1}^z M_i \cdot \sum_{j=1}^z M \cdot j \quad z = 1, 2, 3, 4$$

$$z = 1, 2, 3, 4$$

() $(r-1)(c-1)$ () ()
 $\delta_k = \delta$

k	1	2	3	4	Total
+	72	41	20	9	142
-	87	43	24	9	163
Total	159	84	44	18	305

δ

$$\delta = 142/163 = 0.8711$$

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R/L	1	2	3	4	5	Total
1	22	21	15	10	14	82
2	14	8	15	14	6	57
3	28	9	21	31	15	104
4	9	13	40	29	25	116
5	5	6	12	34	84	141
Total	87	57	103	118	144	500

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R/L	1	2	3	4	5	Total
1	22	17.5	21.5	9.5	9.5	80
2		8	12	13.5	6	57
3			21	35.5	13.5	103.5
4				29	29.5	117
5					84	142.5
						500

$G^2 = 6.211$

$\chi^2 = 14.03$

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:() McNomar

$$\chi^2 = \frac{(b-c)^2}{b+c}, \quad b = \sum_{i>j} X_{ij}, \quad c = \sum_{i<j} X_{ij}$$

$$\chi^2 = 5.14$$

0.01

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(Three Dimension)

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L/R	1	2	3	4	Total
2	21				21
3	15	15			30
4	10	14	31		55
5	14	6	15	25	60
Total	60	35	46	25	166

R/L	1	2	3	4	Total
2	14				14
3	28	9			37
4	9	13	40		62
5	5	6	12	34	57
Total	56	28	52	34	170

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L/R	1	2	3	4	Total
2	17.7				17.7
3	21.8	12.1			33.9
4	9.6	13.7	9.7		59.3
5	9.6	6.1	17.2	9.8	59.1
Total	58.7	31.9	26.9	9.8	170

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R/L	1	2	3	4	Total
2	17.5				17.5
3	21.3	12.2			33.5
4	9.5	13	35.5		58
5	9.5	6	13.5	30	59
Total	58	31	49	30	168

() $G^2 = (10.25)$ () $\chi^2 = (3.37)$

$\delta = 168/170 = 0.9882$

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.()
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I1, I2, I3, I4, I5

K					Total
+1	14	9	40	34	97
-1	21	15	31	25	92
+2	28	13	12		53
-2	15	14	15		44
+3	9	34			43
-3	10	6			16
+4	5				5
-4	14				14
					364

$k = 1,2,3,4$ δk ()

$$\delta_1 = 97/92 = 1.0543 > 1$$

$$\delta_2 = 53/44 = 1.2045 > 1$$

$$\delta_3 = 43/16 = 2.6875 > 1$$

$$\delta_4 = 5/14 = 0.3571 < 1$$

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$k = 1, 2, 3, 4$

TK

G^2

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R/L	1	2	3	4	5	Total
1	22	18.1	12.3	36.4	30.3	119.1
2	17	8	23.5	14.8	14.8	78.1
3	11.7	19.5	21	5.2	10.8	68.2
4	34.7	12.2	13.8	29	1.3	91
5	28.7	12.2	29.1	10.3	84	164.3
Total	102.4	70	99.7	95.7	141.2	500

K	χ^2	G^2
1	78.405	14.471
2	0	0
	78.405	14.471

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χ^2

$G^2 = 14.471$ $\chi^2 = 78.405$

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$$\hat{\gamma}_1 < 1$$

$i \neq j$

$$M_{ij} > M_{ij}$$

$$1 = \hat{\gamma}_2, \hat{\gamma}_2 > 1$$

$$\sum_{i=1}^z M_i \cdot \sum_{j=1}^z M \cdot j$$

$z = 1, 2, 3, 4$

() () () -

() δ_k $(r-1)(c-1)$ $()$ $\delta_k = \delta$

k	1	2	3	4	Total
+	97	53	43	5	198
-	92	44	16	14	166
Total	189	97	56	19	364

δ

$\delta = 198/166 = 1.1927$

-:

()

δ

$\delta = 142/163 = 0.8711$

$\delta = 198/166 = 1.1927$

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$M_i > M_j$
 $k = 1, 2, 3, 4$ $\delta k < 1$

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